Name____ Date__________

7.5 Angle Relationships in Circles

For use with Exploration 7.5

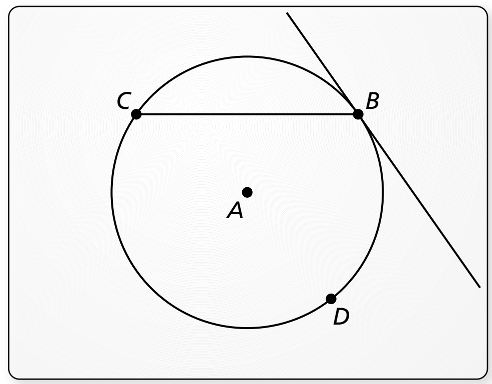
Essential Question When a chord intersects a tangent line or another chord, what relationships exist among the angles and arcs formed?

1 EXPLORATION: Angles Formed by a Chord and Tangent Line

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

Sample



a. Construct a chord in a circle. At one of the endpoints of the chord, construct a tangent line to the circle.

b. Find the measures of the two angles formed by the chord and the tangent line.

c. Find the measures of the two circular arcs determined by the chord.

d. Repeat parts (a)–(c) several times. Record your results in the following table. Then write a conjecture that summarizes the data.

Angle Measure 1	Angle Measure 2	Circular Arc Measure 1	Circular Arc Measure 2

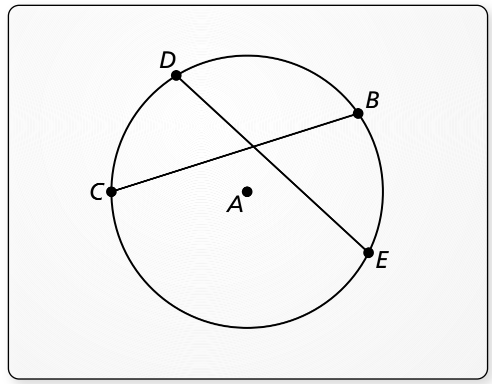
7.5 Angle Relationships in Circles (continued)

2 EXPLORATION: Angles Formed by Intersecting Chords

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

Sample



a. Construct two chords that intersect inside a circle.

b. Find the measure of one of the angles formed by the intersecting chords.

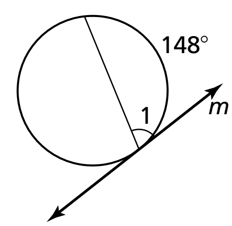
c. Find the measures of the arcs intercepted by the angle in part (b) and its vertical angle. What do you observe?

d. Repeat parts (a)–(c) several times. Record your results in the following table. Then write a conjecture that summarizes the data.

Angle Measure	Arc Measures	Observations

Communicate Your Answer

3. When a chord intersects a tangent line or another chord, what relationships exist among the angles and arcs formed?



4. Line m is tangent to the circle in the figure at the right. Find the measure of $\angle 1$.

5. Two chords intersect inside a circle to form a pair of vertical angles with measures of 55°. Find the sum of the measures of the arcs intercepted by the two angles.

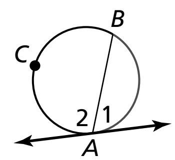
Name____ Date __________

7.5 Practice
For use after Lesson 7.5

Theorems

Tangent and Intersected Chord Theorem

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one-half the measure of its intercepted arc.



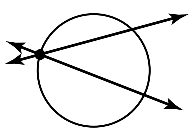
$m\angle 1 = \frac{1}{2}m\overset{\frown}{AB}$ $\quad m\angle 2 = \frac{1}{2}m\overset{\frown}{BCA}$

Notes:

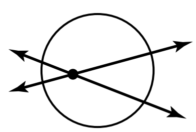
Core Concepts

Intersecting Lines and Circles

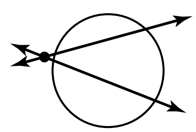
If two nonparallel lines intersect a circle, there are three places where the lines can intersect.



on the circle



inside the circle



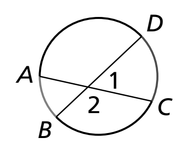
outside the circle

Notes:

Theorems

Angles Inside the Circle Theorem

If two chords intersect *inside* a circle, then the measure of each angle is one-half the *sum* of the measures of the arcs intercepted by the angle and its vertical angle.



