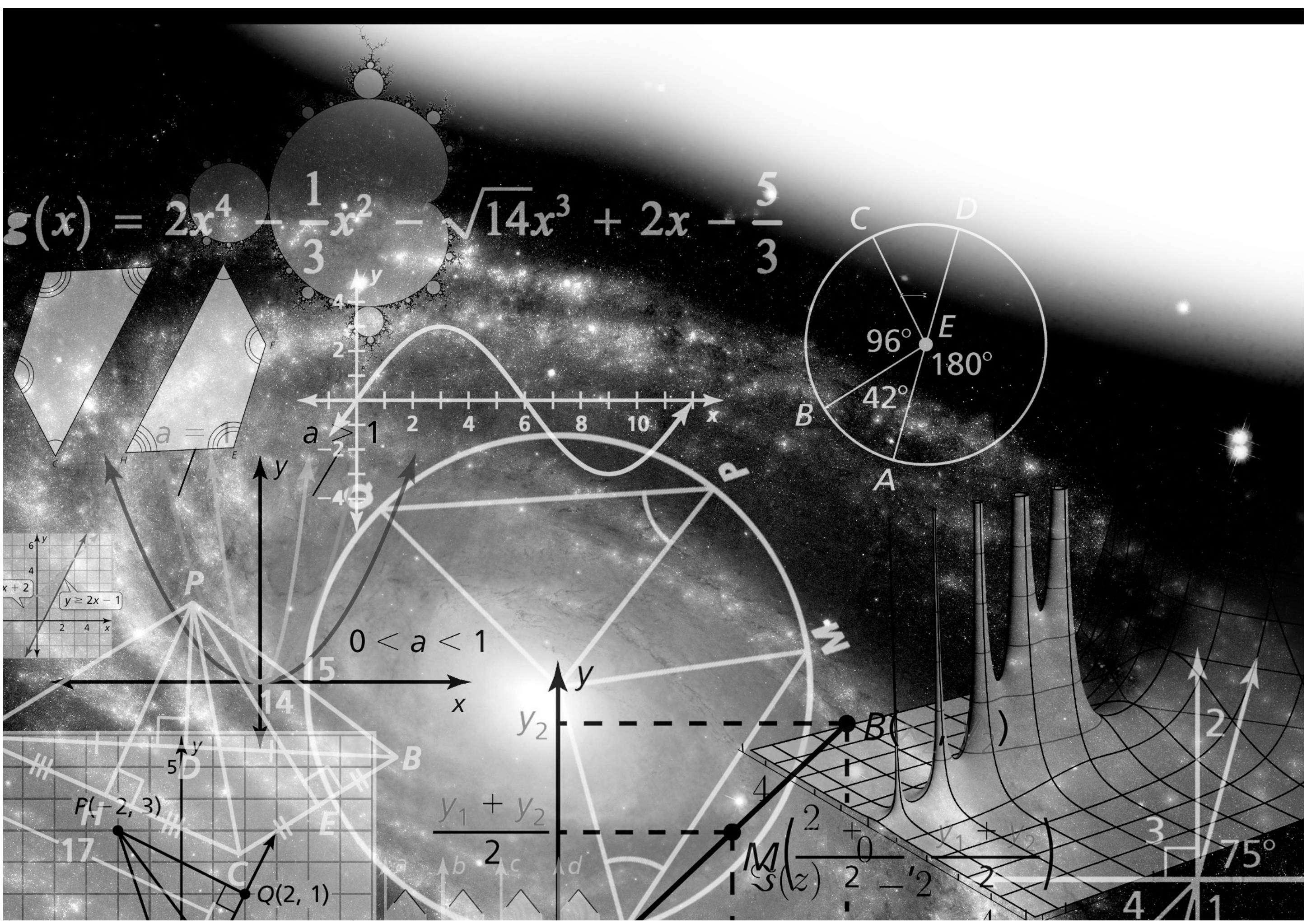
CHAPTER 5

Trigonometric Ratios and Functions



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Chapter 5 Maintaining Mathematical Proficiency

Graph the function.

