

7.6

Segment Relationships in Circles

For use with Exploration 7.6

Essential Question What relationships exist among the segments formed by two intersecting chords or among segments of two secants that intersect outside a circle?

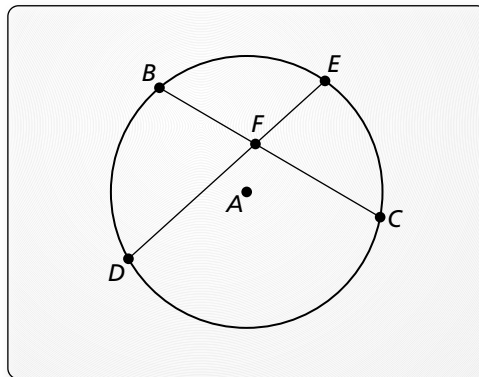
1 EXPLORATION: Segments Formed by Two Intersecting Chords

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

- a. Construct two chords \overline{BC} and \overline{DE} that intersect in the interior of a circle at point F .

Sample



- b. Find the segment lengths BF , CF , DF , and EF and complete the table. What do you observe?

BF	CF	$BF \cdot CF$
DF	EF	$DF \cdot EF$

- c. Repeat parts (a) and (b) several times. Write a conjecture about your results.

7.6 Segment Relationships in Circles (continued)

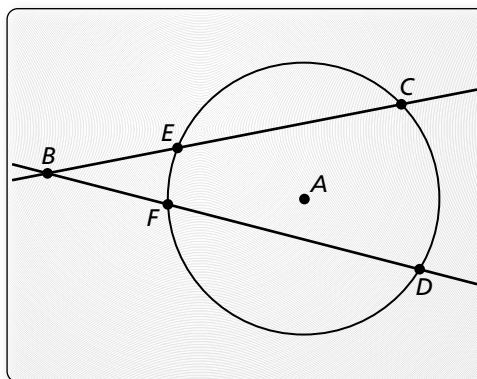
2 EXPLORATION: Secants Intersecting Outside a Circle

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

- a. Construct two secants \overline{BC} and \overline{BD} that intersect at a point B outside a circle, as shown.
- b. Find the segment lengths BE , BC , BF , and BD , and complete the table. What do you observe?

Sample

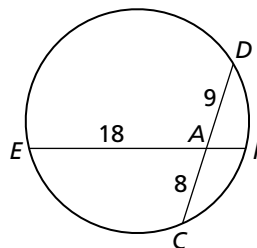


BE	BC	$BE \cdot BC$
BF	BD	$BF \cdot BD$

- c. Repeat parts (a) and (b) several times. Write a conjecture about your results.

Communicate Your Answer

- 3. What relationships exist among the segments formed by two intersecting chords or among segments of two secants that intersect outside a circle?
- 4. Find the segment length AF in the figure at the right.



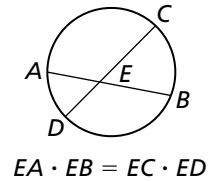
7.6

Practice
For use after Lesson 7.6

Theorems

Segments of Chords Theorem

If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

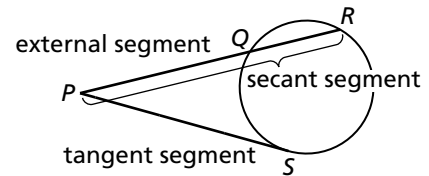


Notes:

Core Concepts

Tangent Segment and Secant Segment

A **tangent segment** is a segment that is tangent to a circle at an endpoint. A **secant segment** is a segment that contains a chord of a circle and has exactly one endpoint outside the circle. The part of a secant segment that is outside the circle is called an **external segment**.



\overline{PS} is a tangent segment.
 \overline{PR} is a secant segment.
 \overline{PQ} is the external segment of \overline{PR} .

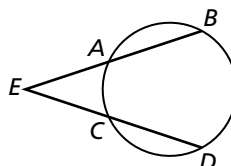
Notes:

7.6 Practice (continued)

Theorems

Segments of Secants Theorem

If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.

