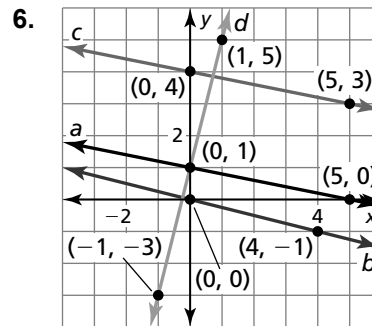
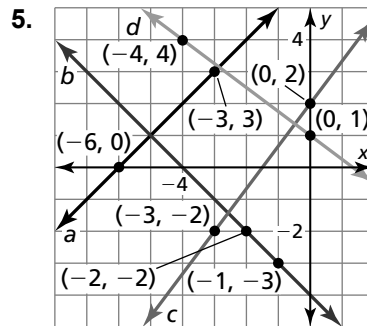
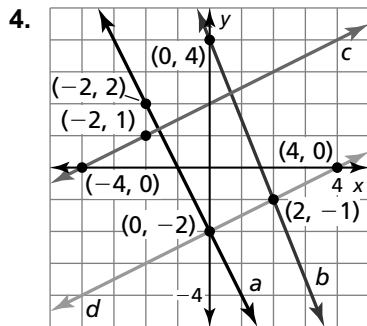


Chapter 6 Maintaining Mathematical Proficiency

Solve the equation by interpreting the expression in parentheses as a single quantity.

1. $5(10 - x) = 100$ 2. $6(x + 8) - 12 = -48$ 3. $3(2 - x) + 4(2 - x) = 56$

Determine which lines are parallel and which are perpendicular.



7. Explain why you can rewrite $4(x - 9) + 5(9 - x) = 11$ as $-(x - 9) = 11$? Then solve the equation.

6.1**Bisectors of Triangles**

For use with Exploration 6.1

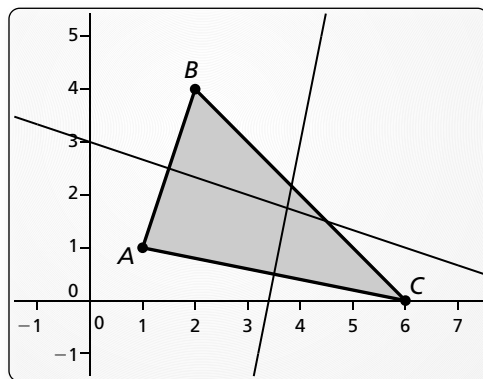
Essential Question What conjectures can you make about the perpendicular bisectors and the angle bisectors of a triangle?

1 EXPLORATION: Properties of the Perpendicular Bisectors of a Triangle

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software. Draw any $\triangle ABC$.

- Construct the perpendicular bisectors of all three sides of $\triangle ABC$. Then drag the vertices to change $\triangle ABC$. What do you notice about the perpendicular bisectors?
- Label a point D at the intersection of the perpendicular bisectors.
- Draw the circle with center D through vertex A of $\triangle ABC$. Then drag the vertices to change $\triangle ABC$. What do you notice?

**Sample**

Points

 $A(1, 1)$ $B(2, 4)$ $C(6, 0)$

Segments

 $BC = 5.66$ $AC = 5.10$ $AB = 3.16$

Lines

 $x + 3y = 9$ $-5x + y = -17$

2 EXPLORATION: Properties of the Angle Bisectors of a Triangle

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

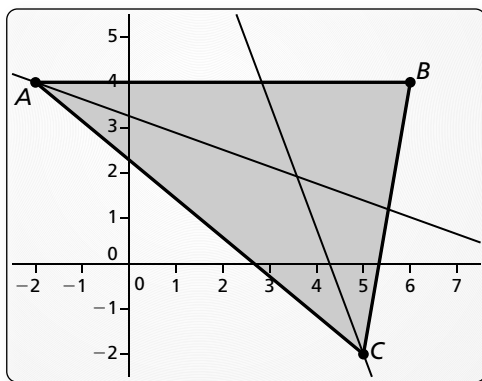
Work with a partner. Use dynamic geometry software. Draw any $\triangle ABC$.

