Name_____ Date _____

6.7 Properties of Trapezoids and Kites

For use with Exploration 6.7

Essential Question What are some properties of trapezoids and kites?

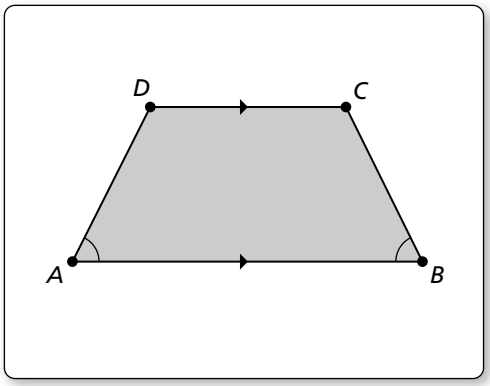
1 EXPLORATION: Making a Conjecture about Trapezoids

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct a trapezoid whose base angles are congruent. Explain your process.

Sample



b. Is the trapezoid isosceles? Justify your answer.

c. Repeat parts (a) and (b) for several other trapezoids. Write a conjecture based on your results.

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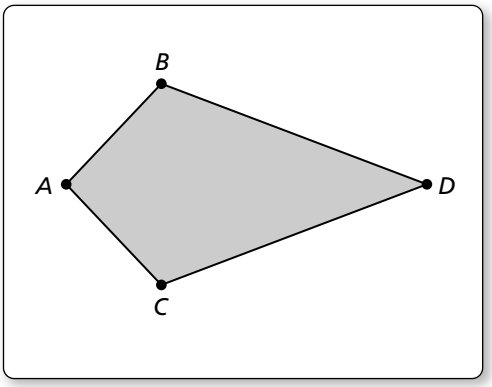
2 EXPLORATION: Discovering a Property of Kites

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct a kite. Explain your process.

Sample



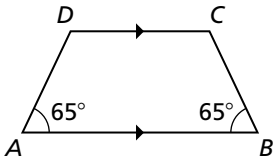
b. Measure the angles of the kite. What do you observe?

c. Repeat parts (a) and (b) for several other kites. Write a conjecture based on your results.

Communicate Your Answer

3. What are some properties of trapezoids and kites?

4. Is the trapezoid at the right isosceles? Explain.



5. A quadrilateral has angle measures of 70°, 70°, 110°, and 110°. Is the quadrilateral a kite? Explain.

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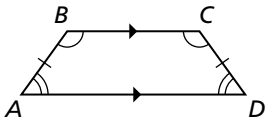
6.7 Practice
For use after Lesson 6.7

Theorems

Isosceles Trapezoid Base Angles Theorem

If a trapezoid is isosceles, then each pair of base angles is congruent.

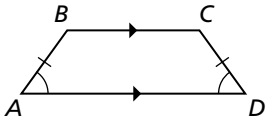
If trapezoid $ABCD$ is isosceles, then $\angle A \cong \angle D$ and $\angle B \cong \angle C$.



Isosceles Trapezoid Base Angles Converse

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

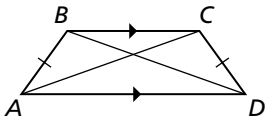
If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid $ABCD$ is isosceles.



Isosceles Trapezoid Diagonals Theorem

A trapezoid is isosceles if and only if its diagonals are congruent.

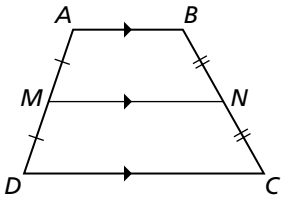
Trapezoid $ABCD$ is isosceles if and only if $\overline{AC} \cong \overline{BD}$.



Trapezoid Midsegment Theorem

The midsegment of a trapezoid is parallel to each base, and its length is one-half the sum of the lengths of the bases.

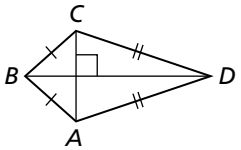
If $\overline{MN}$ is the midsegment of trapezoid $ABCD$, then $\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$, and $MN = \frac{1}{2}(AB + CD)$.



Kite Diagonals Theorem

If a quadrilateral is a kite, then its diagonals are perpendicular.

