Name ______________________________ Date __________

5.4 Graphing Sine and Cosine Functions

For use with Exploration 5.4

Essential Question What are the characteristics of the graphs of the sine and cosine functions?

1 EXPLORATION: Graphing the Sine Function

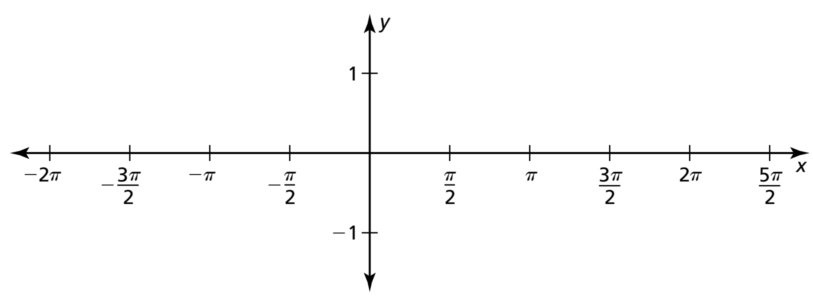
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

a. Complete the table for $y = \sin x$, where x is an angle measure in radians.

x	-2π	$-\frac{7\pi}{4}$	$-\frac{3\pi}{2}$	$-\frac{5\pi}{4}$	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0
y* = sin *x									
x	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π	$\frac{9\pi}{4}$
y* = sin *x									

b. Plot the points (x, y) from part (a). Draw a smooth curve through the points to sketch the graph of $y = \sin x$.



c. Use the graph to identify the x-intercepts, the x-values where the local maximums and minimums occur, and the intervals for which the function is increasing or decreasing over $-2\pi \le x \le 2\pi$. Is the sine function *even*, *odd*, or *neither*?

2 EXPLORATION: Graphing the Cosine Function

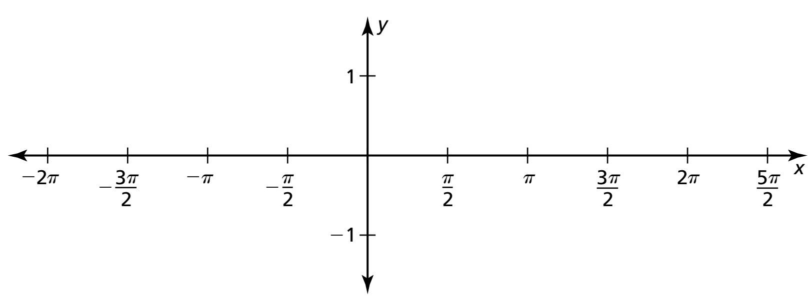
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

a. Complete the table for $y = \cos x$ using the same values of x as those used in Exploration 1.

x	-2π	$-\frac{7\pi}{4}$	$-\frac{3\pi}{2}$	$-\frac{5\pi}{4}$	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0
$y = \cos x$									
x	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π	$\frac{9\pi}{4}$
$y = \cos x$									

b. Plot the points (x, y) from part (a) and sketch the graph of $y = \cos x$



c. Use the graph to identify the x-intercepts, the x-values where the local maximums and minimums occur, and the intervals for which the function is increasing or decreasing over $-2\pi \le x \le 2\pi$. Is the cosine function *even*, *odd*, or *neither*?

Communicate Your Answer

3. What are the characteristics of the graphs of the sine and cosine functions?

4. Describe the end behavior of the graph of $y = \sin x$.

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5.4 Practice
For use after Lesson 5.4

Core Concepts

Characteristics of $y = \sin x$ and $y = \cos x$

- The domain of each function is all real numbers.
- The range of each function is $-1 \le y \le 1$. So, the minimum value of each function is -1 and the maximum value is 1.
- The **amplitude** of the graph of each function is one-half of the difference of the maximum value and the minimum value, or $\frac{1}{2}[1 - (-1)] = 1$.
- Each function is **periodic**, which means that its graph has a repeating pattern. The shortest repeating portion of the graph is called a **cycle**. The horizontal length of each cycle is called the **period**. The graph of each function has a period of 2π.
- The x-intercepts for $y = \sin x$ occur when $x = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$.
- The x-intercepts for $y = \cos x$ occur when $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots$.

Amplitude and Period

The amplitude and period of the graphs of $y = a \sin bx$ and $y = a \cos bx$, where a and b are nonzero real numbers, are as follows:

$$\text{Amplitude} = |a| \qquad\qquad \text{Period} = \frac{2\pi}{|b|}$$

Notes:

Graphing $y = a \sin b(x - h) + k$ and $y = a \cos b(x - h) + k$

To graph $y = a \sin b(x - h) + k$ or $y = a \cos b(x - h) + k$ where $a > 0$ and $b > 0$, follow these steps:

Step 1 Identify the amplitude a, the period $\frac{2\pi}{b}$, the horizontal shift h, and the vertical shift k of the graph.

Step 2 Draw the horizontal line $y = k$, called the **midline** of the graph.

Step 3 Find the five key points by translating the key points of $y = a \sin bx$ or $y = a \cos bx$ horizontally h units and vertically k units.