- Construct the angle bisectors of all three angles of $\triangle ABC$. Then drag the vertices to change $\triangle ABC$. What do you notice about the angle bisectors?

6.1 Bisectors of Triangles (continued)

2 EXPLORATION: Properties of the Angle Bisectors of a Triangle (continued)

- b. Label a point D at the intersection of the angle bisectors.
- c. Find the distance between D and \overline{AB} . Draw the circle with center D and this distance as a radius. Then drag the vertices to change $\triangle ABC$. What do you notice?



Sample

Points

$A(-2, 4)$

$B(6, 4)$

$C(5, -2)$

Segments

$BC = 6.08$

$AC = 9.22$

$AB = 8$

Lines

$0.35x + 0.94y = 3.06$

$-0.94x - 0.34y = -4.02$

Communicate Your Answer

- 3. What conjectures can you make about the perpendicular bisectors and the angle bisectors of a triangle?

6.1

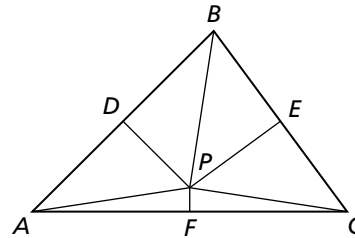
Practice
For use after Lesson 6.1

Theorems

Circumcenter Theorem

The circumcenter of a triangle is equidistant from the vertices of the triangle.

If \overline{PD} , \overline{PE} , and \overline{PF} are perpendicular bisectors, then $PA = PB = PC$.

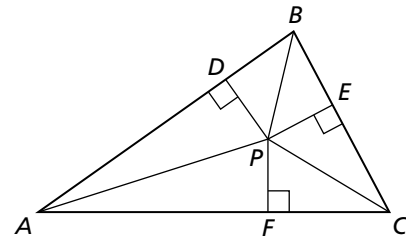


Notes:

Incenter Theorem

The incenter of a triangle is equidistant from the sides of the triangle.

If \overline{AP} , \overline{BP} , and \overline{CP} are angle bisectors of $\triangle ABC$, then $PD = PE = PF$.



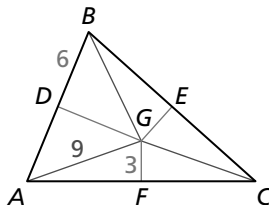
Notes:

Worked-Out Examples

Example #1

The perpendicular bisectors of $\triangle ABC$ intersect at point G and contain \overline{DG} , \overline{EG} , and \overline{FG} . Find the indicated measure.

Find BG .



Because G is the circumcenter of $\triangle ABC$, $AG = BG = CG$. Therefore, because $AG = 9$, $BG = 9$.

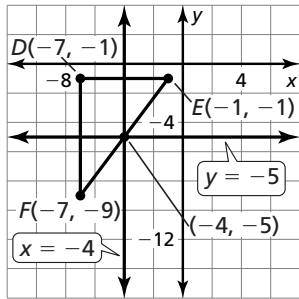
6.1 Practice (continued)

Example #2

Find the coordinates of the circumcenter of the triangle with the given vertices.

$D(-7, -1), E(-1, -1), F(-7, -9)$

Graph $\triangle DEF$.



$$\begin{aligned} \text{midpoint of } \overline{DE} &= \left(\frac{-7 + (-1)}{2}, \frac{-1 + (-1)}{2} \right) \\ &= \left(\frac{-8}{2}, \frac{-2}{2} \right) = (-4, -1) \end{aligned}$$

$$\begin{aligned} \text{midpoint of } \overline{DF} &= \left(\frac{-7 + (-7)}{2}, \frac{-1 + (-9)}{2} \right) \\ &= \left(\frac{-14}{2}, \frac{-10}{2} \right) = (-7, -5) \end{aligned}$$

The equation of the perpendicular bisector of \overline{DE} through its midpoint $(-4, -1)$ is $x = -4$, and the equation of the perpendicular bisector of \overline{DF} through its midpoint $(-7, -5)$ is $y = -5$. The point of intersection of the two perpendicular bisectors is $(-4, -5)$. So, the coordinates of the circumcenter of $\triangle DEF$ are $(-4, -5)$.

