

**7.3****Using Chords**

For use with Exploration 7.3

**Essential Question** What are two ways to determine when a chord is a diameter of a circle?

**1** **EXPLORATION:** Drawing Diameters

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

**Work with a partner.** Use dynamic geometry software to construct a circle of radius 5 with center at the origin. Draw a diameter that has the given point as an endpoint. Explain how you know that the chord you drew is a diameter.

a.  $(4, 3)$

b.  $(0, 5)$

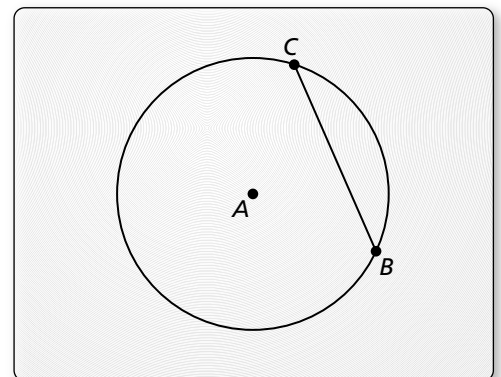
c.  $(-3, 4)$

d.  $(-5, 0)$

**2** **EXPLORATION:** Writing a Conjecture about Chords

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

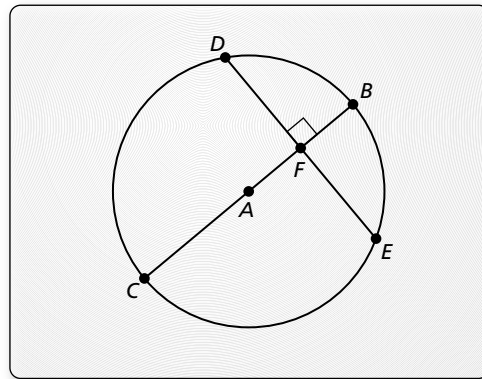
**Work with a partner.** Use dynamic geometry software to construct a chord  $\overline{BC}$  of a circle  $A$ . Construct a chord on the perpendicular bisector of  $\overline{BC}$ . What do you notice? Change the original chord and the circle several times. Are your results always the same? Use your results to write a conjecture.



**7.3 Using Chords (continued)****3 EXPLORATION: A Chord Perpendicular to a Diameter**

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

**Work with a partner.** Use dynamic geometry software to construct a diameter  $\overline{BC}$  of a circle  $A$ . Then construct a chord  $\overline{DE}$  perpendicular to  $\overline{BC}$  at point  $F$ . Find the lengths  $DF$  and  $EF$ . What do you notice? Change the chord perpendicular to  $\overline{BC}$  and the circle several times. Do you always get the same results? Write a conjecture about a chord that is perpendicular to a diameter of a circle.

**Communicate Your Answer**

4. What are two ways to determine when a chord is a diameter of a circle?

**7.3**

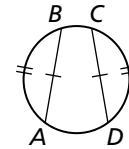
**Practice**  
For use after Lesson 7.3

**Theorems**

**Congruent Corresponding Chords Theorem**

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

**Notes:**

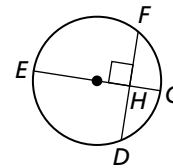


$\widehat{AB} \cong \widehat{CD}$  if and only if  $\overline{AB} \cong \overline{CD}$ .

**Perpendicular Chord Bisector Theorem**

If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

**Notes:**

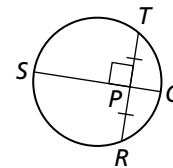


If  $\overline{EG}$  is a diameter and  $\overline{EG} \perp \overline{DF}$ ,  
then  $\overline{HD} \cong \overline{HF}$  and  $\widehat{GD} \cong \widehat{GF}$ .

**Perpendicular Chord Bisector Converse**

If one chord of a circle is a perpendicular bisector of another chord, then the first chord is a diameter.

**Notes:**

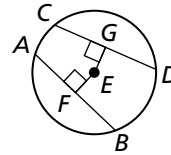


If  $\overline{QS}$  is a perpendicular bisector of  $\overline{TR}$ ,  
then  $\overline{QS}$  is a diameter of the circle.

**7.3 Practice (continued)**

**Equidistant Chords Theorem**

In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.



**Notes:**

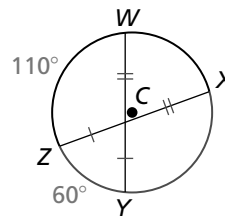
$\overline{AB} \cong \overline{CD}$  if and only if  $EF = EG$ .

**Worked-Out Examples**

**Example #1**

Find the measure of  $\widehat{XYZ}$  in  $\odot C$ .

By segment addition,  $WY = WC + CY$  and  $ZX = ZC + CX$ . Because  $WC = CX$  and  $ZC = YC$ ,  $WY = ZX$ . By arc addition,  $m\widehat{XYZ} = m\widehat{XY} + m\widehat{YZ}$ ,  $m\widehat{WZY} = m\widehat{WZ} + m\widehat{ZY}$ , and  $m\widehat{XYZ} = m\widehat{XWZ}$ . By substitution,  $m\widehat{XY} + m\widehat{YZ} = m\widehat{WZ} + m\widehat{ZY}$ ,  $m\widehat{XY} + 60^\circ = 110^\circ + 60^\circ$ ,  $m\widehat{XY} + 60^\circ = 170^\circ$ ,  $m\widehat{XY} = 110^\circ$ . Therefore,  $m\widehat{XYZ} = 110^\circ + 60^\circ = 170^\circ$ .