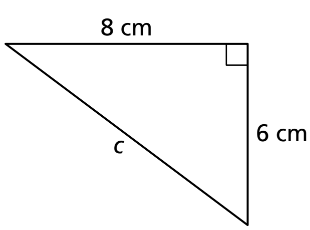
1. $f(x) = (x + 3)(x - 2)^2$

2. $f(x) = \frac{1}{2}(x + 1)(x - 4)^2$

3. $f(x) = \frac{1}{4}(x + 2)(x - 1)(x - 3)$

Find the missing side length of the triangle.

4.



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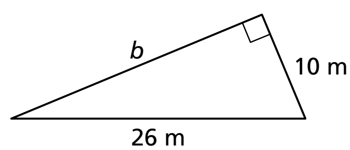

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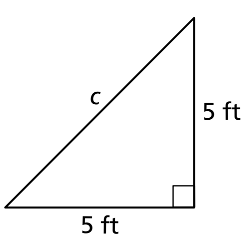

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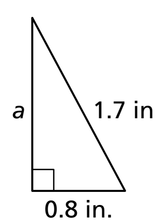

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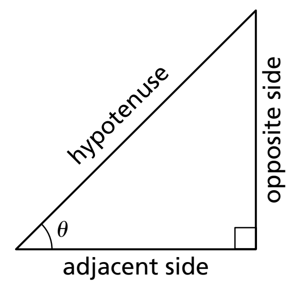
5.1 Right Triangle Trigonometry
For use with Exploration 5.1

Essential Question How can you find a trigonometric function of an acute angle θ?

Consider one of the acute angles θ of a right triangle. Ratios of a right triangle's side lengths are used to define the six *trigonometric functions*, as shown.



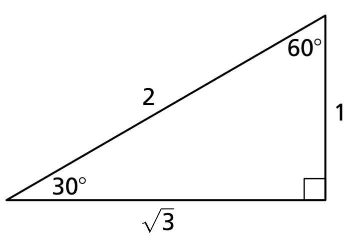
Sine $\sin\theta = \dfrac{\text{opp.}}{\text{hyp.}}$ **Cosine** $\cos\theta = \dfrac{\text{adj.}}{\text{hyp.}}$

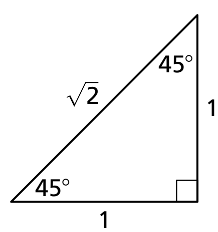
Tangent $\tan\theta = \dfrac{\text{opp.}}{\text{adj.}}$ **Cotangent** $\cot\theta = \dfrac{\text{adj.}}{\text{opp.}}$

Secant $\sec\theta = \dfrac{\text{hyp.}}{\text{adj.}}$ **Cosecant** $\csc\theta = \dfrac{\text{hyp.}}{\text{opp.}}$

1 EXPLORATION: Trigonometric Functions of Special Angles

Work with a partner. Find the exact values of the sine, cosine, and tangent functions for the angles 30°, 45°, and 60° in the right triangles shown.





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2 EXPLORATION: Exploring Trigonometric Identities

Work with a partner.

Use the definitions of the trigonometric functions to explain why each *trigonometric identity* is true.

a. $\sin\theta = \cos(90^\circ - \theta)$

b. $\cos\theta = \sin(90^\circ - \theta)$

c. $\sin\theta = \dfrac{1}{\csc\theta}$

d. $\tan\theta = \dfrac{1}{\cot\theta}$

Use the definitions of the trigonometric functions to complete each trigonometric identity.

