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4.2 Multiplying and Dividing Rational Expressions

For use with Exploration 4.2

Essential Question How can you determine the excluded values in a product or quotient of two rational expressions?

1 EXPLORATION: Multiplying and Dividing Rational Expressions

Work with a partner. Find the product or quotient of the two rational expressions. Then match the product or quotient with its excluded values. Explain your reasoning.

Product or Quotient	Excluded Values
a. $\frac{1}{x-1} \cdot \frac{x-2}{x+1} =$	**A.** $-1, 0,$ and 2
b. $\frac{1}{x-1} \cdot \frac{-1}{x-1} =$	**B.** -2 and 1
c. $\frac{1}{x-2} \cdot \frac{x-2}{x+1} =$	**C.** $-2, 0,$ and 1
d. $\frac{x+2}{x-1} \cdot \frac{-x}{x+2} =$	**D.** -1 and 2
e. $\frac{x}{x+2} \div \frac{x+1}{x+2} =$	**E.** $-1, 0,$ and 1
f. $\frac{x}{x-2} \div \frac{x+1}{x} =$	**F.** -1 and 1
g. $\frac{x}{x+2} \div \frac{x}{x-1} =$	**G.** -2 and -1
h. $\frac{x+2}{x} \div \frac{x+1}{x-1} =$	**H.** 1

2 EXPLORATION: Writing a Product or Quotient

Work with a partner. Write a product or quotient of rational expressions that has the given excluded values. Justify your answer.

a. -1

b. -1 and 3

c. -1, 0, and 3

Communicate Your Answer

3. How can you determine the excluded values in a product or quotient of two rational expressions?

4. Is it possible for the product or quotient of two rational expressions to have *no* excluded values? Explain your reasoning. If it is possible, give an example.

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4.2 Practice
For use after Lesson 4.2

Core Concepts

Simplifying Rational Expressions

Let a, b, and c be expressions with $b \neq 0$ and $c \neq 0$.

Property $\frac{a\cancel{c}}{b\cancel{c}} = \frac{a}{b}$ Divide out common factor c.

Examples $\frac{15}{65} = \frac{3 \bullet \cancel{5}}{13 \bullet \cancel{5}} = \frac{3}{13}$ Divide out common factor 5.

$\frac{4\cancel{(x+3)}}{(x+3)\cancel{(x+3)}} = \frac{4}{x+3}$ Divide out common factor $x + 3$.

Notes:

Multiplying Rational Expressions

Let a, b, c, and d be expressions with $b \neq 0$ and $d \neq 0$.

Property $\frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd}$ Simplify $\frac{ac}{bd}$ if possible.

Example

$$\frac{5x^2}{2xy^2} \bullet \frac{6xy^3}{10y} = \frac{30x^3y^3}{20xy^3} = \frac{\cancel{10} \bullet 3 \bullet \cancel{x} \bullet x^2 \bullet \cancel{y^3}}{\cancel{10} \bullet 2 \bullet \cancel{x} \bullet \cancel{y^3}} = \frac{3x^2}{2}, x \neq 0, y \neq 0$$

Notes:

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4.2 Practice (continued)

Dividing Rational Expressions

Let a, b, c, and d be expressions with $b \neq 0, c \neq 0,$ and $d \neq 0$.

Property $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c} = \frac{ad}{bc}$ Simplify $\frac{ad}{bc}$ if possible.

Example $\frac{7}{x+1} \div \frac{x+2}{2x-3} = \frac{7}{x+1} \bullet \frac{2x-3}{x+2} = \frac{7(2x=3)}{(x+1)(x+2)}, x \neq \frac{3}{2}$

Notes:

Worked-Out Examples

Example #1

Simplify the expression, if possible.

$$\frac{x^2 - 3x - 18}{x^2 - 7x + 6} = \frac{(x-6)(x+3)}{(x-6)(x-1)}$$
$$= \frac{x+3}{x-1}, x \neq 6$$

Example #2

Find the product.

$$\frac{x^2 + 3x - 4}{x^2 + 4x + 4} \bullet \frac{2x^2 + 4x}{x^2 - 4x + 3} = \frac{(x+4)(x-1)}{(x+2)(x+2)} \bullet \frac{2x(x+2)}{(x-1)(x-3)}$$
$$= \frac{2x(x+4)(x-1)(x+2)}{(x+2)(x+2)(x-1)(x-3)}$$
$$= \frac{2x(x+4)}{(x+2)(x-3)}, x \neq 1$$

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4.2 Practice (continued)

Practice A

In Exercises 1–4, simplify the expression, if possible.

