Name____ Date __________

5.3 Trigonometric Functions of Any Angle
For use with Exploration 5.3

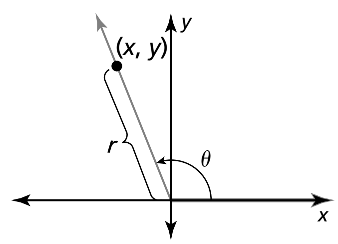
Essential Question How can you use the unit circle to define the trigonometric functions of any angle?

Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$. The six trigonometric functions of θ are defined as shown.

$\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{r}{y}, y \neq 0$

$\cos\theta = \frac{x}{r}$ $\sec\theta = \frac{r}{x}, x \neq 0$

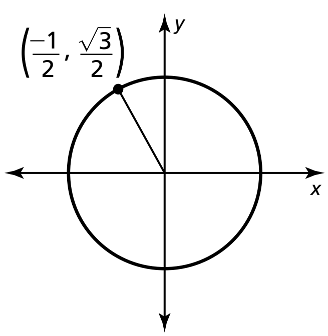
$\tan\theta = \frac{y}{x}, x \neq 0$ $\cot\theta = \frac{x}{y}, y \neq 0$



1 EXPLORATION: Writing Trigonometric Functions

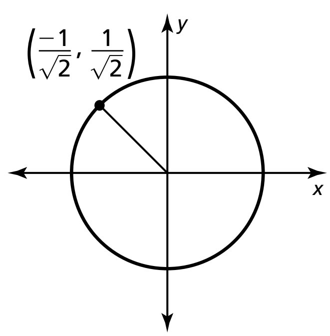
Work with a partner. Find the sine, cosine, and tangent of the angle θ in standard position whose terminal side intersects the unit circle at the point (x, y) shown.

a.



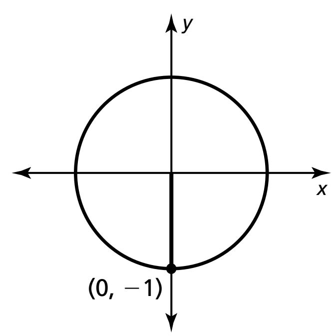
$\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$

b.



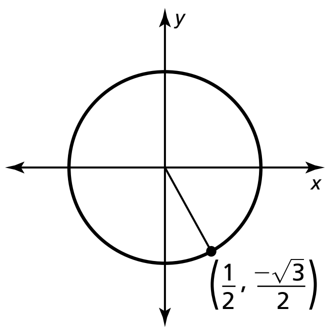
$\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

c.

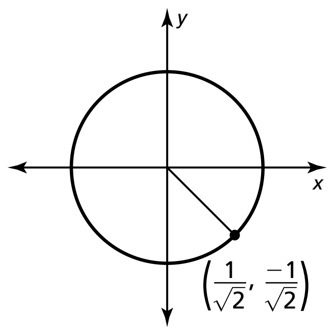


1 EXPLORATION: Writing Trigonometric Functions (continued)

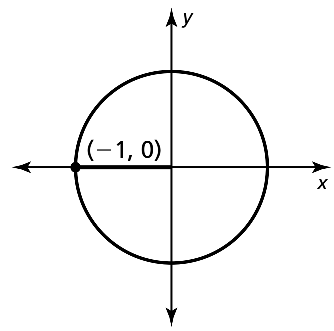
d.



e.



f.



Communicate Your Answer

2. How can you use the unit circle to define the trigonometric functions of any angle?

3. For which angles are each function undefined? Explain your reasoning.

a. tangent

b. cotangent

c. secant

d. cosecant

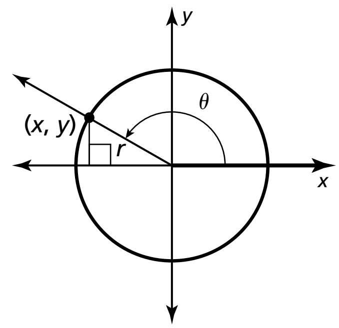
Name______________________________ Date__________

5.3 Practice
For use after Lesson 5.3

Core Concepts

General Definitions of Trigonometric Functions

Let θ be an angle in standard position, and let (x, y) be the point where the terminal side of θ intersects the circle $x^2 + y^2 = r^2$. The six trigonometric functions of θ are defined as shown.



$\sin\theta = \dfrac{y}{r}$ $\qquad$ $\csc\theta = \dfrac{r}{y}, y \neq 0$

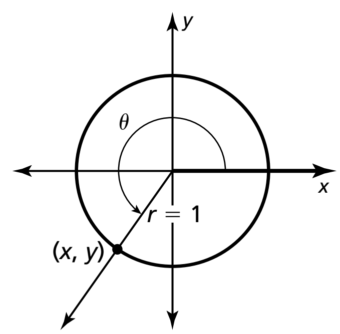
$\cos\theta = \dfrac{x}{r}$ $\qquad$ $\sec\theta = \dfrac{r}{x}, x \neq 0$

$\tan\theta = \dfrac{y}{x}, x \neq 0$ $\qquad$ $\cot\theta = \dfrac{x}{y}, y \neq 0$

These functions are sometimes called *circular functions*.