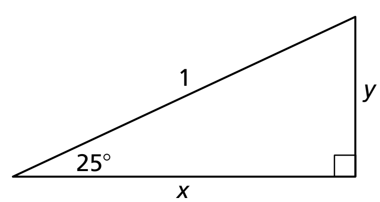
e. $(\sin\theta)^2 + (\cos\theta)^2 =$ _________

f. $(\sec\theta)^2 - (\tan\theta)^2 =$ _________

Communicate Your Answer

3. How can you find a trigonometric function of an acute angle θ?

4. Use a calculator to find the lengths x and y of the legs of the right triangle shown.



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5.1 Practice
For use after Lesson 5.1

Core Concepts

Right Triangle Definitions of Trigonometric Functions

Let θ be an acute angle of a right triangle. The six trigonometric functions of θ are defined as shown.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

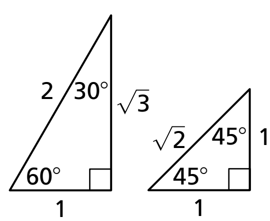
The abbreviations *opp.*, *adj.*, and *hyp.* are often used to represent the side lengths of the right triangle. Note that the ratios in the second row are reciprocals of the ratios in the first row.

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Notes:

Trigonometric Values for Special Angles

The table gives the values of the six trigonometric functions for the angles 30°, 45°, and 60°. You can obtain these values from the triangles shown.



θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

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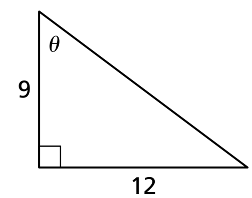
Notes:

Worked-Out Examples

Example #1

Evaluate the six trigonometric functions of the angle θ.



From the Pythagorean Theorem, the length of the hypotenuse is

$$\text{hyp.} = \sqrt{9^2 + 12^2} = \sqrt{225} = 15.$$

Using adj. = 9, opp. = 12, and hyp. = 15, the six trigonometric functions of θ are:

$\sin\theta = \frac{\text{opp.}}{\text{hyp.}} = \frac{12}{15} = \frac{4}{5}$ $\qquad$ $\cos\theta = \frac{\text{adj.}}{\text{hyp.}} = \frac{9}{15} = \frac{3}{5}$

$\tan\theta = \frac{\text{opp.}}{\text{adj.}} = \frac{12}{9} = \frac{4}{3}$ $\qquad$ $\csc\theta = \frac{\text{hyp.}}{\text{opp.}} = \frac{15}{12} = \frac{5}{4}$

$\sec\theta = \frac{\text{hyp.}}{\text{adj.}} = \frac{15}{9} = \frac{5}{3}$ $\qquad$ $\cot\theta = \frac{\text{adj.}}{\text{opp.}} = \frac{9}{12} = \frac{3}{4}$

Example #2

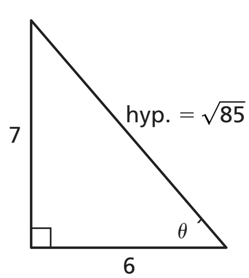
Let θ be an acute angle of a right triangle. Evaluate the other five trigonometric functions of θ.

$\tan\theta = \frac{7}{6}$

Step 1 Draw a right triangle with acute angle θ such that the leg adjacent θ has length 6 and the leg opposite θ has length 7.

Step 2 Find the length of the hypothenuse. By the Pythagorean Theorem, the length of the hypothenuse is

hyp. $= \sqrt{6^2 + 7^2} = \sqrt{85}$.



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Step 3 Find the values of the remaining five trigonometric functions. Because $\tan\theta = \frac{7}{6}$, $\cot\theta = \frac{6}{7}$. The other values are:

$$\sin\theta = \frac{\text{opp.}}{\text{hyp.}} = \frac{7}{\sqrt{85}} = \frac{7\sqrt{85}}{85}$$

$$\cos\theta = \frac{\text{adj.}}{\text{hyp.}} = \frac{6}{\sqrt{85}} = \frac{6\sqrt{85}}{85}$$