1. $\frac{2x^3 - 8x^2}{6x^2}$

2. $\frac{5xy^3 - 2x^2y^2}{x^2y^2}$

3. $\frac{x^2 - 5x + 4}{x^2 - 2x + 1}$

4. $\frac{x^3 + 3x^2}{x^2 - 5x - 24}$

In Exercises 5–10, find the product or the quotient.

5. $\frac{3xy}{xy^2} \bullet \frac{y}{2x}$

6. $\frac{x + y}{7xy} \div \frac{4x}{y}$

7. $\frac{x(x + 1)}{x - 2} \div \frac{(x + 1)(x - 6)}{(x - 6)(x - 9)}$

8. $\frac{x^2 - 2x - 3}{x^2 - 1} \bullet \frac{x^2 - 2x - 63}{x^2 + 4x - 21}$

9. $\frac{x^2 - 2x}{x + 7} \bullet \frac{x^3 + 8}{x^3 - 4x}$

10. $\frac{x^2 + 2x - 15}{x^2 - 3x - 40} \div \frac{x^2 + 8x - 9}{x^2 + x - 72}$

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Practice B

In Exercises 1–6, simplify the expression, if possible.

1. $\dfrac{4x^3}{3x^3 + 7x}$

2. $\dfrac{x^2 + 5x + 6}{x^2 + 2x - 3}$

3. $\dfrac{2x^2 - 5x}{x^2 + 7x + 12}$

4. $\dfrac{x^2 - x - 20}{x^3 + 64}$

5. $\dfrac{x^4 - 16}{5x^3 - 3x^2 + 20x - 12}$

6. $\dfrac{6x^3 - 6x^2 + 5x - 5}{72x^4 - 50}$

In Exercises 7–12, find the product.

7. $\dfrac{x^4(x - 4)}{x + 3} \bullet \dfrac{(x + 3)(x - 2)}{x^5}$

8. $\dfrac{x^2 + 6x}{x - 4} \bullet \dfrac{x^2 - 2x - 8}{x}$

9. $\dfrac{x^2 - 2x}{x + 5} \bullet \dfrac{x^2 + 6x + 5}{3x}$

10. $\dfrac{x^2 - x - 6}{x^2 + 8x + 16} \bullet \dfrac{3x^2 + 12x}{x^2 - 2x - 3}$

11. $\dfrac{x^2 + 3x - 28}{x^2 - 25} \bullet (x^2 - 8x + 15)$

12. $\dfrac{x^2 + 2x - 15}{x^2 - 9} \bullet (x^2 - x - 12)$

In Exercises 13–16, find the quotient.

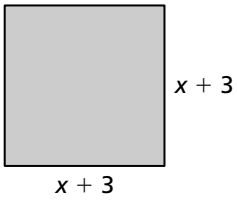
13. $\dfrac{2x^3 + 10x^2}{x^2 + x - 20} \div \dfrac{2x^2}{x - 4}$

14. $\dfrac{x^2 - 10x + 21}{x + 2} \div (x^2 - 14x + 49)$

15. $\dfrac{x^2 - 2x - 3}{x^2 + 2x - 8} \div \dfrac{x^2 + 4x + 3}{x^2 + 6x + 8}$

16. $\dfrac{x^2 + x - 6}{x^2 + 7x + 12} \div \dfrac{x^2 - 5x + 6}{x^2 + x - 12}$

17. Find the ratio of the perimeter to the area of the square shown.



18. Find the expression that makes the following statement true. Assume $x \neq -5$ and $x \neq -3$.

$$\frac{x + 3}{x^2 - 8x + 12} \div \frac{\square}{x^2 + 3x - 10} = \frac{x + 5}{x - 6}$$