Name ____ Date __________

3.7 Comparing Linear, Exponential, and Quadratic Functions

For use with Exploration 3.7

Essential Question How can you compare the growth rates of linear, exponential, and quadratic functions?

1 EXPLORATION: Comparing Speeds

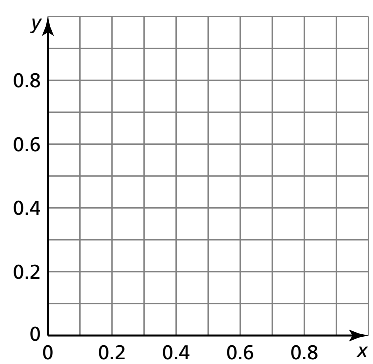
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Three cars start traveling at the same time. The distance traveled in t minutes is y miles. Complete each table and sketch all three graphs in the same coordinate plane. Compare the speeds of the three cars. Which car has a constant speed? Which car is accelerating the most? Explain your reasoning.

t	$y = t$
0	
0.2	
0.4	
0.6	
0.8	
1.0	

t	$y = 2^t - 1$
0	
0.2	
0.4	
0.6	
0.8	
1.0	

t	$y = t^2$
0	
0.2	
0.4	
0.6	
0.8	
1.0	



2 EXPLORATION: Comparing Speeds

Work with a partner. Analyze the speeds of the three cars over the given time periods. The distance traveled in t minutes is y miles. Which car eventually overtakes the others?

t	$y = t$
1.0	
1.5	
2.0	
2.5	
3.0	
3.5	
4.0	
4.5	
5.0	

t	$y = 2^t - 1$
1.0	
1.5	
2.0	
2.5	
3.0	
3.5	
4.0	
4.5	
5.0	

t	$y = t^2$
1.0	
1.5	
2.0	
2.5	
3.0	
3.5	
4.0	
4.5	
5.0	

Communicate Your Answer

3. How can you compare the growth rates of linear, exponential, and quadratic functions?

4. Which function has a growth rate that is eventually much greater than the growth rates of the other two functions? Explain your reasoning.

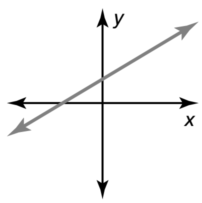
Name ____ Date __________

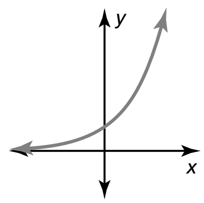
3.7 Practice
For use after Lesson 3.7

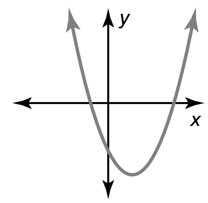
Core Concepts

Linear, Exponential, and Quadratic Functions

Linear Function	Exponential Function	Quadratic Function
$y = mx + b$	$y = ab^x$	$y = ax^2 + bx + c$







Notes:

Differences and Ratios of Functions

You can use patterns between consecutive data pairs to determine which type of function models the data. The differences of consecutive y-values are called *first differences.* The differences of consecutive first differences are called *second differences.*

- **Linear Function** The first differences are constant.
- **Exponential Function** Consecutive y-values have a common *ratio*.
- **Quadratic Function** The second differences are constant.

In all cases, the differences of consecutive x-values need to be constant.

Notes:

Name____ Date__________

3.7 Practice (continued)

Comparing Functions Using Average Rates of Change

- As a and b increase, the average rate of change between $x = a$ and $x = b$ of an increasing exponential function $y = f(x)$ will eventually exceed the average rate of change between $x = a$ and $x = b$ of an increasing quadratic function $y = g(x)$ or an increasing linear function $y = h(x)$. So, as x increases, $f(x)$ will eventually exceed $g(x)$ or $h(x)$.
- As a and b increase, the average rate of change between $x = a$ and $x = b$ of an increasing quadratic function $y = g(x)$ will eventually exceed the average rate of change between $x = a$ and $x = b$ of an increasing linear function $y = h(x)$. So, as x increases, $g(x)$ will eventually exceed $h(x)$.

Notes:

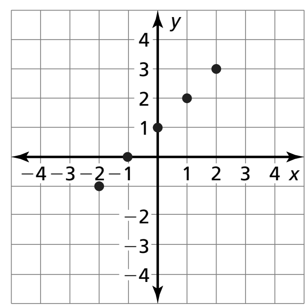
Worked-Out Examples

Example #1

Plot the points. Tell whether the points appear to represent a linear, an exponential, or a quadratic function.

(–2, –1), (–1, 0), (1, 2), (2, 3), (0, 1)

The points appear to lie on a straight line. So, they appear to represent a linear function.



Example #2

Tell whether the data represent a linear, an exponential, or a quadratic function. Then write the function.

(–3, 8), (–2, 4), (–1, 2), (0, 1), (1, 0.5)

Consecutive y-values have a common ratio of $\frac{1}{2}$. So, the table represents an exponential function with $b = \frac{1}{2}$. When $x = 0$, $y = 1$. So, $a = 1$.