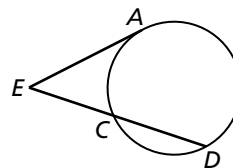


$$EA \cdot EB = EC \cdot ED$$

Notes:

Segments of Secants and Tangents Theorem

If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.



$$EA^2 = EC \cdot ED$$

Notes:

Worked-Out Examples

Example #1

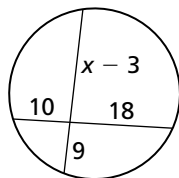
Find the value of x .

$$9(x - 3) = 10 \cdot 18$$

$$9x - 27 = 180$$

$$9x = 207$$

$$x = 23$$



Example #2

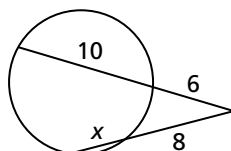
Find the value of x .

$$6 \cdot 16 = 8(8 + x)$$

$$96 = 64 + 8x$$

$$32 = 8x$$

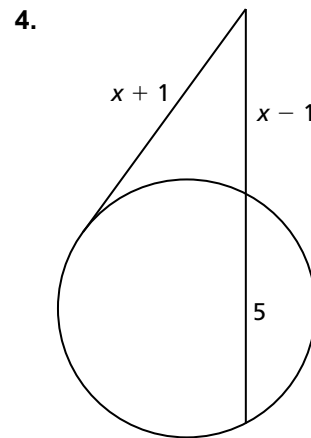
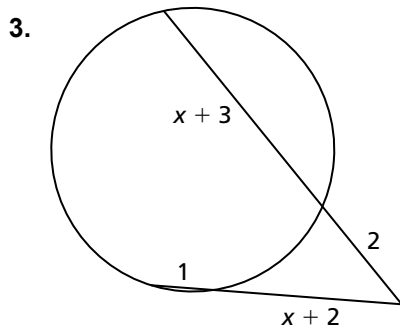
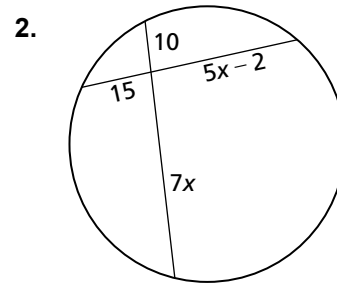
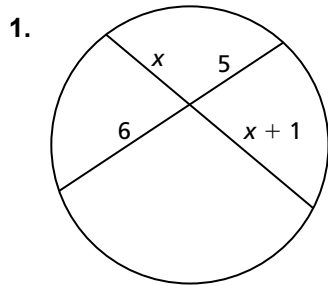
$$x = 4$$



7.6 Practice (continued)

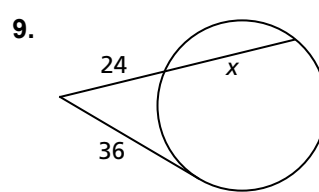
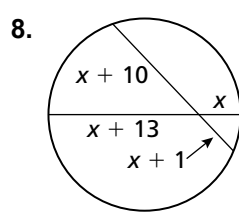
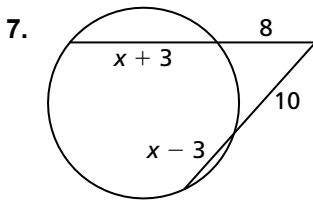
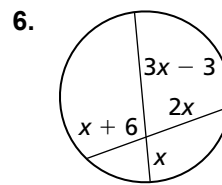
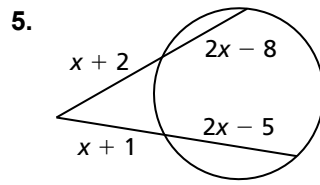
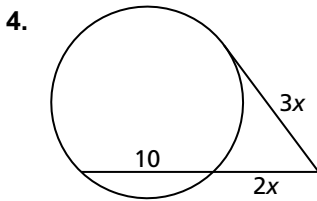
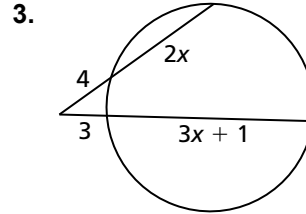
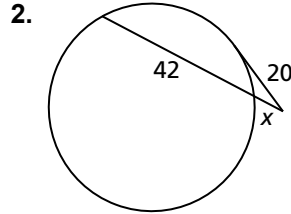
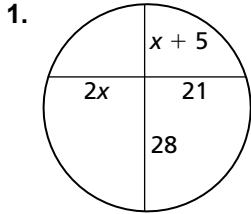
Practice A

In Exercises 1–4, find the value of x .

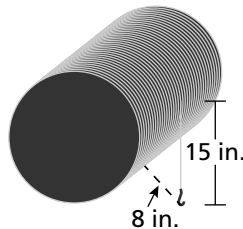


Practice B

In Exercises 1–9, find the value of x .



10. A large industrial winch is shown. There are 15 inches of cable hanging free off of the spool and the distance from the end of the cable to the spool is 8 inches. What is the diameter of the spool?



11. The diagram shows a cross-section of a large storm drain pipe with a small amount of standing water. The distance across the surface of the water is 48 inches and the water is 4.25 inches deep at its deepest point. What is the diameter of the storm drain pipe?

