

8.3

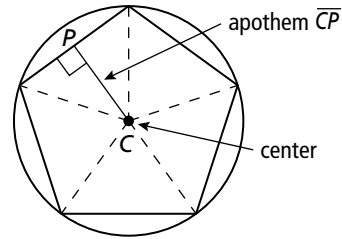
Areas of Polygons

For use with Exploration 8.3

Essential Question How can you find the area of a regular polygon?

The **center of a regular polygon** is the center of its circumscribed circle.

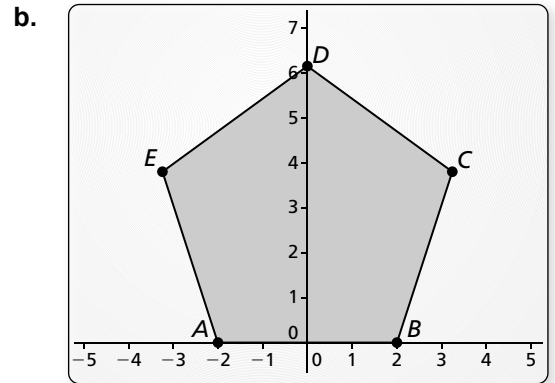
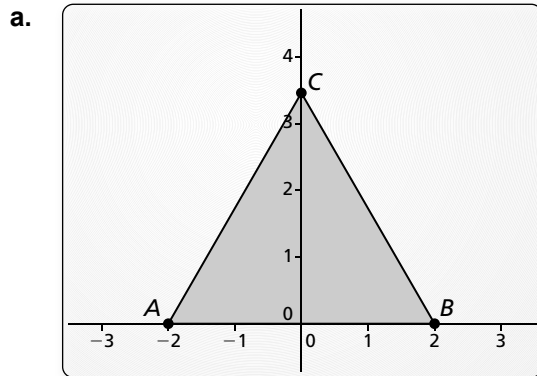
The distance from the center to any side of a regular polygon is called the **apothem of a regular polygon**.



1 EXPLORATION: Finding the Area of a Regular Polygon

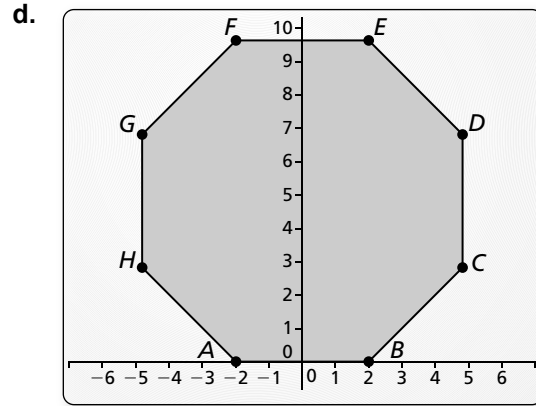
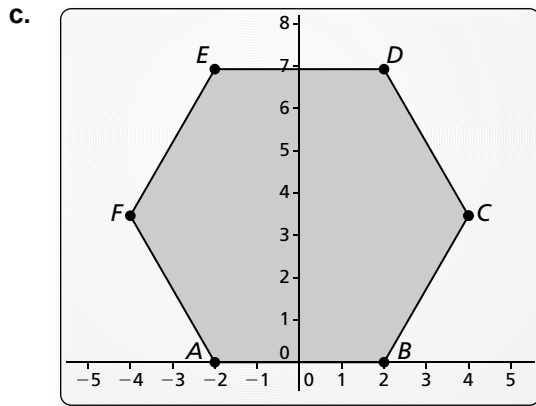
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to construct each regular polygon with side lengths of 4, as shown. Find the apothem and use it to find the area of the polygon. Describe the steps that you used.



8.3 Areas of Polygons (continued)

1 EXPLORATION: Finding the Area of a Regular Polygon (continued)



2 EXPLORATION: Writing a Formula for Area

Work with a partner. Generalize the steps you used in Exploration 1 to develop a formula for the area of a regular polygon.

Communicate Your Answer

3. How can you find the area of a regular polygon?

4. Regular pentagon $ABCDE$ has side lengths of 6 meters and an apothem of approximately 4.13 meters. Find the area of $ABCDE$.

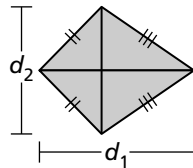
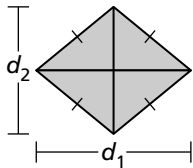
8.3

Practice
For use after Lesson 8.3

Core Concepts

Area of a Rhombus or Kite

The area of a rhombus or kite with diagonals d_1 and d_2 is $\frac{1}{2}d_1d_2$.

