

6.5**Proving That a Quadrilateral Is a Parallelogram**

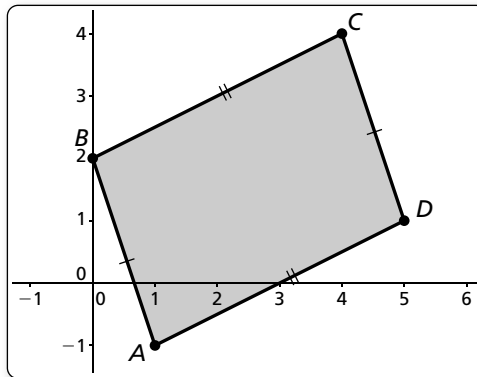
For use with Exploration 6.5

Essential Question How can you prove that a quadrilateral is a parallelogram?

1 EXPLORATION: Proving That a Quadrilateral Is a Parallelogram

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

**Sample**

Points

$A(1, -1)$

$B(0, 2)$

$C(4, 4)$

$D(5, 1)$

Segments

$AB = 3.16$

$BC = 4.47$

$CD = 3.16$

$DA = 4.47$

- Construct any quadrilateral $ABCD$ whose opposite sides are congruent.
- Is the quadrilateral a parallelogram? Justify your answer.
- Repeat parts (a) and (b) for several other quadrilaterals. Then write a conjecture based on your results.
- Write the converse of your conjecture. Is the converse true? Explain.

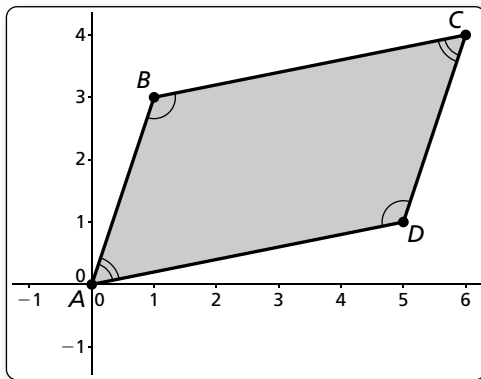
6.5 Proving That a Quadrilateral Is a Parallelogram (continued)

2 EXPLORATION: Proving That a Quadrilateral Is a Parallelogram

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

- a. Construct any quadrilateral $ABCD$ whose opposite angles are congruent.
- b. Is the quadrilateral a parallelogram? Justify your answer.



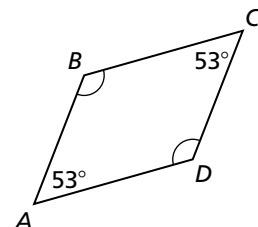
Sample

Points	Angles
$A(0, 0)$	$\angle A = 60.26^\circ$
$B(1, 3)$	$\angle B = 119.74^\circ$
$C(6, 4)$	$\angle C = 60.26^\circ$
$D(5, 1)$	$\angle D = 119.74^\circ$

- c. Repeat parts (a) and (b) for several other quadrilaterals. Then write a conjecture based on your results.
- d. Write the converse of your conjecture. Is the converse true? Explain.

Communicate Your Answer

- 3. How can you prove that a quadrilateral is a parallelogram?
- 4. Is the quadrilateral at the right a parallelogram? Explain your reasoning.



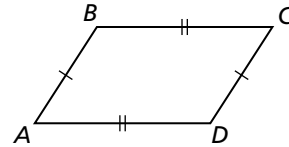
6.5**Practice**

For use after Lesson 6.5

Theorems**Parallelogram Opposite Sides Converse**

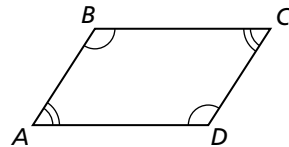
If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$, then $ABCD$ is a parallelogram.

**Notes:****Parallelogram Opposite Angles Converse**

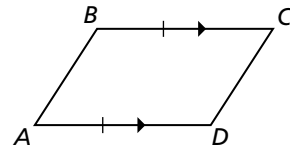
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.

**Notes:****Opposite Sides Parallel and Congruent Theorem**

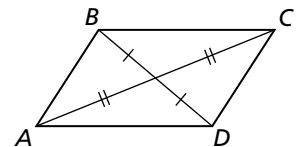
If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.

If $\overline{BC} \parallel \overline{AD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.

**Notes:****Parallelogram Diagonals Converse**

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

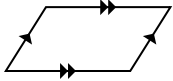
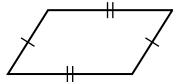
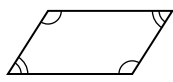
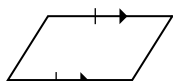
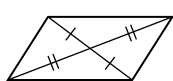
If \overline{BD} and \overline{AC} bisect each other, then $ABCD$ is a parallelogram.

**Notes:**

6.5 Practice (continued)

Core Concepts

Ways to Prove a Quadrilateral Is a Parallelogram

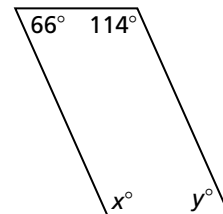
<p>1. Show that both pairs of opposite sides are parallel. (<i>Definition</i>)</p>	
<p>2. Show that both pairs of opposite sides are congruent. (<i>Parallelogram Opposite Sides Converse</i>)</p>	
<p>3. Show that both pairs of opposite angles are congruent. (<i>Parallelogram Opposite Angles Converse</i>)</p>	
<p>4. Show that one pair of opposite sides are congruent and parallel. (<i>Opposite Sides Parallel and Congruent Theorem</i>)</p>	
<p>5. Show that the diagonals bisect each other. (<i>Parallelogram Diagonals Converse</i>)</p>	

Worked-Out Examples

Example #1

Find the values of x and y that make the quadrilateral a parallelogram.

