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4.4 Solving Rational Equations

For use with Exploration 4.4

Essential Question How can you solve a rational equation?

1 EXPLORATION: Solving Rational Equations

Work with a partner. Match each equation with the graph of its related system of equations. Explain your reasoning. Then use the graph to solve the equation.

a. $\frac{2}{x-1} = 1$

b. $\frac{2}{x-2} = 2$

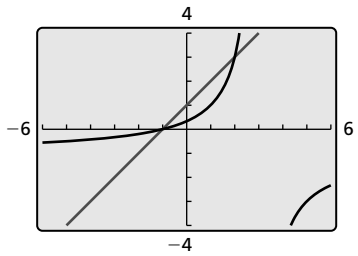
c. $\frac{-x-1}{x-3} = x+1$

d. $\frac{2}{x-1} = x$

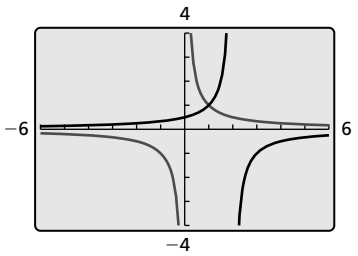
e. $\frac{1}{x} = \frac{-1}{x-2}$

f. $\frac{1}{x} = x^2$

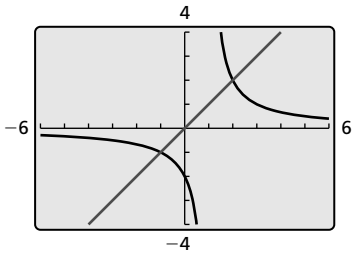
A.



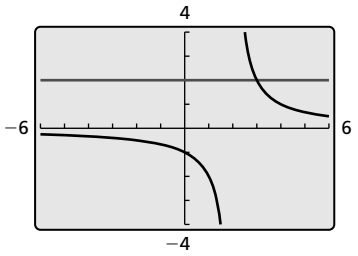
B.



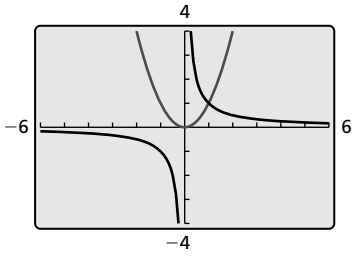
C.



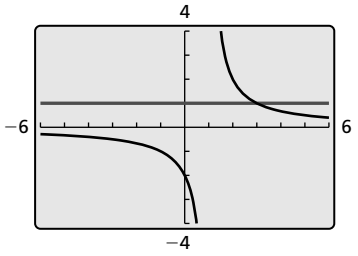
D.



E.



F.



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2 EXPLORATION: Solving Rational Equations

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Look back at the equations in Explorations 1(d) and 1(e). Suppose you want a more accurate way to solve the equations than using a graphical approach.

a. Show how you could use a *numerical approach* by creating a table. For instance, you might use a spreadsheet to solve the equations.

b. Show how you could use an *analytical approach*. For instance, you might use the method you used to solve proportions.

Communicate Your Answer

3. How can you solve a rational equation?

4. Use the method in either Exploration 1 or 2 to solve each equation.

a. $\frac{x+1}{x-1} = \frac{x-1}{x+1}$ **b.** $\frac{1}{x+1} = \frac{1}{x^2+1}$ **c.** $\frac{1}{x^2-1} = \frac{1}{x-1}$

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4.4 Practice

For use after Lesson 4.4

Notes:

Worked-Out Examples

Example #1

Solve the equation by cross multiplying. Check your solution(s).

$$\frac{x^2 - 3}{x + 2} = \frac{x - 3}{2}$$

$$2(x^2 - 3) = (x + 2)(x - 3)$$

$$2x^2 - 6 = x^2 - x - 6$$

$$x^2 + x = 0$$

$$x(x + 1) = 0$$

$$x = 0 \text{ or } x = -1$$

Check

$$\frac{0^2 - 3}{0 + 2} \stackrel{?}{=} \frac{0 - 3}{2}$$

$$\frac{-3}{2} = \frac{-3}{2} \checkmark$$

$$\frac{(-1)^2 - 3}{-1 + 2} \stackrel{?}{=} \frac{-1 - 3}{2}$$

$$\frac{-2}{1} \stackrel{?}{=} \frac{-4}{2}$$

$$-2 = -2 \checkmark$$

The solutions are $x = -1$ and $x = 0$.

