CHAPTER 8

Circumference, Area, and Volume

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10 cm

Find the missing dimension.

3. A rectangle has an area of 25 square inches and a length of 10 inches. What is the width of the rectangle?

4. A triangle has an area of 32 square centimeters and a base of 8 centimeters. What is the height of the triangle?

1

8.1

Circumference and Arc Length For use with Exploration 8.1

Essential Question How can you find the length of a circular arc?

EXPLORATION: Finding the Length of a Circular Arc

Work with a partner. Find the length of each gray circular arc.

a. entire circle



b. one-fourth of a circle



c. one-third of a circle



d. five-eighths of a circle



8.1 Circumference and Arc Length (continued)



EXPLORATION: Using Arc Length

Work with a partner. The rider is attempting to stop with the front tire of the motorcycle in the painted rectangular box for a skills test. The front tire makes exactly one-half additional revolution before stopping. The diameter of the tire is 25 inches. Is the front tire still in contact with the painted box? Explain.



Communicate Your Answer

3. How can you find the length of a circular arc?

4. A motorcycle tire has a diameter of 24 inches. Approximately how many inches does the motorcycle travel when its front tire makes three-fourths of a revolution?

Core Concepts

Circumference of a Circle

The circumference *C* of a circle is $C = \pi d$ or $C = 2\pi r$, where *d* is the diameter of the circle and *r* is the radius of the circle.

Notes:



Arc Length

In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360° .

$$\frac{\text{Arc length of } \widehat{AB}}{2\pi r} = \frac{m\widehat{AB}}{360^{\circ}}, \text{ or}$$
Arc length of $\widehat{AB} = \frac{m\widehat{AB}}{360^{\circ}} \cdot 2\pi r$

Notes:



8.1 Practice (continued)

Converting Between Degrees and Radians

Degrees to radians

Multiply degree measure by

$$\frac{2\pi \text{ radians}}{360^\circ}$$
, or $\frac{\pi \text{ radians}}{180^\circ}$.

Notes:

Worked-Out Examples

Example #1

Find the indicated measure.

Arc length of \widehat{AB}

Arc length of $\widehat{AB} = \frac{45^{\circ}}{360^{\circ}} \cdot 8\pi$ = $\frac{1}{8} \cdot 8\pi$ ≈ 3.14

The arc length of \widehat{AB} is about 3.14 feet.



Convert the angle measure.

Convert 70° to radians.

$$70^\circ \cdot \left(\frac{\pi}{180^\circ}\right) = \frac{70^\circ \pi}{180^\circ} = \frac{7\pi}{18}$$
 radian

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Radians to degrees

Multiply radian measure by

 $\frac{360^{\circ}}{2\pi \text{ radians}}$, or $\frac{180^{\circ}}{\pi \text{ radians}}$.

C P A B ft B 8.1 **Practice** (continued)

Practice A

In Exercises 1–5, find the indicated measure.

- 1. diameter of a circle with a circumference of 10 inches
- 2. circumference of a circle with a radius of 3 centimeters
- **3.** radius of a circle with a circumference of 8 feet
- 4. circumference of a circle with a diameter of 2.4 meters
- **5.** arc length of \widehat{AC}



In Exercises 6 and 7, convert the angle measure.

6. Convert 60° to radians.

7. Convert
$$\frac{5\pi}{6}$$
 radians to degrees.

Practice B

In Exercises 1 and 2, find the indicated measure.

- 1. exact diameter of a circle with a circumference of 36 meters
- 2. exact circumference of a circle with a radius of 5.4 feet
- **3.** Find the circumference of a circle inscribed in a square with a side length of 14 centimeters.

In Exercises 4–9, use the diagram of circle *D* with $\angle EDF \cong \angle FDG$ to find the indicated measure.

- **4.** $m \widehat{EFG}$
- 5. $m \widehat{EHG}$
- **6.** arc length of \widehat{EFG}
- **7.** arc length of \widehat{EHG}
- 8. $m \widehat{EHF}$
- **9.** arc length of \widehat{FEG}

In Exercises 10–12, find the indicated measure.

10. *mAB*

11. circumference of $\bigcirc F$

Ε







Н

80°

7 m

F

G



14. Convert $\frac{5\pi}{6}$ radians to degrees.

In Exercises 13 and 14, convert the angle measure.

- **13.** Convert 105° to radians.
- **15.** The chain of a bicycle travels along the front and rear sprockets, as shown in the figure. The circumferences of the rear sprocket and the front sprocket are 12 inches and 20 inches, respectively.
 - **a.** How long is the chain? Round your answer to the nearest tenth.
 - **b.** On a chain, the teeth are spaced in $\frac{1}{2}$ -inch intervals. About how many teeth are there on this chain?