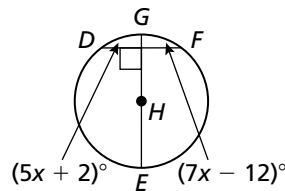


**Example #2**

Find the value of  $x$ .

By the Perpendicular Chord Bisector Theorem:

$$\begin{aligned} 5x + 2 &= 7x - 12 \\ -2x + 2 &= -12 \\ -2x &= -14 \\ x &= 7 \end{aligned}$$

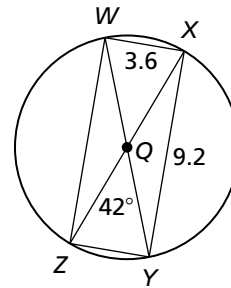


**7.3 Practice (continued)**

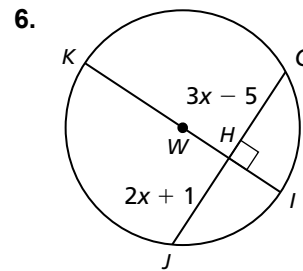
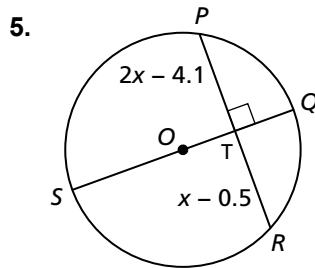
**Practice A**

In Exercises 1–4, find the measure of the arc or chord in  $\odot Q$ .

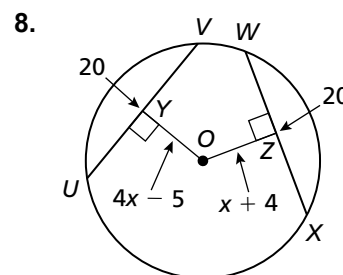
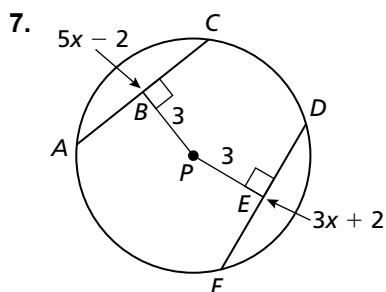
- 1.  $m\widehat{WX}$
- 2.  $YZ$
- 3.  $WZ$
- 4.  $m\widehat{XY}$



In Exercises 5 and 6, find the value of  $x$ .



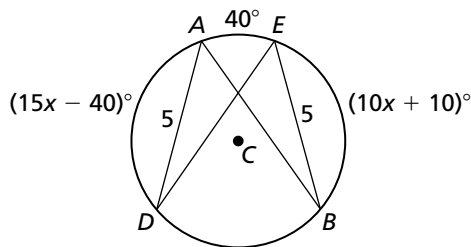
In Exercises 7 and 8, find the radius of the circle.



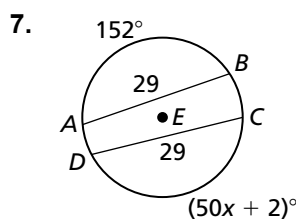
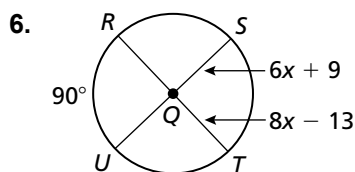
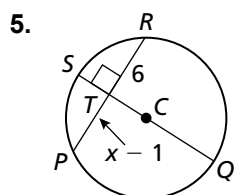
## Practice B

In Exercises 1–4, use the diagram of  $\odot C$ .

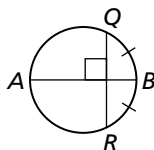
1. Explain why  $\widehat{AD} \cong \widehat{BE}$ .
2. Find the value of  $x$ .
3. Find  $m\widehat{AD}$  and  $m\widehat{BE}$ .
4. Find  $m\widehat{BD}$ .



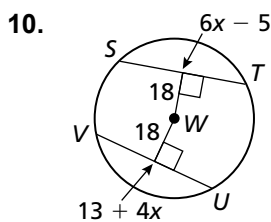
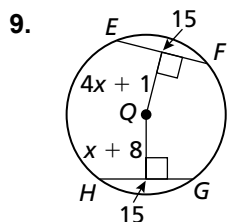
In Exercises 5–7, find the value of  $x$ .



8. Determine whether  $\overline{AB}$  is a diameter of the circle. Explain your reasoning.



In Exercises 9 and 10, find the radius of  $\odot C$ .

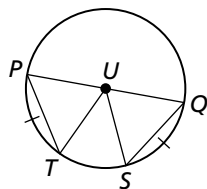


11. Copy and complete the proof.

**Given**  $\overline{PQ}$  is a diameter of  $\odot U$ .

$$\widehat{PT} \cong \widehat{QS}$$

**Prove**  $\triangle PUT \cong \triangle QUS$



**STATEMENTS**

1.  $\overline{PQ}$  is a diameter of  $\odot U$ .
2. \_\_\_\_\_
3.  $\overline{UP} \cong \overline{UQ} \cong \overline{UT} \cong \overline{US}$
4.  $\triangle PUT \cong \triangle QUS$

**REASONS**

1. \_\_\_\_\_
2. Congruent Corresponding Chords Theorem
3. \_\_\_\_\_
4. \_\_\_\_\_

12. Briefly explain what other congruence theorem you could use to prove that  $\triangle PUT \cong \triangle QUS$  in Exercise 11.