Example #2

Identify the LCD of the rational expressions in the equation.

$$\frac{2}{x + 1} + \frac{x}{x + 4} = \frac{1}{2}$$

The LCD is $2(x + 1)(x + 4)$.

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4.4 Practice (continued)

Practice A

In Exercises 1–4, solve the equation by cross multiplying. Check your solution(s).

1. $\frac{2}{x+8} = \frac{5}{2x-7}$

2. $\frac{x}{x+1} = \frac{-4}{x}$

3. $\frac{x+1}{x-3} = \frac{x+2}{x-6}$

4. $\frac{-2}{x-3} = \frac{x+9}{x+21}$

In Exercises 5–12, solve the equation by using the LCD. Check your solution(s).

5. $\frac{4}{7} - \frac{1}{x} = 6$

6. $\frac{3}{x+1} + \frac{4}{x+2} = \frac{15}{x+2}$

7. $\frac{12}{x+4} - \frac{7}{x} = \frac{22}{x^2+4x}$

8. $3 - \frac{18}{x-1} = -\frac{12}{x}$

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4.4 Practice (continued)

9. $\frac{2}{x-5} + \frac{3}{x} = \frac{10}{x^2 - 5x}$

10. $\frac{x+6}{x-4} - \frac{30}{x^2 - 5x + 4} = \frac{3}{x-1}$

11. $\frac{x}{x-5} + \frac{2}{x+2} = \frac{11}{x^2 - 3x - 10}$

12. $\frac{x-2}{x-4} - \frac{2}{x-1} = \frac{12}{x^2 - 5x + 4}$

In Exercises 13 and 14, determine whether the inverse of *f* is a function. Then find the inverse.

13. $f(x) = \frac{8}{x-3}$

14. $f(x) = \frac{12}{x} + 9$

15. You can complete the yard work at your friend's home in 5 hours. Working together, you and your friend can complete the yard work in 3 hours. How long would it take your friend to complete the yard work when working alone?

Let t be the time (in hours) your friend would take to complete the yard work when working alone.

	Work Rate	Time	Work Done
You	$\frac{1 \text{ yard}}{5 \text{ hours}}$	3 hours	
Friend		3 hours	

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Practice B

In Exercises 1–3, solve the equation by cross multiplying. Check your solution(s).

1. $\frac{3}{x+2} = \frac{5}{x-2}$

2. $\frac{2}{x-4} = \frac{x-3}{x-1}$

3. $\frac{x-5}{4} = \frac{x^2-5}{x+4}$

4. So far in soccer practice, you have made 10 out of 32 goal attempts. Solve the equation $0.45 = \frac{10+x}{32+x}$ to find the number x of consecutive goals you need to make to raise your goal average to 0.45.

In Exercises 5 and 6, identify the LCD of the rational expressions in the equation.

5. $\frac{6}{x+3} + \frac{x}{x+2} = \frac{4}{5}$

6. $\frac{6}{x-8} - \frac{2x}{3x-2} = \frac{9}{4}$

In Exercises 7–12, solve the equation by using the LCD. Check your solution(s).

7. $\frac{3}{4x} + \frac{1}{8} = \frac{7}{4x}$

8. $\frac{5}{x-6} + \frac{1}{x} = \frac{x-1}{x-6}$

9. $\frac{x-4}{x-5} + 5 = \frac{4x}{x}$

10. $\frac{16}{x^2-4x} - \frac{8}{x-4} = \frac{4}{x}$

11. $\frac{x+1}{x+2} + \frac{1}{x} = \frac{2x+1}{x+2}$

12. $\frac{4}{x} - 1 = \frac{4}{x+2}$

13. Describe and correct the error in the first step of solving the equation.

$$\frac{3}{x+2} + 5 = \frac{1}{x}$$

$$(x+2) \bullet \frac{3}{x+2} + (x+2) \bullet 5 = (x+2) \bullet \frac{1}{x}$$

14. You can kayak around a certain island in 3 hours. Kayaking together, you and your friend can kayak around the island in 1.4 hours. Let t be the time (in hours) your friend would take to kayak around the island when kayaking alone. Write and solve an equation to find how long your friend would take to kayak around the island when kayaking alone.

$\left(\textit{Hint: } \left(\text{Work done}\right) = \left(\text{Work rate}\right) \times \left(\text{Time}\right)\right)$

In Exercises 15 and 16, determine whether the inverse of *f* is a function. Then find the inverse.

15. $f(x) = \frac{3}{2x-7}$

16. $f(x) = \frac{1}{x^3} + 9$