Name ______________________________ Date __________

6.2 Medians and Altitudes of Triangles

For use with Exploration 6.2

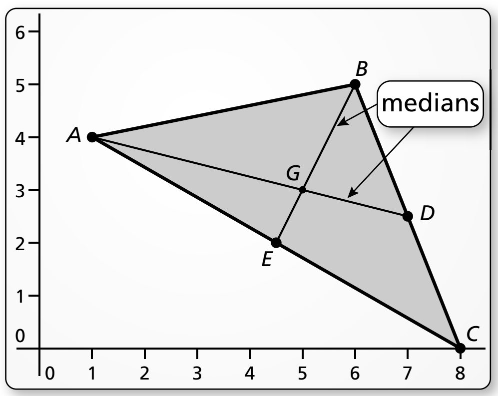
Essential Question What conjectures can you make about the medians and altitudes of a triangle?

1 EXPLORATION: Finding Properties of the Medians of a Triangle

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software. Draw any $\triangle ABC$.

a. Plot the midpoint of $\overline{BC}$ and label it D. Draw $\overline{AD}$, which is a *median* of $\triangle ABC$. Construct the medians to the other two sides of $\triangle ABC$.



Sample
Points
$A(1, 4)$
$B(6, 5)$
$C(8, 0)$
$D(7, 2.5)$
$E(4.5, 2)$
$G(5, 3)$

b. What do you notice about the medians? Drag the vertices to change $\triangle ABC$. Use your observations to write a conjecture about the medians of a triangle.

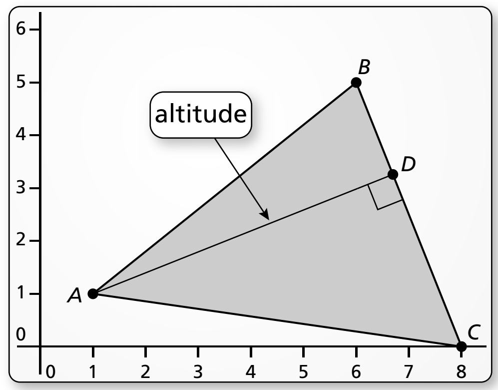
c. In the figure above, point G divides each median into a shorter segment and a longer segment. Find the ratio of the length of each longer segment to the length of the whole median. Is this ratio always the same? Justify your answer.

2 EXPLORATION: Finding Properties of the Altitudes of a Triangle

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software. Draw any $\triangle ABC$.

a. Construct the perpendicular segment from vertex A to $\overline{BC}$. Label the endpoint D. $\overline{AD}$ is an *altitude* of $\triangle ABC$.



b. Construct the altitudes to the other two sides of $\triangle ABC$. What do you notice?

c. Write a conjecture about the altitudes of a triangle. Test your conjecture by dragging the vertices to change $\triangle ABC$.

Communicate Your Answer

3. What conjectures can you make about the medians and altitudes of a triangle?

4. The length of median $\overline{RU}$ in $\triangle RST$ is 3 inches. The point of concurrency of the three medians of $\triangle RST$ divides $\overline{RU}$ into two segments. What are the lengths of these two segments?

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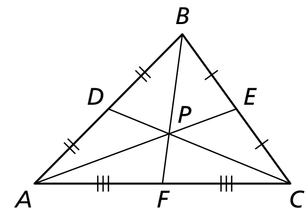
6.2 Practice
For use after Lesson 6.2

Theorems

Centroid Theorem



The centroid of a triangle is two-thirds of the distance from each vertex to the midpoint of the opposite side.

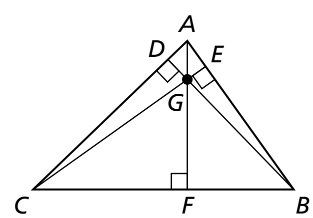
The medians of $\triangle ABC$ meet at point P, and

$AP = \frac{2}{3}AE$, $BP = \frac{2}{3}BF$, and $CP = \frac{2}{3}CD$.

Notes:

Core Concepts

Orthocenter



The lines containing the altitudes of a triangle are concurrent. This point of concurrency is the **orthocenter** of the triangle.

The lines containing $\overline{AF}$, $\overline{BD}$, and $\overline{CE}$ meet at the orthocenter G of $\triangle ABC$.

Notes:

Name____ Date__________

6.2 **Practice (continued)**

Worked-Out Examples

Example #1

Point *P* is the centroid of △*LMN*. Find *PN* and *QP*.

