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4.3 Adding and Subtracting Rational Expressions

For use with Exploration 4.3

Essential Question How can you determine the domain of the sum or difference of two rational expressions?

1 EXPLORATION: Adding and Subtracting Rational Expressions

Work with a partner. Find the sum or difference of the two rational expressions. Then match the sum or difference with its domain. Explain your reasoning.

Sum or Difference	Domain
a. $\frac{1}{x-1}+\frac{3}{x-1}=$	**A.** all real numbers except -2
b. $\frac{1}{x-1}+\frac{1}{x}=$	**B.** all real numbers except -1 and 1
c. $\frac{1}{x-2}+\frac{1}{2-x}=$	**C.** all real numbers except 1
d. $\frac{1}{x-1}+\frac{-1}{x+1}=$	**D.** all real numbers except 0
e. $\frac{x}{x+2}-\frac{x+1}{2+x}=$	**E.** all real numbers except -2 and 1
f. $\frac{x}{x-2}-\frac{x+1}{x}=$	**F.** all real numbers except 0 and 1
g. $\frac{x}{x+2}-\frac{x}{x-1}=$	**G.** all real numbers except 2
h. $\frac{x+2}{x}-\frac{x+1}{x}=$	**H.** all real numbers except 0 and 2

2 EXPLORATION: Writing a Sum or Difference

Work with a partner. Write a sum or difference of rational expressions that has the given domain. Justify your answer.

a. all real numbers except -1

b. all real numbers except -1 and 3

c. all real numbers except -1, 0, and 3

Communicate Your Answer

3. How can you determine the domain of the sum or difference of two rational expressions?

4. Your friend found a sum as follows. Describe and correct the error(s).

$$\frac{x}{x+4} + \frac{3}{x-4} = \frac{x+3}{2x}$$

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4.3 Practice
For use after Lesson 4.3

Core Concepts

Adding or Subtracting with Like Denominators

Let a, b, and c be expressions with $c \neq 0$.

Addition

$$\frac{a}{c} + \frac{b}{c} = \frac{a + b}{c}$$

Subtraction

$$\frac{a}{c} - \frac{b}{c} = \frac{a - b}{c}$$

Notes:

Adding or Subtracting with Unlike Denominators

Let a, b, c, and d be expressions with $c \neq 0$ and $d \neq 0$.

Addition

$$\frac{a}{c} + \frac{b}{d} = \frac{ad}{cd} + \frac{bc}{cd} = \frac{ad + bc}{cd}$$

Subtraction

$$\frac{a}{c} - \frac{b}{d} = \frac{ad}{cd} - \frac{bc}{cd} = \frac{ad - bc}{cd}$$

Notes:

Simplifying Complex Fractions

Method 1 If necessary, simplify the numerator and denominator by writing each as a single fraction. Then divide by multiplying the numerator by the reciprocal of the denominator.

Method 2 Multiply the numerator and the denominator by the LCD of *every* fraction in the numerator and denominator. Then simplify.

Notes:

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4.3 Practice (continued)

Worked-Out Examples

Example #1

Find the sum or difference.

$$\frac{4x^2}{2x-1} - \frac{1}{2x-1} = \frac{4x^2-1}{2x-1} = \frac{(2x-1)(2x+1)}{2x-1}$$
$$= 2x+1, x \neq \frac{1}{2}$$

Example #2

Find the sum or difference.

$$\frac{12}{x^2+5x-24} + \frac{3}{x-3} = \frac{12}{(x+8)(x-3)} + \frac{3}{x-3}$$
$$= \frac{12}{(x+8)(x-3)} + \frac{3(x+8)}{(x+8)(x-3)}$$
$$= \frac{12+3(x+8)}{(x+8)(x-3)}$$
$$= \frac{3(x+12)}{(x+8)(x-3)}$$

Practice A

In Exercises 1–4, find the sum or difference.

1. $\frac{1}{x-1} - \frac{5}{x-1}$

2. $\frac{4x}{3x-5} + \frac{x}{3x-5}$

3. $\frac{6x}{x+4} + \frac{24}{x+4}$

4. $\frac{2x^2}{x-7} - \frac{14x}{x-7}$

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4.3 Practice (continued)

In Exercises 5–7, find the least common multiple of the expressions.

5. $9x^3, 3x^2 - 21x$

6. $x + 5, 2x^2 + 11x + 5$

7. $x^2 + 5x + 6, x^2 - 3x - 18$

In Exercises 8–11, find the sum or the difference.

8. $\frac{3}{2x} + \frac{11}{5x}$

9. $\frac{15}{x - 2} + \frac{3}{x + 8}$

10. $\frac{3x}{2x + 1} + \frac{10}{2x^2 - 5x - 3}$

11. $\frac{x}{x - 7} - \frac{2}{x + 1} - \frac{8x}{x^2 - 6x - 7}$

In Exercises 12 and 13, simplify the complex fraction.

12. $\dfrac{\frac{x}{10} - 3}{5 + \frac{1}{x}}$

13. $\dfrac{\frac{12}{x^2 - 7x - 44}}{\frac{2}{x - 11} + \frac{1}{x + 4}}$

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Practice B

In Exercises 1–3, find the sum or difference.

1. $\frac{x}{25x^2} - \frac{5}{25x^2}$

2. $\frac{2x^2}{x+6} + \frac{8x}{x+6}$

3. $\frac{3x}{x-4} - \frac{12}{x-4}$

In Exercises 4–7, find the least common multiple of the expressions.

4. $36x^2,\ 9x^2 - 18x$

5. $x^2 - 100,\ x - 10$

6. $25x^2 - 4,\ 3x^2 - 10x - 8$

7. $x^2 + 7x - 18,\ x + 9$

8. Describe and correct the error in finding and simplifying the sum.

$$\frac{4}{7x} + \frac{5}{x^3} = \frac{4(x^3)}{7x(x^3)} + \frac{5(7x)}{x^3(7x)} = \frac{4x^3 + 35x}{7x^4}$$

In Exercises 9–12, find the sum or difference.

9. $\frac{7}{x-5} + \frac{4x}{x+1}$

10. $\frac{7}{x^2 - 5x - 24} + \frac{3}{x-8}$

11. $\frac{x^2 - 3}{x^2 - 6x - 16} - \frac{x+5}{x+2}$

12. $\frac{x-2}{x-3} + \frac{3}{x} + \frac{6x}{2x+1}$

In Exercises 13 and 14, tell whether the statement is *always*, *sometimes*, or *never* true. Explain.

13. The LCD of two rational functions is one of the denominators when the other denominator is a factor.

14. The LCD of two rational functions will have a degree equal to that of the denominator with the higher degree.

In Exercises 15–18, rewrite the function in the form $g(x) = \frac{a}{x-h} + k$. Graph the function. Describe the graph of g as a transformation of the graph of $f(x) = \frac{a}{x}$.

15. $g(x) = \frac{5x+3}{x+4}$

16. $g(x) = \frac{9x}{x+12}$

17. $g(x) = \frac{5x-4}{x}$

18. $g(x) = \frac{8x+13}{x-6}$