Step 4 Draw the graph through the five translated key points.

Notes:

Worked-Out Examples

Example #1

Identify the amplitude and period of the function. Then graph the function and describe the graph of *g* as a transformation of the graph of its parent function.

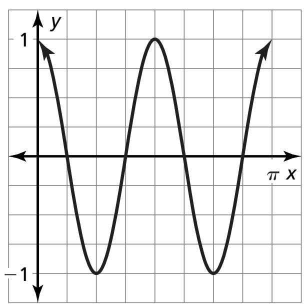
$g(x) = \cos 4x$

The function is of the form $g(x) = a \cos bx$, where $a = 1$ and $b = 4$. So, the amplitude is $a = 1$ and the period is $\frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}$.

Intercepts: $\left(\frac{\pi}{8}, 0\right)$; $\left(\frac{3\pi}{8}, 0\right)$

Maximum: $(0, 1)$; $\left(\frac{\pi}{2}, 1\right)$

Minimum: $\left(\frac{\pi}{4}, -1\right)$



The graph of g is a horizontal shrink by a factor of $\frac{1}{4}$ of the graph of $f(x) = \cos x$.

Example #2

Identify the amplitude and period of the function. Then graph the function and describe the graph of *g* as a transformation of the graph of its parent function.

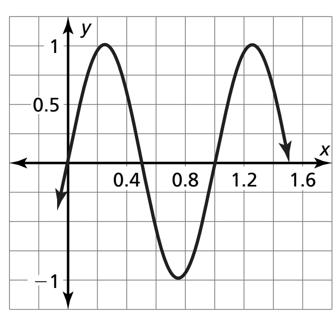
$g(x) = \sin 2\pi x$

The function is of the form $g(x) = a \sin bx$, where $a = 1$ and $b = 2\pi$. So, the amplitude is $a = 1$ and the period is $\frac{2\pi}{b} = \frac{2\pi}{2\pi} = 1$.

Intercepts: (0, 0); (0.5, 0); (1, 0)

Maximum: (0.25, 1)

Minimum: (0.75, −1)



The graph of g is a horizontal shrink by a factor of $\frac{1}{2\pi}$ of the graph of $f(x) = \sin x$.

Practice A

In Exercises 1–4, identify the amplitude and period of the function. Then graph the function and describe the graph of *g* as a transformation of the graph of its parent function.

1. $g(x) = \sin 2x$

2. $g(x) = \frac{1}{3}\cos 2x$

3. $g(x) = 4\sin 2\pi x$

4. $g(x) = \frac{1}{2}\cos 3\pi x$

In Exercises 5 and 6, graph the function.

5. $g(x) = \sin \frac{1}{2}(x - \pi) + 1$

6. $g(x) = \cos\left(x + \frac{\pi}{2}\right) - 3$

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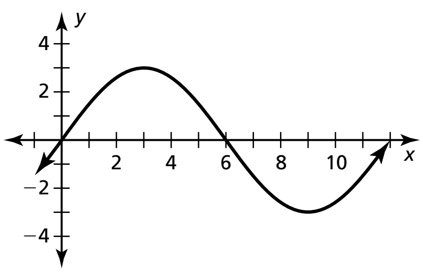
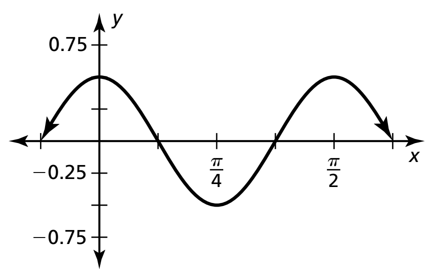
Practice B

In Exercises 1 and 2, identify the amplitude and period of the graph of the function.

1. 

2. 

In Exercises 3–6, identify the amplitude and period of the function. Then graph the function and describe the graph of *g* as a transformation of the graph of its parent function.

3. $g(x) = 4 \sin x$

4. $g(x) = \cos \pi x$

5. $g(x) = 5 \sin 4x$

6. $g(x) = \frac{1}{4} \cos 2x$

7. Write an equation of the form $y = a \cos bx$, where $a > 0$ and $b > 0$, so that the graph has the given amplitude and period.

   a. amplitude: 1
   period: 3

   b. amplitude: 3
   period: 4

   c. amplitude: 12
   period: 2π

   d. amplitude: $\frac{1}{3}$
   period: π

In Exercises 8–11, graph the function.

8. $g(x) = \cos x + 3$

9. $g(x) = 2 \sin x - 1$

10. $g(x) = \sin \frac{1}{2}(x - \pi) - 2$

11. $g(x) = \cos \frac{1}{2}(x + \pi) - 4$

In Exercises 12 and 13, write a rule for *g* that represents the indicated transformations of the graph of *f*.

12. $f(x) = \frac{1}{2} \cos 3x$; translation 2 units up, followed by a reflection in the line $y = 2$

13. $f(x) = \frac{1}{3} \sin \pi x$; translation 3 units down, followed by a reflection in the line $y = -3$