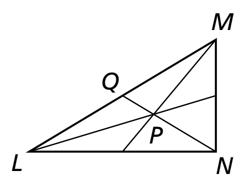
$QN = 30$

$PN = \frac{2}{3}QN$ | $QP = \frac{1}{3} \cdot QN$

$PN = \frac{2}{3} \cdot 30$ | $QP = \frac{1}{3} \cdot 30$

$PN = \frac{60}{3} = 20$ | $QP = 10$

$PN = 20$ units | $QP = 10$ units



Example #2

Point D is the centroid of △*ABC*. Find *CD* and *CE*.

$DE = 15$

$DE = \frac{1}{3} \cdot CE$

$15 = \frac{1}{3} \cdot CE$

$3 \cdot 15 = 3 \cdot \frac{1}{3} \cdot CE$

$45 = CE$

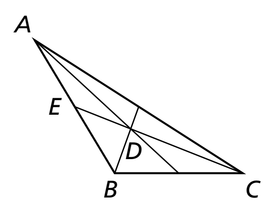
$CE = 45$ units

$CD = \frac{2}{3} \cdot CE$

$CD = \frac{2}{3} \cdot 45$

$CD = \frac{90}{3} = 30$

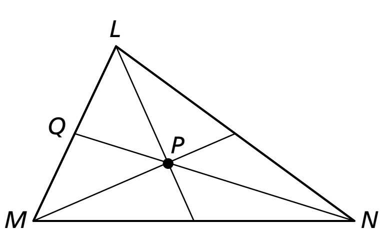
$CD = 30$ units



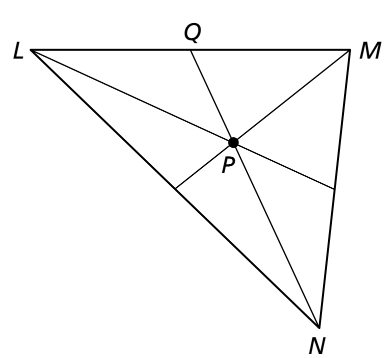
Practice A

In Exercises 1–3, point *P* is the centroid of △*LMN*. Find *PN* and *QP*.

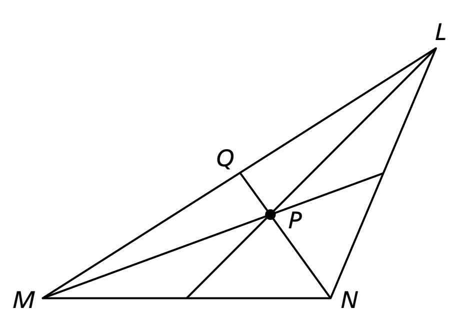
1. $QN = 33$



2. $QN = 45$



3. $QN = 39$



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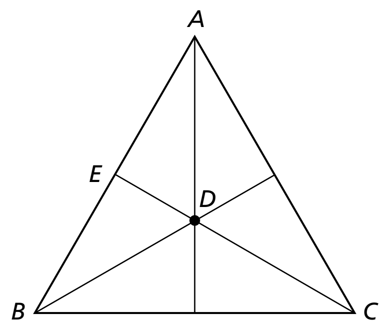
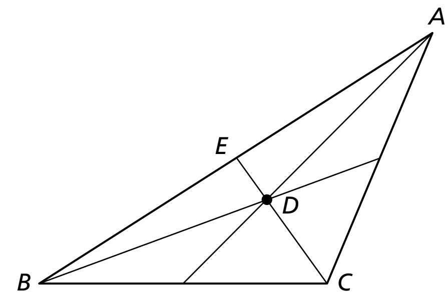
6.2 Practice (continued)

In Exercises 4 and 5, point *D* is the centroid of $\triangle ABC$. Find *CD* and *CE*.

4. $DE = 7$

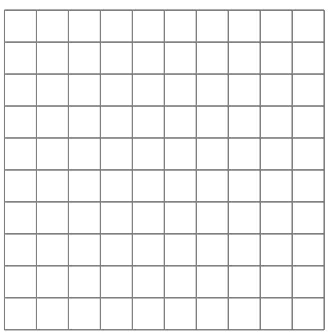


5. $DE = 12$

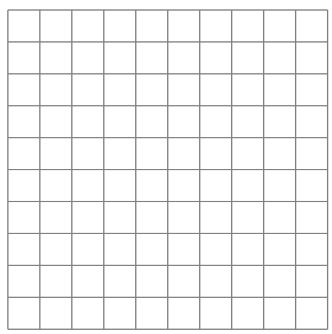


In Exercises 6–8, find the coordinates of the centroid of the triangle with the given vertices.

