Name____ Date __________

5.5 Graphing Other Trigonometric Functions
For use with Exploration 5.5

Essential Question What are the characteristics of the graph of the tangent function?

1 EXPLORATION: Graphing the Tangent Function

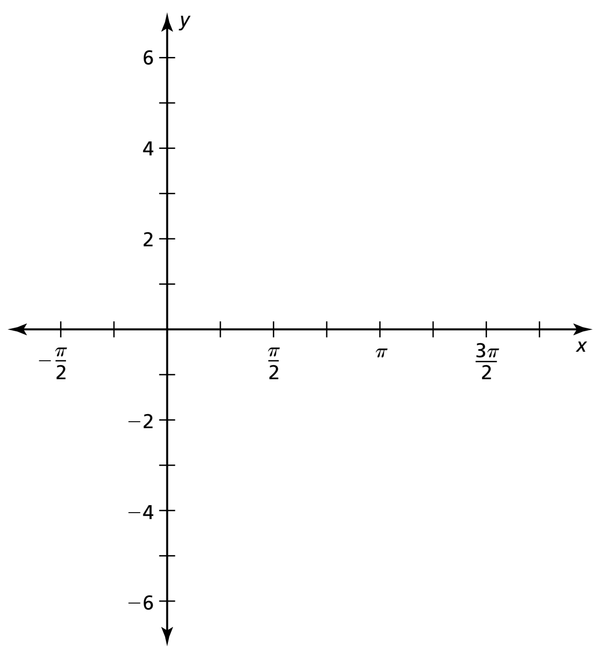
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

a. Complete the table for $y = \tan x$, where x is an angle measure in radians.

x	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y* = tan *x									
x	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$
y* = tan *x									

b. The graph of $y = \tan x$ has vertical asymptotes at x-values where $\tan x$ is undefined. Plot the points (x, y) from part (a). Then use the asymptotes to sketch the graph of $y = \tan x$.



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1 EXPLORATION: Graphing the Tangent Function (continued)

c. For the graph of $y = \tan x$, identify the asymptotes, the x-intercepts, and the intervals for which the function is increasing or decreasing over $-\frac{\pi}{2} \le x \le \frac{3\pi}{2}$. Is the tangent function *even*, *odd*, or *neither*?

Communicate Your Answer

2. What are the characteristics of the graph of the tangent function?

3. Describe the asymptotes of the graph of $y = \cot x$ on the interval $-\frac{\pi}{2} < x < \frac{3\pi}{2}$.

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5.5 Practice

For use after Lesson 5.5

Core Concepts

Characteristics of $y = \tan x$ and $y = \cot x$

The functions $y = \tan x$ and $y = \cot x$ have the following characteristics.

- The domain of $y = \tan x$ is all real numbers except odd multiples of $\frac{\pi}{2}$. At these x-values, the graph has vertical asymptotes.
- The domain of $y = \cot x$ is all real numbers except multiples of π. At these x-values, the graph has vertical asymptotes.
- The range of each function is all real numbers. So, the functions do not have maximum or minimum values, and the graphs do not have an amplitude.
- The period of each graph is π.
- The x-intercepts for $y = \tan x$ occur when $x = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$.
- The x-intercepts for $y = \cot x$ occur when $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots$.

Notes:

Period and Vertical Asymptotes of $y = a \tan bx$ and $y = a \cot bx$

The period and vertical asymptotes of the graphs of $y = a \tan bx$ and $y = a \cot bx$, where a and b are nonzero real numbers, are as follows.

- The period of the graph of each function is $\frac{\pi}{|b|}$.
- The vertical asymptotes for $y = a \tan bx$ occur at odd multiples of $\frac{\pi}{2|b|}$.
- The vertical asymptotes for $y = a \cot bx$ occur at multiples of $\frac{\pi}{|b|}$.

Notes:

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5.5 Practice (continued)

Characteristics of $y = \sec x$ and $y = \csc x$

The functions $y = \sec x$ and $y = \csc x$ have the following characteristics.

- The domain of $y = \sec x$ is all real numbers except odd multiples of $\frac{\pi}{2}$. At these x-values, the graph has vertical asymptotes.
- The domain of $y = \csc x$ is all real numbers except multiples of π. At these x-values, the graph has vertical asymptotes.
- The range of each function is $y \le -1$ and $y \ge 1$. So, the graphs do not have an amplitude.
- The period of each graph is 2π.

Notes:

Worked-Out Examples

Example #1

Graph one period of the function. Describe the graph of *g* as a transformation of the graph of its parent function.

$g(x) = 4 \cot \frac{1}{2} x$

The function is of the form $g(x) = a \cot bx$, where $a = 4$ and $b = \frac{1}{2}$. So, the period is $\frac{\pi}{|b|} = \frac{\pi}{\frac{1}{2}} = 2\pi$.