If quadrilateral $ABCD$ is a kite, then $\overline{AC} \perp \overline{BD}$.



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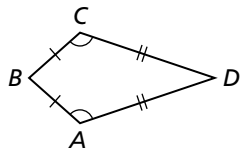
6.7 Practice (continued)

Kite Opposite Angles Theorem

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.



If quadrilateral $ABCD$ is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A \cong \angle C$ and $\angle B \ncong \angle D$.

Notes:

Worked-Out Examples

Example #1

Show that the quadrilateral with the given vertices is a trapezoid. Then decide whether it is isosceles.

$H(1, 9)$, $J(4, 2)$, $K(5, 2)$, $L(8, 9)$

Slope of $\overline{HL} = \frac{9 - 9}{8 - 1} = \frac{0}{7} = 0$

Slope of $\overline{JK} = \frac{2 - 2}{5 - 4} = \frac{0}{1} = 0$

Slope of $\overline{HJ} = \frac{9 - 2}{1 - 4} = -\frac{7}{3}$

Slope of $\overline{LK} = \frac{9 - 2}{8 - 5} = \frac{7}{3}$

$$HJ = \sqrt{(1 - 4)^2 + (9 - 2)^2} = \sqrt{(-3)^2 + (7)^2}$$
$$= \sqrt{9 + 49} = \sqrt{58}$$
$$LK = \sqrt{(8 - 5)^2 + (9 - 2)^2} = \sqrt{(3)^2 + (7)^2}$$
$$= \sqrt{9 + 49} = \sqrt{58}$$

The slope of $\overline{HL}$ equals the slope of $\overline{JK}$, and the slope of $\overline{HJ}$ is not equal to the slope of $\overline{LK}$. Because $HJKL$ has exactly one pair of parallel sides, it is a trapezoid. Also, $HJ = LK$. So, $HJKL$ is an isosceles trapezoid.

Example #2

Find $m\angle G$

$$m\angle G + m\angle H + m\angle E + m\angle F = 360°$$
$$x° + 100° + x° + 40° = 360°$$
$$2x + 140 = 360$$
$$2x = 220$$
$$x = 110$$

So, $m\angle G = 110°$.

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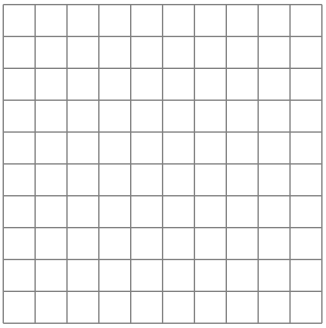
6.7 Practice (continued)

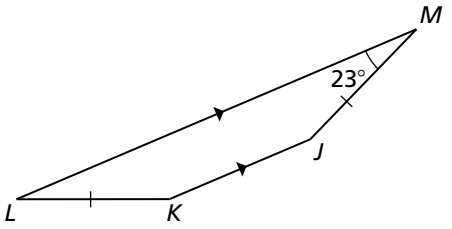
Practice A

1. Show that the quadrilateral with vertices at $Q(0, 3)$, $R(0, 6)$, $S(-6, 0)$, and $T(-3, 0)$ is a trapezoid. Decide whether the trapezoid is isosceles. Then find the length of the midsegment of the trapezoid.



In Exercises 2 and 3, find $m\angle K$ and $m\angle L$.

2.



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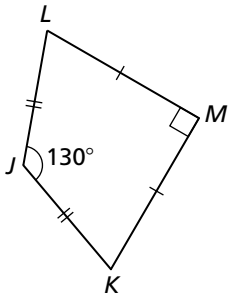
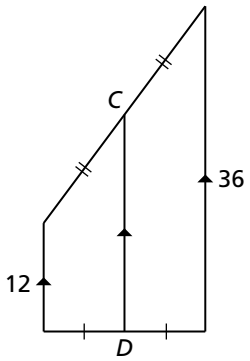
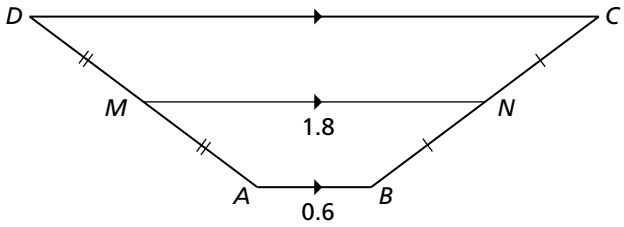


In Exercises 4 and 5, find CD.