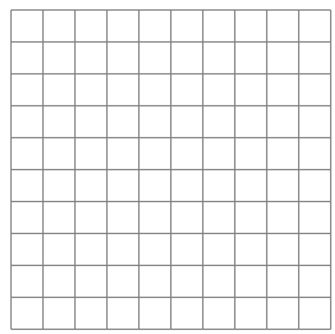
6. $A(-2, -1), B(1, 8), C(4, -1)$



7. $D(-5, 4), E(-3, -2), F(-1, 4)$

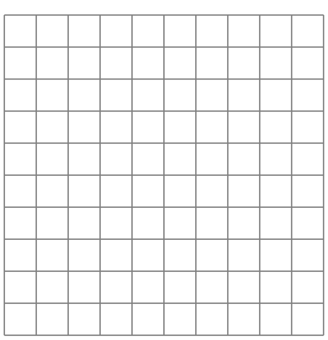


8. $J(8, 7), K(20, 5), L(8, 3)$

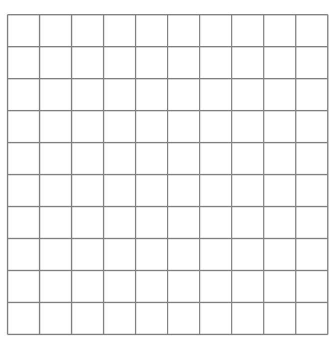


In Exercises 9–11, tell whether the orthocenter is *inside, on,* or *outside* the triangle. Then find the coordinates of the orthocenter.

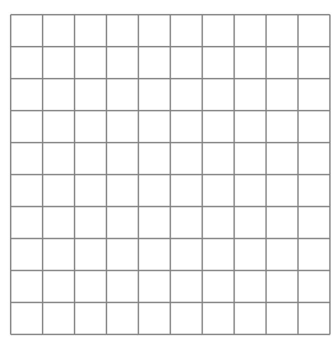
9. $X(3, 6), Y(3, 0), Z(11, 0)$



10. $L(-4, -4), M(1, 1), N(6, -4)$



11. $P(3, 4), Q(11, 4), R(9, -2)$



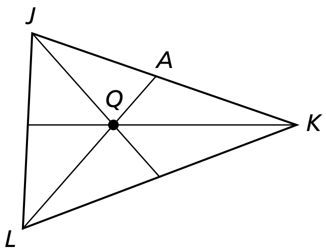
Name ____ Date _________

Practice B

In Exercises 1–3, point *Q* is the centroid of $\triangle JKL$. Use the given information to find the indicated segment lengths.

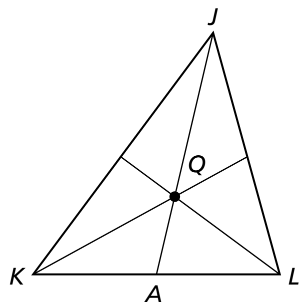
1. $AQ = 21$

Find QL and AL.



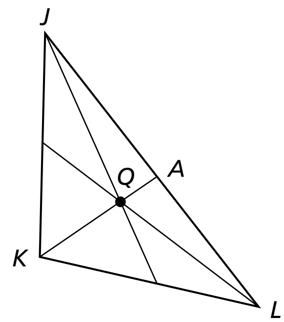
2. $JA = 72$

Find JQ and QA.



3. $KQ = 10$

Find QA and KA.



4. Find the coordinates of the centroid of the triangle with the vertices $A(-6, 8)$, $B(-3, 1)$, and $C(0, 3)$.

In Exercises 5 and 6, tell whether the orthocenter is *inside*, *on*, or *outside* the triangle. Then find the coordinates of the orthocenter.

5. $Q(-1, 5)$, $R(4, 3)$, $S(-1, -2)$

6. $L(4, 6)$, $M(-3, 2)$, $N(-2, -6)$

7. Given two vertices and the centroid of a triangle, how many possible locations are there for the third vertex? Explain your reasoning.

8. Given two vertices and the orthocenter of a triangle, how many possible locations are there for the third vertex? Explain your reasoning.

9. The centroid of a triangle is at $(2, -1)$ and vertices at $(3, -5)$ and $(-7, -4)$. Find the third vertex of the triangle.

10. The orthocenter of a triangle is at the origin, and two of the vertices of the triangle are at $(-5, 0)$ and $(3, 4)$. Find the third vertex of the triangle.

11. Your friend claims that it is possible to draw an equilateral triangle for which the circumcenter, incenter, centroid, and orthocenter are not all the same point. Do you agree? Explain your reasoning.

12. Your friend claims that when the median from one vertex of a triangle is the same as the altitude from the same vertex, the median divides the triangle into two congruent triangles. Do you agree? Explain your reasoning.

13. Can the circumcenter and the incenter of an obtuse triangle be the same point? Explain.