$y = ab^x$

$y = 1\left(\frac{1}{2}\right)^x$

$y = \left(\frac{1}{2}\right)^x$

So, the exponential function is $y = \left(\frac{1}{2}\right)^x$.

+1 +1 +1 +1

x	-3	-2	-1	0	1
y	8	4	2	1	0.5

$\times\frac{1}{2}$ $\times\frac{1}{2}$ $\times\frac{1}{2}$ $\times\frac{1}{2}$

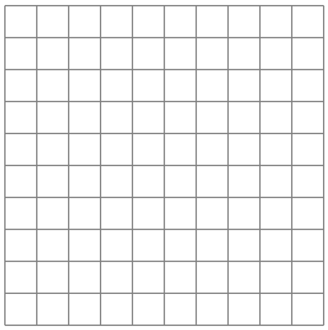
Name ____ Date __________

3.7 **Practice** **(continued)**

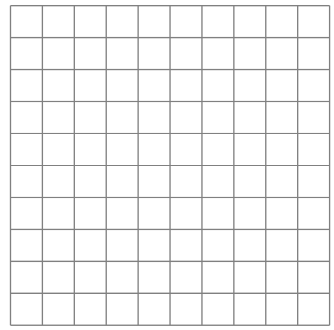
Extra Practice

In Exercises 1–4, plot the points. Tell whether the points appear to represent a *linear*, an *exponential*, or a *quadratic* function.

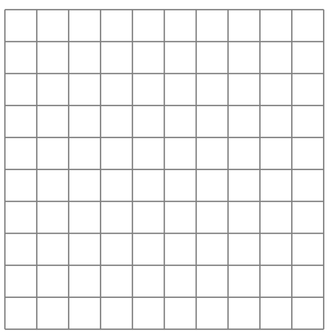
1. $(-3, 2), (-2, 4), (-4, 4), (-1, 8), (-5, 8)$



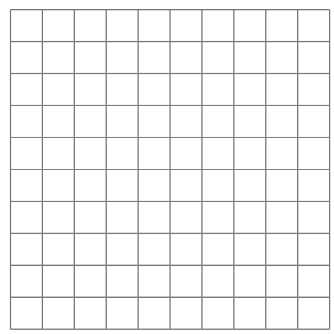
2. $(-3, 1), (-2, 2), (-1, 4), (0, 8), (2, 14)$



3. $(4, 0), (2, 1), (0, 3), (-1, 6), (-2, 10)$



4. $(2, -4), (0, -2), (-2, 0), (-4, 2), (-6, 4)$



In Exercises 5 and 6, tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function.

5.

x	−2	−1	0	1	2
y	7	4	1	−2	−5

6.

x	−2	−1	0	1	2
y	6	2	0	2	6

In Exercises 7 and 8, tell whether the data represent a *linear*, an *exponential*, or a *quadratic* function. Then write the function.

7. $(-2, -4), (-1, -1), (0, 2), (1, 5), (2, 8)$

8. $(-2, -9), (-1, 0), (0, 3), (1, 0), (2, -9)$

Name____ Date__________

3.7 Practice (continued)

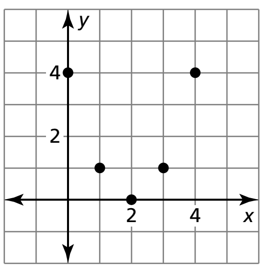
9. A ball is dropped from a height of 305 feet. The table shows the height h (in feet) of the ball t seconds after being dropped. Let the time t represent the independent variable. Tell whether the data can be modeled by a *linear*, an *exponential*, or a *quadratic* function. Explain.

Time, *t*	0	1	2	3	4
Height, *h*	305	289	241	161	49

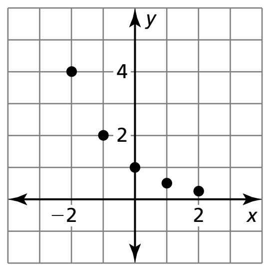
Practice B

In Exercises 1 and 2, tell whether the points appear to represent a *linear*, an *exponential*, or a *quadratic* function.

1.



2.



In Exercises 3–6, plot the points. Tell whether the points appear to represent a *linear*, an *exponential*, or a *quadratic* function.

3. $\left(2, \frac{1}{9}\right), \left(1, \frac{1}{3}\right), (0, 1), (-1, 3), (-2, 9)$

4. $(-1, 3), (0, 0), (1, -1), (2, 0), (3, 3)$

5. $(-4, -2), (-2, -1), (0, 0), (2, 1), (4, 2)$

6. $(-3, -2), (-2, -1), (-1, 0), (0, 1), (1, 2)$

In Exercises 7–10, tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function.

7.

x	−3	−2	−1	0	1	2
y	0.9	0.4	0.1	0	0.1	0.4

8.

x	1	2	3	4	5	6
y	1	−1	−3	−5	−7	−9

9.

x	1	2	3	4	5	6
y	9	4	1	0	1	4

10.

x	−1	0	1	2	3
y	6	3	$\frac{3}{2}$	$\frac{3}{4}$	$\frac{3}{8}$

11. Write a function that has constant second differences of 4.