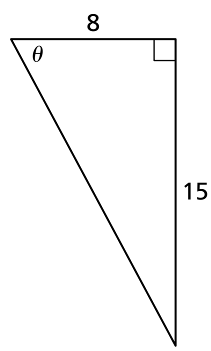
$$\csc\theta = \frac{\text{hyp.}}{\text{opp}} = \frac{\sqrt{85}}{7}$$

$$\sec\theta = \frac{\text{hyp.}}{\text{adj.}} = \frac{\sqrt{85}}{6}$$

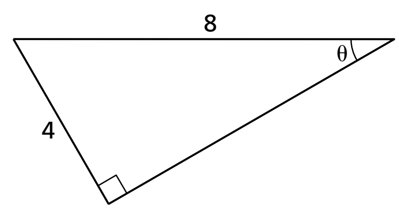
Practice A

In Exercises 1 and 2, evaluate the six trigonometric functions of the angle θ.

1.



2.



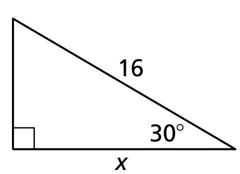
In Exercises 3 and 4, let θ be an acute angle of a right triangle. Evaluate the other five trigonometric functions of θ.

3. $\tan\theta = 1$

4. $\sin\theta = \frac{3}{19}$

In Exercises 5 and 6, find the value of x for the right triangle.

5.



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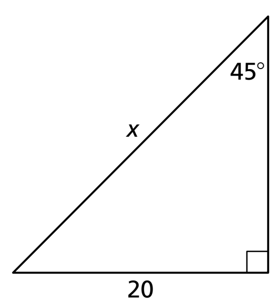

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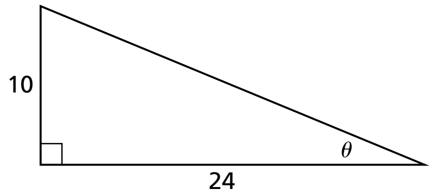
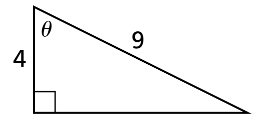
Practice B

In Exercises 1 and 2, evaluate the six trigonometric functions of the angle θ.

1.



2.



3. Evaluate the six trigonometric functions of the angle $90° - \theta$ in Exercise 1. Describe the relationships you notice.

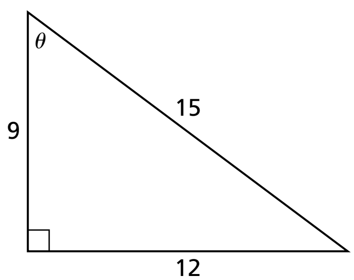
In Exercises 4–6, let θ be an acute angle of a right triangle. Evaluate the other five trigonometric functions of θ.

4. $\cos\theta = \frac{5}{11}$

5. $\cot\theta = \frac{7}{8}$

6. $\sec\theta = \frac{11}{9}$

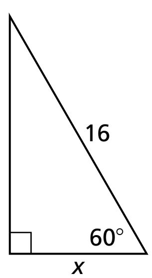
7. Describe and correct the error in finding $\csc\theta$ of the triangle below.



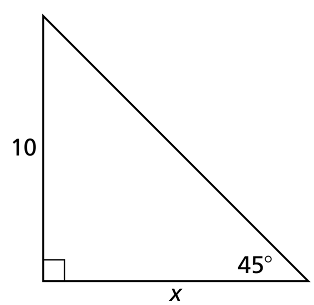
$$\sec\theta = \frac{\text{adj.}}{\text{hyp.}} = \frac{9}{15} = \frac{3}{5}$$

In Exercises 8 and 9, find the value of *x* for the right triangle.

8.



9.



10. A cable is attached to the top of a pole and mounted to the ground 3 feet from the base of the pole. The angle of elevation from the mounting to the top of the pole is $78°$. Estimate the height of the pole. Round your answer to the nearest tenth.

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