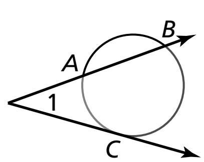
$m\angle 1 = \frac{1}{2}(m\overset{\frown}{DC} + m\overset{\frown}{AB})$,

$m\angle 2 = \frac{1}{2}(m\overset{\frown}{AD} + m\overset{\frown}{BC})$

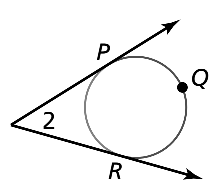
Notes:

Angles Outside the Circle Theorem

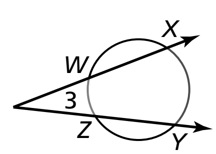
If a tangent and a secant, two tangents, or two secants intersect *outside* a circle, then the measure of the angle formed is one-half the *difference* of the measures of the intercepted arcs.



$m\angle 1 = \frac{1}{2}(m\overset{\frown}{BC} - m\overset{\frown}{AC})$



$m\angle 2 = \frac{1}{2}(m\overset{\frown}{PQR} - m\overset{\frown}{PR})$



$m\angle 3 = \frac{1}{2}(m\overset{\frown}{XY} - m\overset{\frown}{WZ})$

Notes:

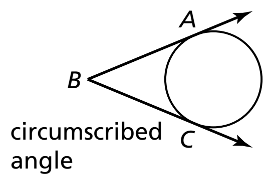
Name ______________________________ Date __________

7.5 **Practice (continued)**

Core Concepts

Circumscribed Angle

A **circumscribed angle** is an angle whose sides are tangent to a circle.

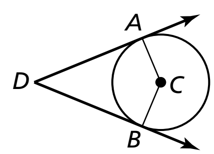


Notes:

Theorems

Circumscribed Angle Theorem

The measure of a circumscribed angle is equal to 180° minus the measure of the central angle that intercepts the same arc.



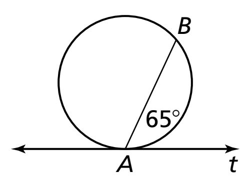
$m\angle ADB = 180° - m\angle ACB$

Notes:

Worked-Out Examples

Example #1

Line *t* is tangent to the circle. Find the indicated measure.

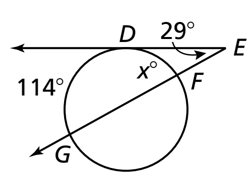


$m\widehat{AB}$

$m\widehat{AB} = 2 \cdot 65° = 130°$

Example #2

Find the value of *x*.



$$m\angle E = \frac{1}{2}\left(m\widehat{GD} - m\widehat{DF}\right)$$

$$29° = \frac{1}{2}(114° - x°)$$

$$58 = 114 - x$$

$$-56 = -x$$

$$56 = x$$

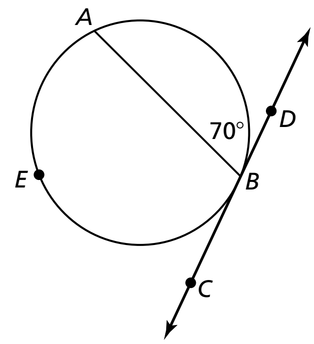
Practice A

In Exercises 1–3, $\overleftrightarrow{CD}$ is tangent to the circle. Find the indicated measure.

1. $m\angle ABC$

2. $m\widehat{AB}$

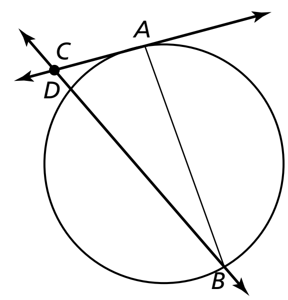
3. $m\widehat{AEB}$



In Exercises 4 and 5, $m\widehat{ADB} = 220°$ and $m\angle B = 21°$. Find the indicated measure.

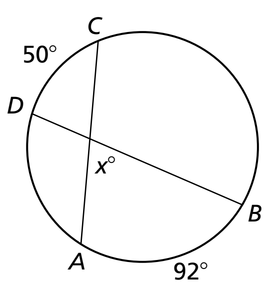
4. $m\widehat{AB}$

5. $m\angle ACB$



In Exercises 6–9, find the value of *x*.

6.



