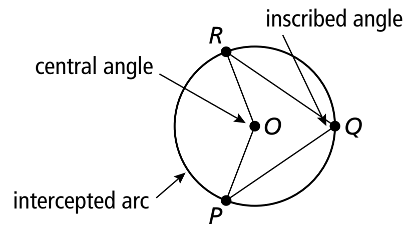
Name ____ Date __________

7.4 Inscribed Angles and Polygons

For use with Exploration 7.4

Essential Question How are inscribed angles related to their intercepted arcs? How are the angles of an inscribed quadrilateral related to each other?

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. An arc that lies between two lines, rays, or segments is called an **intercepted arc**. Recall that a polygon is an inscribed polygon when all its vertices lie on a circle.



1 EXPLORATION: Inscribed Angles and Central Angles

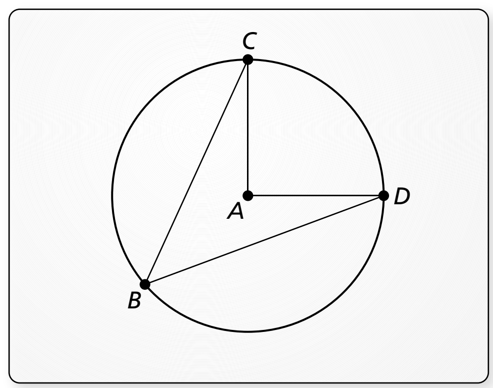
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct an inscribed angle in a circle. Then construct the corresponding central angle.

b. Measure both angles. How is the inscribed angle related to its intercepted arc?

Sample



c. Repeat parts (a) and (b) several times. Record your results in the following table. Write a conjecture about how an inscribed angle is related to its intercepted arc.

Measure of Inscribed Angle	Measure of Central Angle	Relationship

Name____ Date__________

2 EXPLORATION: A Quadrilateral with Inscribed Angles

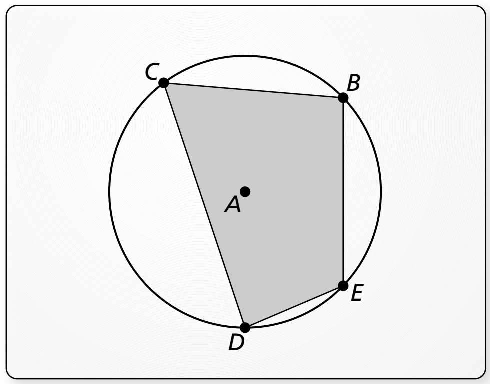
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct a quadrilateral with each vertex on a circle.

b. Measure all four angles. What relationships do you notice?

Sample



c. Repeat parts (a) and (b) several times. Record your results in the following table. Then write a conjecture that summarizes the data.

Angle Measure 1	Angle Measure 2	Angle Measure 3	Angle Measure 4

Communicate Your Answer

3. How are inscribed angles related to their intercepted arcs? How are the angles of an inscribed quadrilateral related to each other?

4. Quadrilateral *EFGH* is inscribed in $\odot C$, and $m\angle E = 80°$. What is $m\angle G$? Explain.

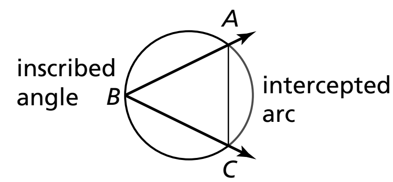
Name ____ Date __________

7.4 Practice
For use after Lesson 7.4

Core Concepts

Inscribed Angle and Intercepted Arc

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. An arc that lies between two lines, rays, or segments is called an **intercepted arc**. If the endpoints of a chord or arc lie on the sides of an inscribed angle, then the chord or arc is said to **subtend** the angle.



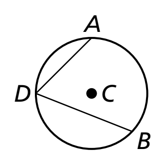
$\angle B$ intercepts $\overset{\frown}{AC}$.
$\overset{\frown}{AC}$ subtends $\angle B$.
$\overline{AC}$ subtends $\angle B$.

Notes:

Theorems

Measure of an Inscribed Angle Theorem

The measure of an inscribed angle is one-half the measure of its intercepted arc.

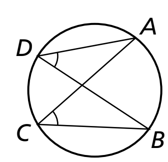


$m\angle ADB = \frac{1}{2}m\overset{\frown}{AB}$

Notes:

Inscribed Angles of a Circle Theorem

If two inscribed angles of a circle intercept the same arc, then the angles are congruent.



$\angle ADB \cong \angle ACB$

Notes:

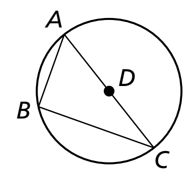
Name______________________________ Date__________

7.4 Practice (continued)

Theorems

Inscribed Right Triangle Theorem

If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.

