Name _____ Date _____

6.6 Properties of Special Parallelograms

For use with Exploration 6.6

Essential Question What are the properties of the diagonals of rectangles, rhombuses, and squares?

1 EXPLORATION: Identifying Special Quadrilaterals

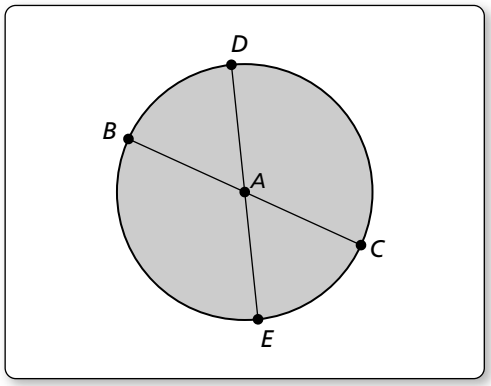
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Draw a circle with center A.

b. Draw two diameters of the circle. Label the endpoints B, C, D, and E.

Sample



c. Draw quadrilateral $BDCE$.

d. Is $BDCE$ a parallelogram? rectangle? rhombus? square? Explain your reasoning.

e. Repeat parts (a) – (d) for several other circles. Write a conjecture based on your results.

Name_____ Date _____

6.6 Properties of Special Parallelograms (continued)

2 EXPLORATION: Identifying Special Quadrilaterals

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

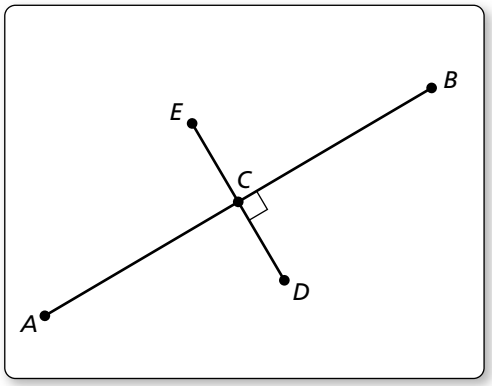
Work with a partner. Use dynamic geometry software.

a. Construct two segments that are perpendicular bisectors of each other. Label the endpoints *A, B, D,* and *E*. Label the intersection *C*.

b. Draw quadrilateral *AEBD*.

c. Is *AEBD* a parallelogram? rectangle? rhombus? square? Explain your reasoning.

Sample

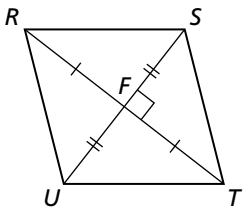


d. Repeat parts (a) – (c) for several other segments. Write a conjecture based on your results.

Communicate Your Answer

3. What are the properties of the diagonals of rectangles, rhombuses, and squares?

4. Is *RSTU* a parallelogram? rectangle? rhombus? square? Explain your reasoning.



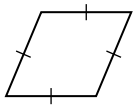
5. What type of quadrilateral has congruent diagonals that bisect each other?

Name _____ Date _____

6.6 Practice
For use after Lesson 6.6

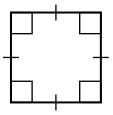
Core Concepts

Rhombuses, Rectangles, and Squares



A **rhombus** is a parallelogram with four congruent sides.

A **rectangle** is a parallelogram with four right angles.



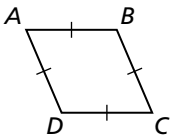
A **square** is a parallelogram with four congruent sides and four right angles.

Notes:

Rhombus Corollary

A quadrilateral is a rhombus if and only if it has four congruent sides.

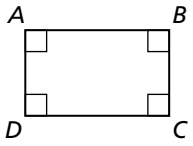
$ABCD$ is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.



Rectangle Corollary

A quadrilateral is a rectangle if and only if it has four right angles.

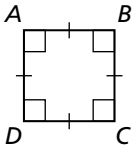
$ABCD$ is a rectangle if and only if $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.



Square Corollary

A quadrilateral is a square if and only if it is a rhombus and a rectangle.

$ABCD$ is a square if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

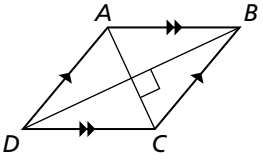


Notes:

Rhombus Diagonals Theorem

A parallelogram is a rhombus if and only if its diagonals are perpendicular.

$\square ABCD$ is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.



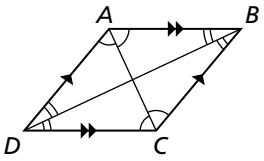
Notes:

Name_____ Date _____

6.6 Practice (continued)

Rhombus Opposite Angles Theorem

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

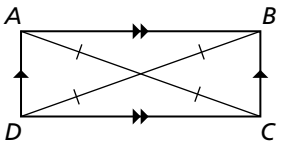


$\square ABCD$ is a rhombus if and only if $\overline{AC}$ bisects $\angle BCD$ and $\angle BAD$, and $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$.

Notes:

Rectangle Diagonals Theorem

A parallelogram is a rectangle if and only if its diagonals are congruent.



$\square ABCD$ is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.

Notes:

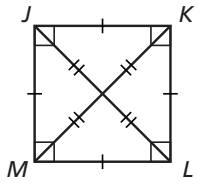
Worked-Out Examples

Example #1

For any rhombus JKLM, decide whether the statement is always or sometimes true. Draw a diagram and explain your reasoning.