Intercept: $\left(\frac{\pi}{2b}, 0\right) = \left(\frac{\pi}{2\left(\frac{1}{2}\right)}, 0\right) = (\pi, 0)$

Asymptotes: $x = 0$; $x = \frac{\pi}{|b|} = \frac{\pi}{\frac{1}{2}}$, or $x = 2\pi$

Halfway points: $\left(\frac{\pi}{4b}, a\right) = \left(\frac{\pi}{4\left(\frac{1}{2}\right)}, 4\right) = \left(\frac{\pi}{2}, 4\right)$;

$$\left(\frac{3\pi}{4b}, -a\right) = \left(\frac{3\pi}{4\left(\frac{1}{2}\right)}, -4\right) = \left(\frac{3\pi}{2}, -4\right)$$

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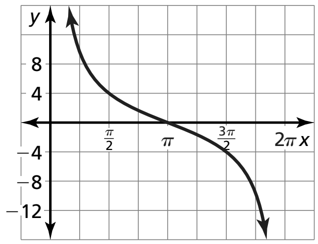


The graph of g is a horizontal stretch by a factor of 2 and a vertical stretch by a factor of 4 of the graph of $f(x) = \cot x$.

Example #2

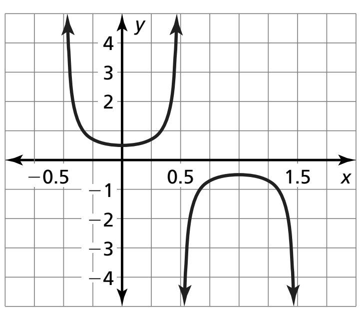
Graph one period of the function. Describe the graph of *g* as a transformation of the graph of its parent function.

$g(x) = \frac{1}{2} \sec \pi x$

Step 1 Graph the function $y = \frac{1}{2} \cos \pi x$. The period is $\frac{2\pi}{\pi} = 2$.

Step 2 Graph asymptotes of g. Because the asymptotes of g occur when $\frac{1}{2} \cos \pi x = 0$, graph $x = -\frac{1}{2}$, $x = \frac{1}{2}$, and $x = \frac{3}{2}$.

Step 3 Plot points on g, such as $\left(0, \frac{1}{2}\right)$ and $\left(1, -\frac{1}{2}\right)$. Then use the asymptotes to sketch the curve.



The graph of g is a horizontal shrink by a factor of $\frac{1}{\pi}$ and a vertical shrink by a factor of $\frac{1}{2}$ of the graph of $f(x) = \sec x$.

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Practice A

In Exercises 1–6, graph one period of the function. Describe the graph of *g* as a transformation of the graph of its parent function.

1. $g(x) = \tan 2x$

2. $g(x) = 2 \cot \frac{1}{2}x$

3. $g(x) = \frac{1}{4} \tan \frac{\pi}{4}x$

4. $g(x) = \frac{1}{2} \cot 3x$

5. $g(x) = 2 \sec 2x$

6. $g(x) = \csc 2\pi x$

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Practice B

In Exercises 1–4, graph one period of the function. Describe the graph of *g* as a transformation of the graph of its parent function.

1. $g(x) = 2\tan 4x$

2. $g(x) = 3\cot \frac{1}{2}x$

3. $g(x) = \frac{1}{4}\tan 2\pi x$

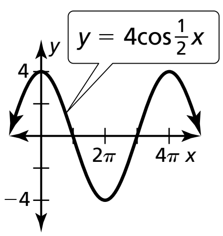
4. $g(x) = \frac{1}{3}\cot \pi x$

5. Describe and correct the error in describing the transformation of $f(x) = \tan x$ represented by $g(x) = 4\tan \frac{1}{2}x$.

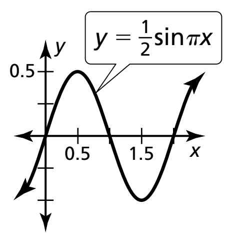
> ✗ A vertical stretch by a factor of 4 and a horizontal shrink by a factor of $\frac{1}{2}$

6. Use the given graph to graph each function.

a. $f(x) = 4\sec \frac{1}{2}x$



b. $f(x) = \frac{1}{2}\csc \pi x$



In Exercises 7–10, graph one period of the function. Describe the graph of *g* as a transformation of the graph of its parent function.

7. $g(x) = \frac{1}{3}\csc \pi x$

8. $g(x) = \frac{1}{2}\sec 6x$

9. $g(x) = \sec \frac{\pi}{2}x$

10. $g(x) = \csc \frac{\pi}{3}x$

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