The Unit Circle

The circle $x^2 + y^2 = 1$, which has center (0, 0) and radius 1, is called the **unit circle**. The values of sin θ and cos θ are simply the y-coordinate and x-coordinate, respectively, of the point where the terminal side of θ intersects the unit circle.



$\sin\theta = \dfrac{y}{r} = \dfrac{y}{1} = y \qquad \cos\theta = \dfrac{x}{r} = \dfrac{x}{1} = x$

Notes:

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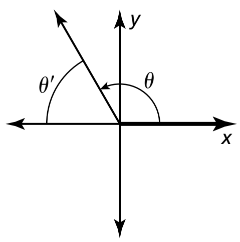
5.3 Practice (continued)

Reference Angle Relationships

Let θ be an angle in standard position. The **reference angle** for θ is the acute angle θ' formed by the terminal side of θ and the x-axis. The relationship between θ and θ' is shown below for nonquadrantal angles θ such that $90° < \theta < 360°$ or, in radians, $\frac{\pi}{2} < \theta < 2\pi$.

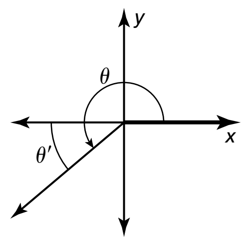
Quadrant II



Degrees: $\theta' = 180° - \theta$

Radians: $\theta' = \pi - \theta$

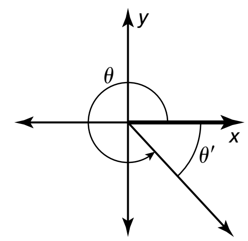
Quadrant III



Degrees: $\theta' = \theta - 180°$

Radians: $\theta' = \theta - \pi$

Quadrant IV



Degrees: $\theta' = 360° - \theta$

Radians: $\theta' = 2\pi - \theta$

Notes:

Evaluating Trigonometric Functions

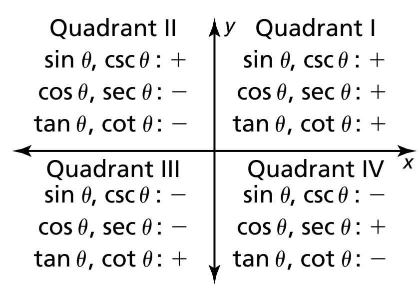
Use these steps to evaluate a trigonometric function for any angle θ:

Step 1 Find the reference angle θ'.

Step 2 Evaluate the trigonometric function for θ'.

Step 3 Determine the sign of the trigonometric function value from the quadrant in which θ lies.

Signs of Function Values



Notes:

Worked-Out Examples

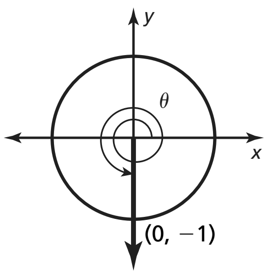
Example #1

Use the unit circle to evaluate the six trigonometric functions of θ.

$\theta = \frac{7\pi}{2}$

Draw a unit circle with the angle $\theta = \frac{7\pi}{2}$ in standard position.

Identify the point where the terminal side of θ intersects the unit circle. The terminal side of θ intersects the unit circle at $(0, -1)$.



Find the values of the six trigonometric functions. Let $x = 0$ and $y = -1$ to evaluate the trigonometric functions.

$\sin \theta = \frac{y}{r} = \frac{-1}{1} = -1$ $\qquad$ $\csc \theta = \frac{r}{y} = \frac{1}{-1} = -1$

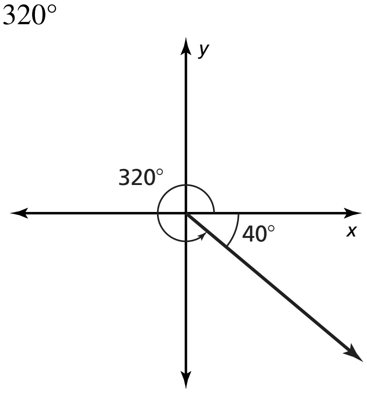
$\cos \theta = \frac{x}{r} = \frac{0}{1} = 0$ $\qquad$ $\sec \theta = \frac{r}{x} = \frac{1}{0}$ undefined

$\tan \theta = \frac{y}{x} = \frac{-1}{0}$ undefined $\qquad$ $\cot \theta = \frac{x}{y} = \frac{0}{-1} = 0$

Example #2

Sketch the angle. Then find its reference angle.