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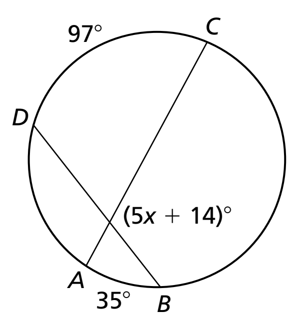

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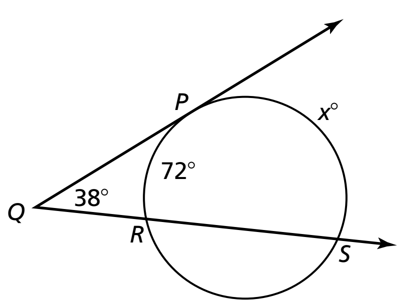

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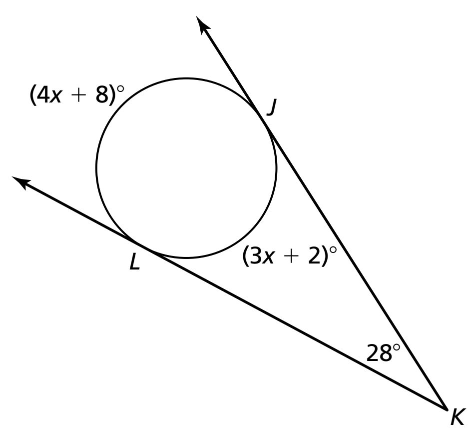

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Practice B

In Exercises 1–6, use the diagram to find the measure of the angle.

1. $m\angle CAF$

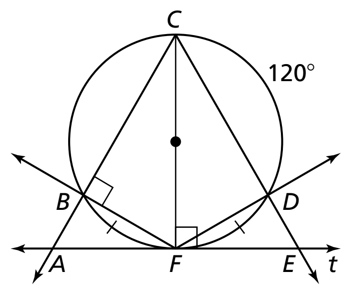
2. $m\angle AFB$

3. $m\angle CEF$

4. $m\angle CFB$

5. $m\angle DCF$

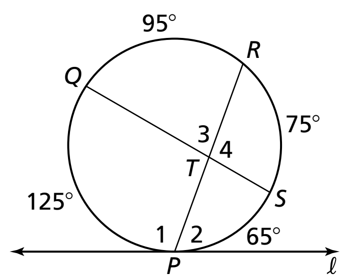
6. $m\angle BCD$



7. In the diagram, ℓ is tangent to the circle at P. Which relationship is *not* true? Explain.

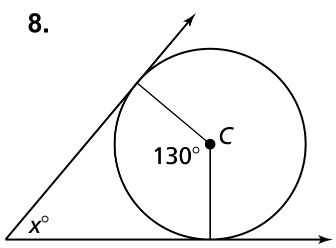
A. $m\angle 1 = 110°$ **B.** $m\angle 2 = 70°$

C. $m\angle 3 = 80°$ **D.** $m\angle 4 = 90°$



In Exercises 8–10, find the value of *x*.

8.



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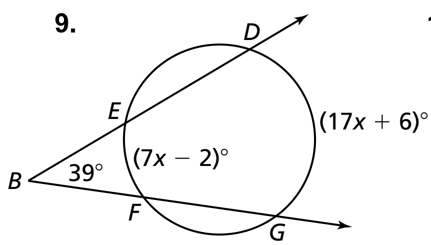
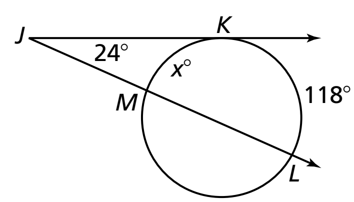


10.

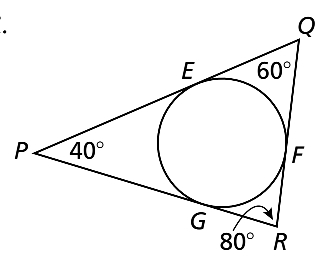


11. In the diagram, the circle is inscribed in $\triangle PQR$.

a. Find $m\overset{\frown}{EF}$.

b. Find $m\overset{\frown}{FG}$.

c. Find $m\overset{\frown}{GE}$.



12. A plane at point U is flying at an altitude of 7 miles above Earth. What is the measure of arc TV that represents the part of Earth you can see from the airplane? The radius of Earth is about 4000 miles.