$x = 114$ and $y = 66$ by the Parallelogram Opposite Angles Converse.



Example #2

Find the value of x that makes the quadrilateral a parallelogram.

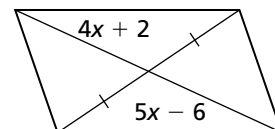
By the Parallelogram Diagonals Converse:

$$4x + 2 = 5x - 6$$

$$2 = x - 6$$

$$8 = x$$

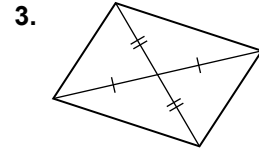
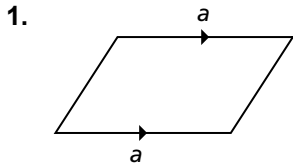
So, $x = 8$.



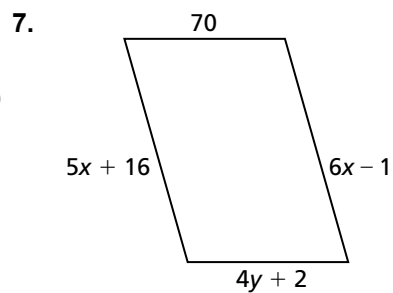
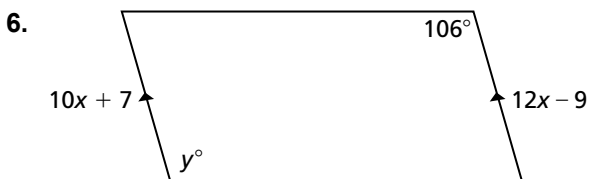
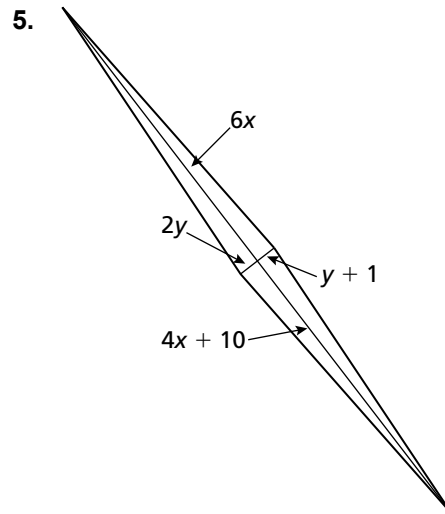
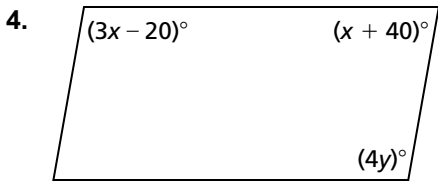
6.5 Practice (continued)

Practice A

In Exercises 1–3, state which theorem you can use to show that the quadrilateral is a parallelogram.

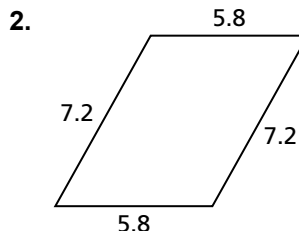
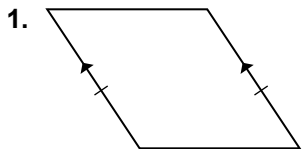


In Exercises 4–7, find the values of x and y that make the quadrilateral a parallelogram.

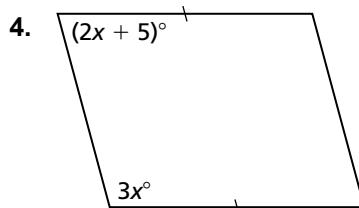
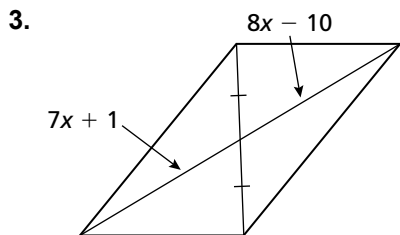


Practice B

In Exercises 1 and 2, state which theorem you can use to show that the quadrilateral is a parallelogram.



In Exercises 3 and 4, find the value of x that makes the quadrilateral a parallelogram.



In Exercises 5 and 6, graph the quadrilateral with the given vertices in a coordinate plane. Then show that the quadrilateral is a parallelogram.

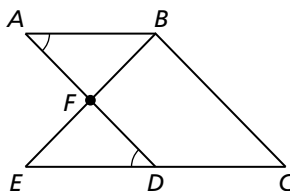
5. $W(-3, -1), X(-3, 4), Y(3, 2), Z(3, -3)$ 6. $A(-4, 0), B(2, 2), C(5, -1), D(-1, -3)$

7. Use the diagram to write a two-column proof.

Given $\angle A \cong \angle FDE$

F is the midpoint of \overline{AD} .

D is the midpoint of \overline{CE} .



Prove $ABCD$ is a parallelogram.

8. A quadrilateral has two pairs of congruent angles.

Can you determine whether the quadrilateral is a parallelogram? Explain your reasoning.

9. An octagon star is shown in the figure on the right.

a. Find $m\angle FCG$, $m\angle BCF$, and $m\angle D$.

b. State which theorem you can use to show that the quadrilateral is a parallelogram.

c. The length of \overline{AB} is three times the length of \overline{AD} . Write an expression for the perimeter of parallelogram $ABCD$ in terms of the variable x .