$\overline{JL} \cong \overline{KM}$

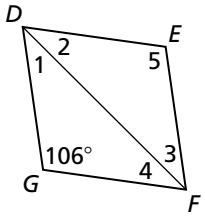
$\overline{JL}$ is sometimes congruent to $\overline{KM}$. Some rhombuses are squares.



Example #2

Find the measures of the numbered angles in rhombus DEFG.

By the Parallelogram Consecutive Angles Theorem, $m\angle EDG = 180° - 106° = 74°$. So, by the Rhombus Opposite Angles Theorem, $m\angle 1 = m\angle 2 = 37°$. By the definition of a parallelogram, $\overline{DE} \parallel \overline{GF}$. So, $m\angle 3 = 37°$, by the Alternate Interior Angles Theorem. By the Rhombus Opposite Angles Theorem, $m\angle 4 = 37°$. By the Parallelogram Opposite Angles Theorem, $m\angle 5 = 106°$.



Name _____ Date _____

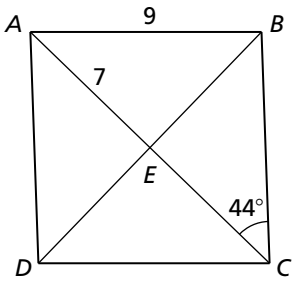
6.6 Practice (continued)

Practice A

1. For any rhombus *MNOP,* decide whether the statement $\overline{MO} \cong \overline{NP}$ is *always* or *sometimes* true. Draw a diagram and explain your reasoning.

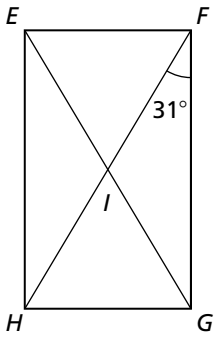
2. For any rectangle *PQRS,* decide whether the statement $\angle PQS \cong \angle RSQ$ is *always* or *sometimes* true. Draw a diagram and explain your reasoning.

In Exercises 3–5, the diagonals of rhombus *ABCD* intersect at *E*. Given that $m\angle BCA = 44°$, $AB = 9$, and $AE = 7$, find the indicated measure.



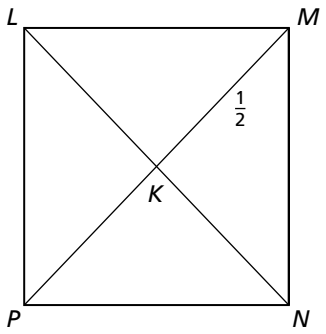
3. BC **4.** AC **5.** $m\angle ADC$

In Exercises 6–8, the diagonals of rectangle *EFGH* intersect at *I*. Given that $m\angle HFG = 31°$ and $EG = 17$, find the indicated measure.



6. $m\angle FHG$ **7.** HF **8.** $m\angle EFH$

In Exercises 9–11, the diagonals of square *LMNP* intersect at *K*. Given that $MK = \frac{1}{2}$, find the indicated measure.



9. PK **10.** $m\angle PKN$ **11.** $m\angle MNK$

Name _____ Date _____

Practice B

In Exercises 1 and 2, decide whether quadrilateral *JKLM* is a rectangle, a rhombus, or a square. Give all names that apply. Explain your reasoning.

1. $J(3, 5), K(7, 6), L(6, 2), M(2, 1)$

2. $J(-4, -1), K(-1, 5), L(5, 2), M(2, -4)$

In Exercises 3–7, the diagonals of rhombus *ABCD* intersect at *M*. Given that $m\angle MAB = 53°$, $MB = 16$, and $AM = 12$, find the indicated measure.

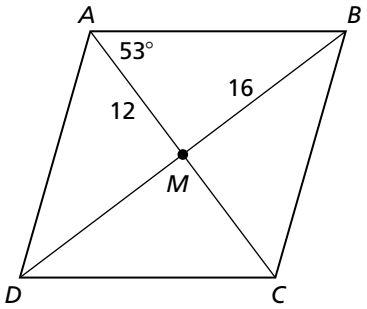
3. $m\angle AMD$

4. $m\angle ADM$

5. $m\angle ACD$

6. DM

7. AC

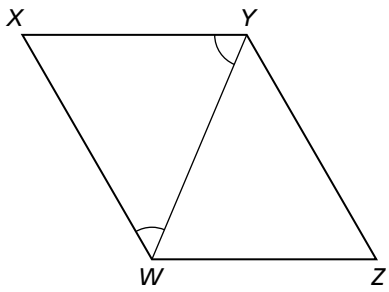


8. Find the point of intersection of the diagonals of the rhombus with vertices $(-1, 2)$, $(3, 4)$, $(5, 8)$, and $(1, 6)$.

9. Use the figure to write a two-column proof.

Given $WXYZ$ is a parallelogram.
$\angle XWY \cong \angle XYW$

Prove $WXYZ$ is a rhombus.



10. Your friend claims that you can transform every rhombus into a square using a similarity transformation. Is your friend correct? Explain your reasoning.

11. A quadrilateral has four congruent angles. Is the quadrilateral a parallelogram? Explain your reasoning.

12. A quadrilateral has two consecutive right angles. If the quadrilateral is not a rectangle, can it still be a parallelogram? Explain your reasoning.

13. Will a diagonal of a rectangle ever divide the rectangle into two isosceles triangles? Explain your reasoning.