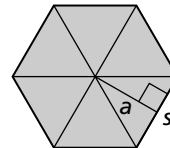


Notes:

Area of a Regular Polygon

The area of a regular n -gon with side length s is one-half the product of the apothem a and the perimeter P .

$$A = \frac{1}{2}aP, \text{ or } A = \frac{1}{2}a \cdot ns$$



Notes:

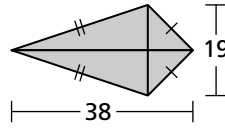
8.3 Practice (continued)

Worked-Out Examples

Example #1

Find the area of the kite or rhombus.

$$\begin{aligned}
 A &= \frac{1}{2} d_1 d_2 \\
 &= \frac{1}{2} \cdot 38 \cdot 19 \\
 &= 361
 \end{aligned}$$

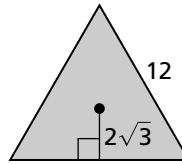


The area of the kite is 361 square units.

Example #2

Find the area of the regular polygon.

$$\begin{aligned}
 A &= \frac{1}{2} a \cdot ns \\
 &= \frac{1}{2} \cdot 2\sqrt{3} \cdot 3 \cdot 12 \\
 &= 36\sqrt{3} \approx 62.35
 \end{aligned}$$

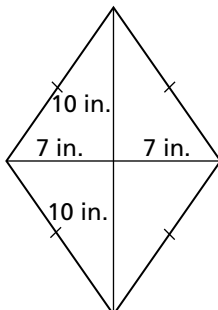


The area of the triangle is about 62.35 square units.

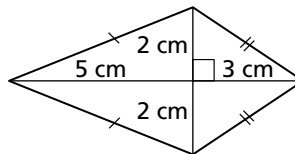
Practice A

In Exercises 1 and 2, find the area of the kite or rhombus.

1.



2.



8.3 Practice (continued)

3. Find the measure of a central angle of a regular polygon with 8 sides.

4. The central angles of a regular polygon are 40° . How many sides does the polygon have?

5. A regular pentagon has a radius of 4 inches and a side length of 3 inches.
 - a. Find the apothem of the pentagon.

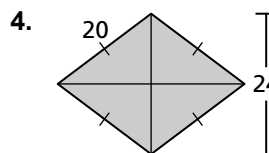
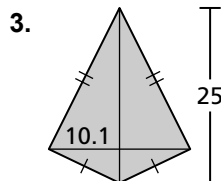
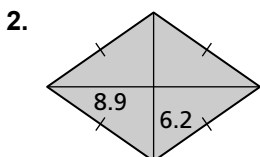
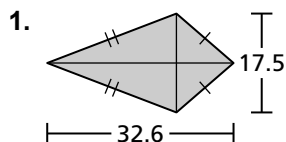
 - b. Find the area of the pentagon.

6. A regular hexagon has an apothem of 10 units.
 - a. Find the radius of the hexagon and the length of one side.

 - b. Find the area of the hexagon.

Practice B

In Exercises 1–4, find the area of the kite or rhombus.



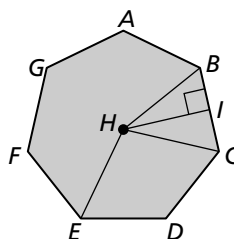
In Exercises 5–8, find the given angle measure for regular heptagon $ABCDEFGH$. Round your answer to the nearest tenth of a degree, if necessary.

5. $m\angle BHC$

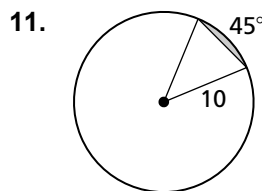
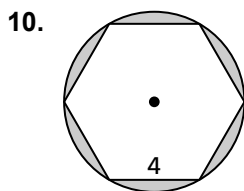
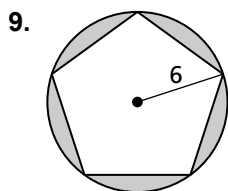
6. $m\angle BHI$

7. $m\angle IBH$

8. $m\angle EHB$



In Exercises 9–11, find the area of the shaded region.



12. The area of a kite is 384 square feet. One diagonal is three times as long as the other diagonal. Find the length of each diagonal.
13. The area of a rhombus is 484 square millimeters. One diagonal is one-half as long as the other diagonal. Find the length of each diagonal.
14. You are laying concrete around a gazebo that is a regular octagon with a radius of 8 feet. The concrete will form a circle that extends 15 feet from the vertices of the octagon.
 - a. Sketch a diagram that represents this situation.
 - b. What is the area of the concrete to the nearest square foot?
15. The perimeter of a regular 11-gon is 16.5 meters. Is this enough information to find the area? If so, find the area and explain your reasoning. If not, explain why not.