4.

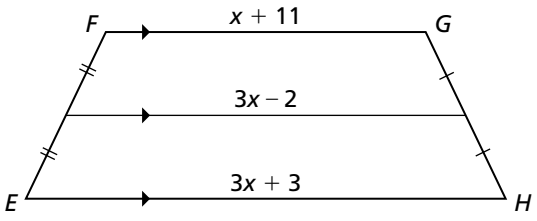


5.



In Exercises 6 and 7, find the value of x.

6.



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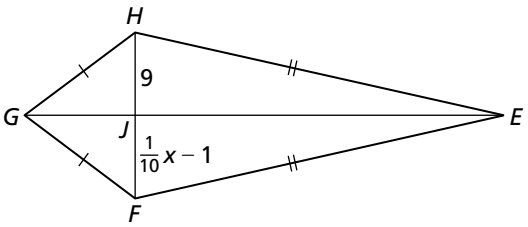


Name_____ Date _____

Practice B

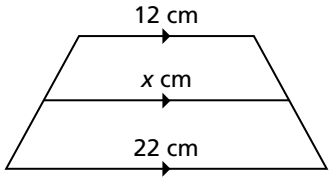
In Exercises 1 and 2, show that the quadrilateral with the given vertices is a trapezoid. Then decide whether it is isosceles.

1. $T(-1, -2)$, $U(-1, 3)$, $V(3, 4)$, $W(3, -3)$

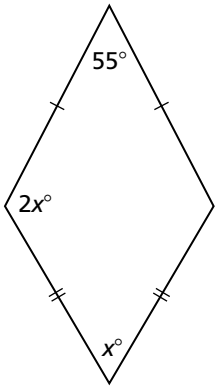
2. $P(0, 0)$, $Q(2, 4)$, $R(5, 4)$, $S(5, 0)$

In Exercises 3 and 4, find the value of *x*.

3.

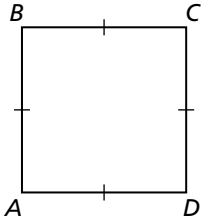


4.



In Exercises 5 and 6, give the most specific name for the quadrilateral. Explain your reasoning.

5.



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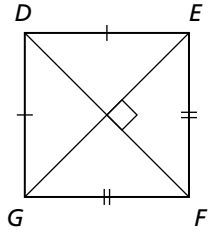
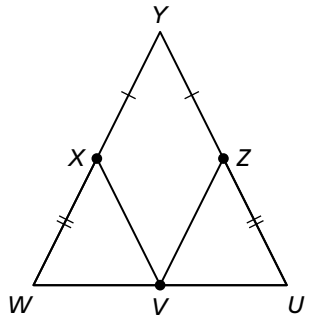


7. Use the diagram to write a two-column proof.

Given $VXYZ$ is a kite.
$\overline{XY} \cong \overline{YZ}$, $\overline{WX} \cong \overline{UZ}$

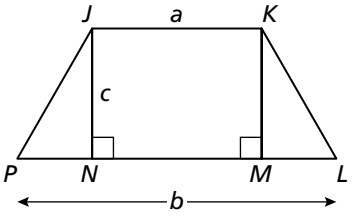
Prove $\triangle WXV \cong \triangle UZV$



8. Three vertices of a trapezoid are given by $(3, -6)$, $(3, -2)$, and $(6, -8)$. Find the fourth vertex such that the trapezoid is an isosceles trapezoid.

9. Is it possible to have a concave kite? Explain your reasoning.

10. The diagram shows isosceles trapezoid $JKLP$ with base lengths a and b, and height c.



a. Explain how you know $JKMN$ is a rectangle. Write the area of $JKMN$.

b. Write the formula for the area of $\triangle JNP$.

c. Write and simplify the formula for the area of trapezoid $JKLP$.