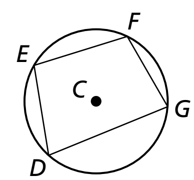


$m\angle ABC = 90°$ if and only if $\overline{AC}$ is a diameter of the circle.

Notes:

Inscribed Quadrilateral Theorem

A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.



D, E, F, and G lie on $\odot C$ if and only if $m\angle D + m\angle F = m\angle E + m\angle G = 180°$.

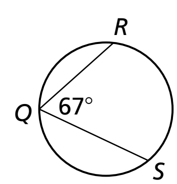
Notes:

Worked-Out Examples

Example #1

Find the indicated measure.

$m\overset{\frown}{RS}$



$m\overset{\frown}{RS} = 2 \bullet m\angle RQS$

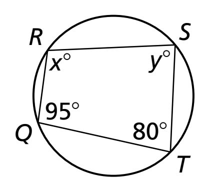
$m\overset{\frown}{RS} = 2 \bullet 67°$

$m\overset{\frown}{RS} = 134°$

Example #2

Find the value of each variable.

By the Inscribed Quadrilateral Theorem:



$m\angle QRS + m\angle STQ = 180°$

$x° + 80° = 180°$

$x = 100$

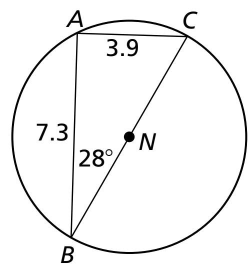
$m\angle RST + m\angle TQR = 180°$

$y° + 95° = 180°$

$x = 85$

Practice A

In Exercises 1–5, use the diagram to find the indicated measure.



1. $m\angle A$

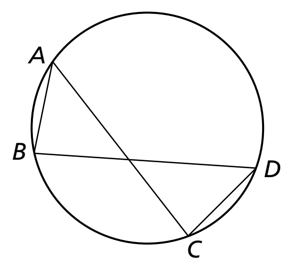
2. $m\angle C$

3. BC

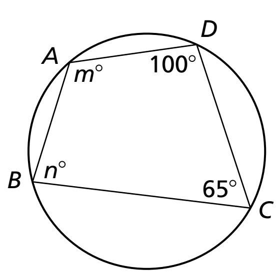
4. $m\overset{\frown}{AC}$

5. $m\overset{\frown}{AB}$

6. Name two pairs of congruent angles.



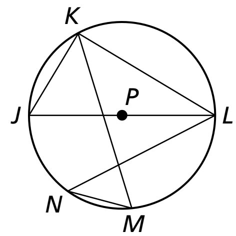
7. Find the value of each variable.



Name ______________________________ Date __________

Practice B

In Exercises 1–8, find the measure of the indicated arc or angle in $\odot P$ given $m\overset{\frown}{LM} = 84°$ and $m\overset{\frown}{KN} = 116°$.



1. $m\angle JKL$

2. $m\angle MKL$

3. $m\angle KMN$

4. $m\angle JKM$

5. $m\angle KLN$

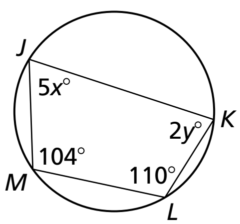
6. $m\angle LNM$

7. $m\overset{\frown}{MJ}$

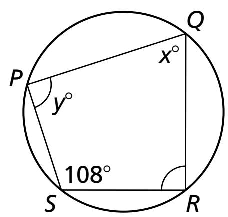
8. $m\overset{\frown}{LKJ}$

In Exercises 9–11, find the value of each variable.

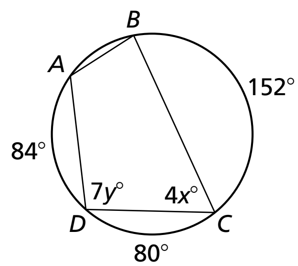
9.



10.



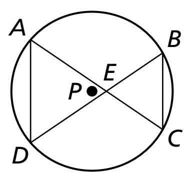
11.



12. Copy and complete the proof.

Given $\odot P$

Prove $\triangle AED \sim \triangle BEC$



STATEMENTS	REASONS
1. $\odot P$	1. Given
2. ______________	2. Vertical Angles Congruence Theorem
3. $\angle CAD \cong \angle DBC$	3. ______________
4. $\triangle AED \sim \triangle BEC$	4. ______________

13. Your friend claims that the angles $\angle ADB$ and $\angle BCA$ could be used in Step 3 of Exercise 12. Is your friend correct? Explain your reasoning.

14. Determine whether $\overline{AB}$ is a diameter of the circle. Explain your reasoning.