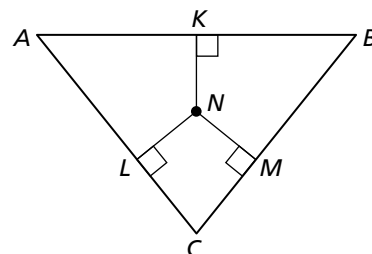
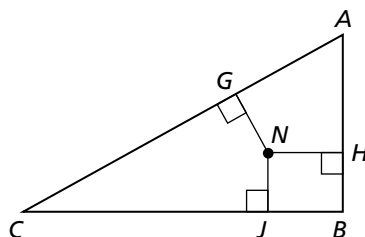
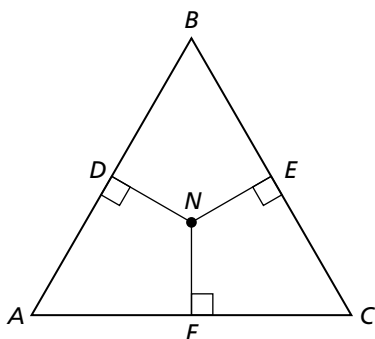
Practice A

In Exercises 1–3, N is the incenter of $\triangle ABC$. Use the given information to find the indicated measure.

1. $ND = 2x - 5$
 $NE = -2x + 7$
 Find NF .

2. $NG = x - 1$
 $NH = 2x - 6$
 Find NJ .

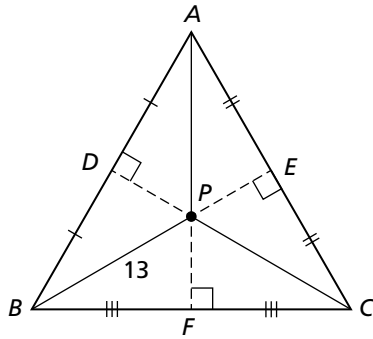
3. $NK = x + 10$
 $NL = -2x + 1$
 Find NM .



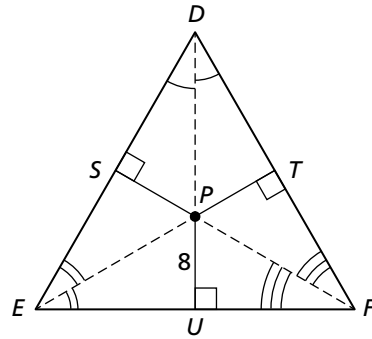
6.1 Practice (continued)

In Exercises 4–7, find the indicated measure.

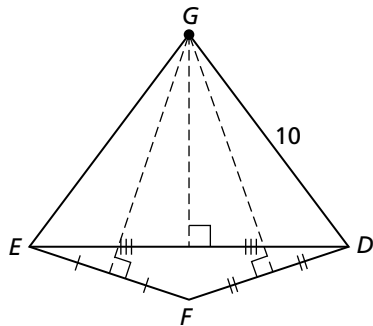
4. PA



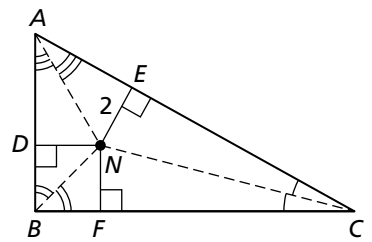
5. PS



6. GE

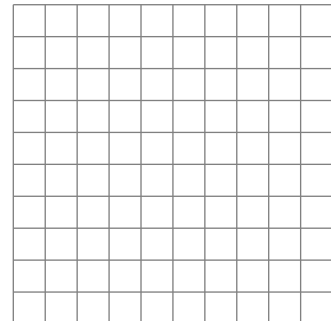
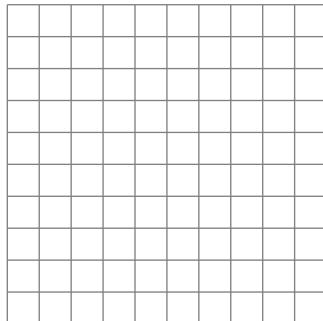
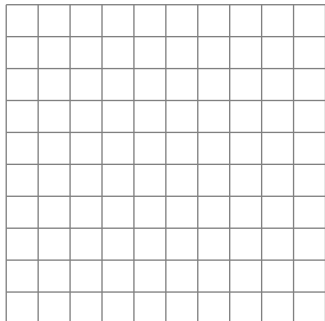


7. NF



In Exercises 8–10, find the coordinates of the circumcenter of the triangle with the given vertices.

