Name _____ Date _____

3.6 Solving Exponential and Logarithmic Equations

For use with Exploration 3.6

Essential Question How can you solve exponential and logarithmic equations?

1 EXPLORATION: Solving Exponential and Logarithmic Equations

Work with a partner. Match each equation with the graph of its related system of equations. Explain your reasoning. Then use the graph to solve the equation.

a. $e^x = 2$

b. $\ln x = -1$

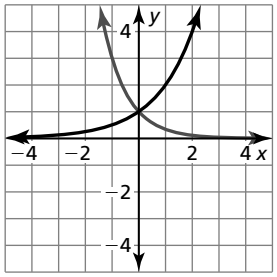
c. $2^x = 3^{-x}$

d. $\log_4 x = 1$

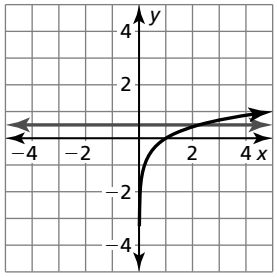
e. $\log_5 x = \frac{1}{2}$

f. $4^x = 2$

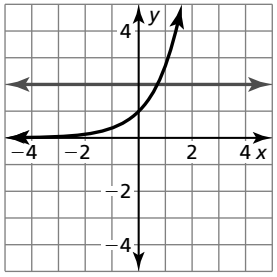
A.



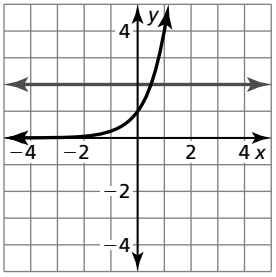
B.



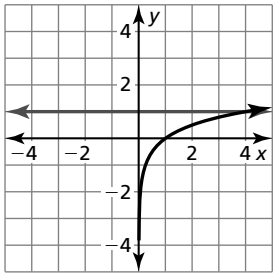
C.



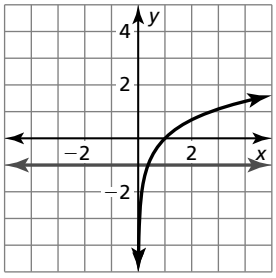
D.



E.



F.



Name_____ Date _____

2 EXPLORATION: Solving Exponential and Logarithmic Equations

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Look back at the equations in Explorations 1(a) and 1(b). Suppose you want a more accurate way to solve the equations than using a graphical approach.

a. Show how you could use a *numerical approach* by creating a table. For instance, you might use a spreadsheet to solve the equations.

b. Show how you could use an *analytical approach*. For instance, you might try solving the equations by using the inverse properties of exponents and logarithms.

Communicate Your Answer

3. How can you solve exponential and logarithmic equations?

4. Solve each equation using any method. Explain your choice of method.

a. $16^x = 2$

b. $2^x = 4^{2x+1}$

c. $2^x = 3^{x+1}$

d. $\log x = \frac{1}{2}$

e. $\ln x = 2$

f. $\log_3 x = \frac{3}{2}$

Name _____ Date _____

3.6 Practice

For use after Lesson 3.6

Core Concepts

Property of Equality for Exponential Equations

Algebra If b is a positive real number other than 1, then $b^x = b^y$ if and only if $x = y$.

Example If $3^x = 3^5$, then $x = 5$. If $x = 5$, then $3^x = 3^5$.

Notes:

Property of Equality for Logarithmic Equations

Algebra If $b, x,$ and y are positive real numbers with $b \neq 1$, then $\log_b x = \log_b y$ if and only if $x = y$.

Example If $\log_2 x = \log_2 7$, then $x = 7$. If $x = 7$, then $\log_2 x = \log_2 7$.

Notes:

Worked-Out Examples

Example #1

Solve the equation.

$$\begin{aligned} 3e^{4x} + 9 &= 15 \\ 3e^{4x} &= 6 \\ e^{4x} &= 2 \\ \ln e^{4x} &= \ln 2 \\ 4x &= \ln 2 \\ x &= \frac{1}{4}\ln 2 \\ x &\approx 0.173 \end{aligned}$$

Name_____ Date _____

3.6 Practice (continued)

Example #2

Solve the equation.