$320°$



The terminal side lies in Quadrant IV. So, the reference angle is $360° - 320° = 40°$.

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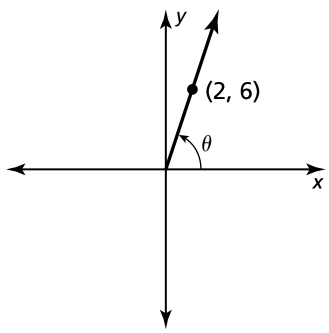
5.3 Practice (continued)

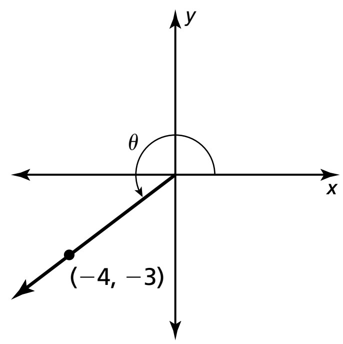
Practice A

In Exercises 1 and 2, evaluate the six trigonometric functions of θ.

1.



2.



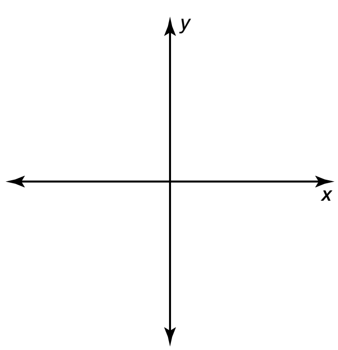
In Exercises 3 and 4, use the unit circle to evaluate the six trigonometric functions of θ.

3. $\theta = -90^\circ$

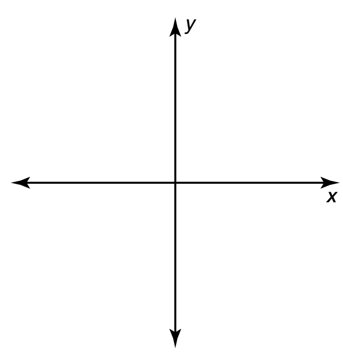
4. $\theta = 4\pi$

In Exercises 5 and 6, sketch the angle. Then find its reference angle.

5. -310°



6. $\dfrac{27\pi}{10}$



7. Evaluate the function csc 150° without using a calculator.

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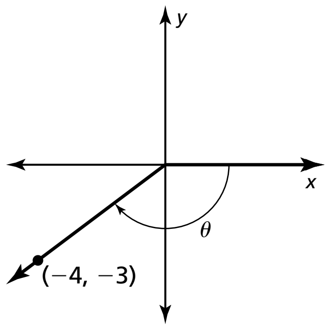
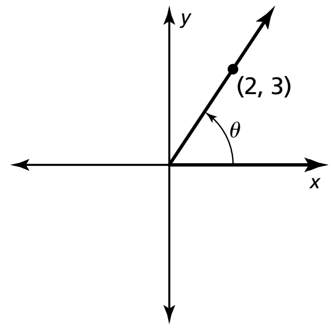
Practice B

In Exercises 1–4, evaluate the six trigonometric functions of θ.

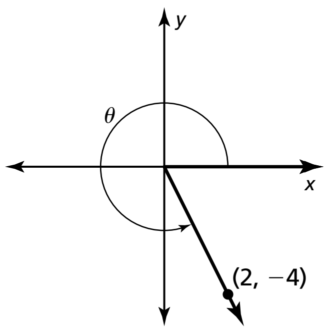
1.



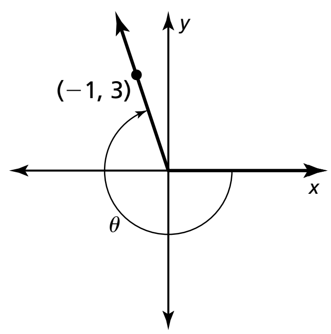
2.



3.



4.



In Exercises 5–7, use the unit circle to evaluate the six trigonometric functions of θ.

5. 5π **6.** -720° **7.** $-\frac{5\pi}{2}$

In Exercises 8–13, find the angle's reference angle.

8. -250° **9.** 110° **10.** -310°

11. $\frac{13\pi}{4}$ **12.** $\frac{11\pi}{6}$ **13.** $-\frac{13\pi}{3}$

In Exercises 14–16, evaluate the function without using a calculator.

14. $\cot 240^\circ$ **15.** $\sin 315^\circ$ **16.** $\sec\left(-\frac{5\pi}{6}\right)$

17. The horizontal distance d (in feet) traveled by a projectile launched at an angle θ and with an initial speed v (in feet per second) is given by $d = \frac{v^2}{32} \sin 2\theta.$ To win a shot-put competition, your last throw must travel a horizontal distance of at least 15 feet. You release the shot put at a 45° angle with an initial speed of 22 feet per second. Do you win the competition? Justify your answer.