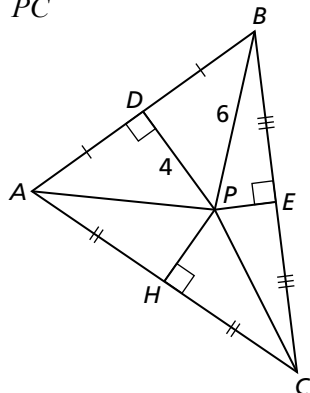
8. $A(-2, -2), B(-2, 4), C(6, 4)$ 9. $D(3, 5), E(3, 1), F(9, 5)$ 10. $J(4, -7), K(4, -3), L(-6, -3)$



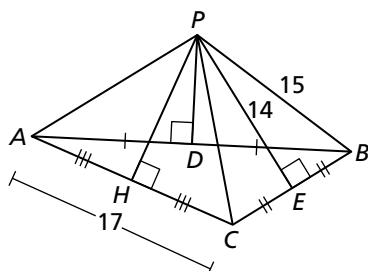
Practice B

In Exercises 1–3, find the indicated measure. Tell which theorem you used.

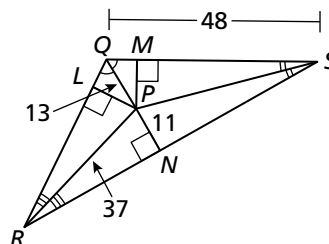
1. PC



2. AP



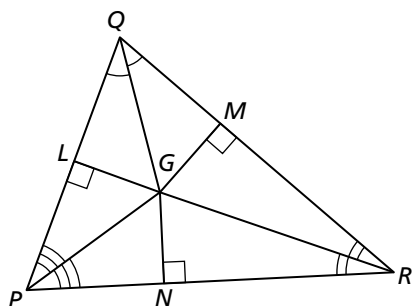
3. MP



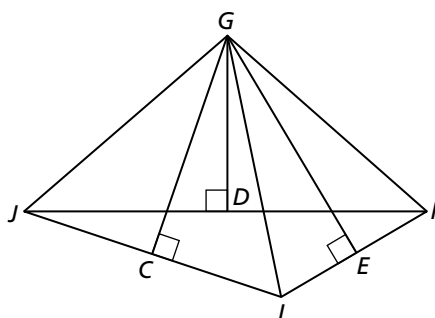
4. Find the coordinates of the circumcenter of the triangle with the vertices $A(4, 12)$, $B(14, 6)$, and $C(-6, 2)$.

In Exercises 5 and 6, use the diagram and the given information to find the indicated measures.

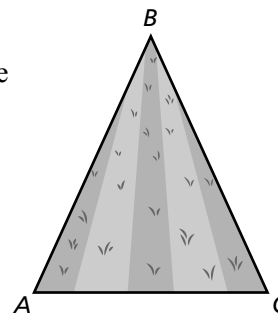
5. $LG = 6x - 14$, $NG = -3x + 22$
Find MG and NG .



6. $GL = 4x - 2$, $GE = 3x + 2$, $GK = 2x + 8$
Find GJ and GE .



7. You are using a rotary sprinkler to water the triangular lawn.
- Explain how to locate the sprinkler the same distance from each side of the triangular lawn.
 - Explain how to locate the sprinkler the same distance from each vertex of the triangular lawn.
 - Which is closer to vertex B , the *incenter* or the *circumcenter*? Explain your reasoning.



8. Explain when the circumcenter of a triangle lies outside the triangle.

9. In the figure at the right, what value of x makes G the incenter of $\triangle JKL$?