$\log_2(3x - 4) = \log_2 5$

$3x - 4 = 5$

$3x = 9$

$x = 3$

Check

$\log_2(3 \cdot 3 - 4) \stackrel{?}{=} \log_2 5$

$\log_2 5 \stackrel{?}{=} \log_2 5$ ✓

Practice A

In Exercises 1–6, solve the equation.

1. $2^{2x+4} = 2^{5x-8}$

2. $4^{2x-1} = 8^{x+2}$

3. $3^{x+3} = 5$

4. $\left(\frac{1}{5}\right)^{3x-2} = \sqrt{25^x}$

5. $12e^{1-x} = 500$

6. $-14 + 3e^x = 11$

In Exercises 7–11, solve the equation. Check for extraneous solutions.

7. $2 = \log_3(4x)$

8. $\ln(x^2 + 3) = \ln(4)$

9. $\log_8(x^2 - 5) = \frac{2}{3}$

10. $\ln x + \ln(x + 2) = \ln(x + 6)$

11. $\log_2(x + 5) - \log_2(x - 2) = 3$

Name _____ Date _____

3.6 Practice (continued)

12. Solve the inequality $\log x \leq \frac{1}{2}$.

13. Your parents buy juice for your graduation party and leave it in their hot car. When they take the cans out of the car and move them to the basement, the temperature of the juice is 80°F. The room temperature of the basement is 60°F, and the cooling rate of the juice is $r = 0.0147$. Using Newton's Law of Cooling, how long will it take to cool the juice to 63°F?

14. Earthquake intensity is measured by the formula $R = \log\left(\frac{I}{I_0}\right)$ where R is the Richter scale rating of an earthquake, I is the intensity of the earthquake, and I_0 is the intensity of the smallest detectable wave. In 1906, an earthquake in San Francisco had an estimated measure of 7.8 on the Richter scale. In the same year, another earthquake had an intensity level four times stronger than the San Francisco earthquake giving it a Richter scale rating of $R_2 = \log\left(\frac{4I}{I_0}\right)$. What was the Richter scale rating on a scale of 1–10 of the other earthquake?

Name _____ Date _____

Practice B

In Exercises 1–6, solve the equation.

1. $9^{3x-5} = 81^{3x+2}$

2. $7^{x} = 32$

3. $9^{3x+6} = \left(\frac{1}{3}\right)^{8-x}$

4. $6^{4x} = 13$

5. $2e^{3x} + 6 = 10$

6. $4e^{2x} - 7 = 1$

7. Fifty grams of radium are stored in a container. The amount R (in grams) of radium present after t years can be modeled by $R = 50e^{-0.00043t}$.

a. After how many years will only 20 grams of radium be present?

b. Seventy-five grams of radium are stored in a different container. The amount R (in grams) of radium present after t years can be modeled by $R = 75e^{-0.00043t}$. Will it take *more years* or *fewer years* for only 20 grams of the radium in this container to be present, compared to the answer in part (a)? Explain.

In Exercises 8–13, solve the equation.

8. $\ln(5x - 2) = \ln(x + 6)$

9. $\log(3x + 5) = \log 6$

10. $\log_2(3x + 12) = 4$

11. $\log_3(3x + 7) = \log_3(10x)$

12. $\log_2(x^2 - 2x + 1) = 4$

13. $\log_3(x^2 + x + 7) = 3$

In Exercises 14–17, solve the equation. Check for extraneous solutions.

14. $\ln x + \ln(x - 2) = 5$

15. $\log_5 2x^2 + \log_5 8 = 2$

16. $\log_3(-x) + \log_3(x + 8) = 2$

17. $\log_2(x + 2) + \log_2(x + 5) = 4$

In Exercises 18–20, solve the inequality.

18. $e^{x-2} < 8$

19. $\ln x > 5$

20. $-2 \log_3 x + 2 \leq 10$

21. You deposit \$2000 in Account A, which pays 2.25% annual interest compounded monthly. You deposit another \$2000 in Account B, which pays 3% annual interest compounded monthly. When is the sum of the balance in both accounts at least \$5000?