CHAPTER 9

Geometric Modeling
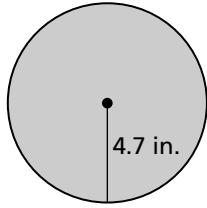
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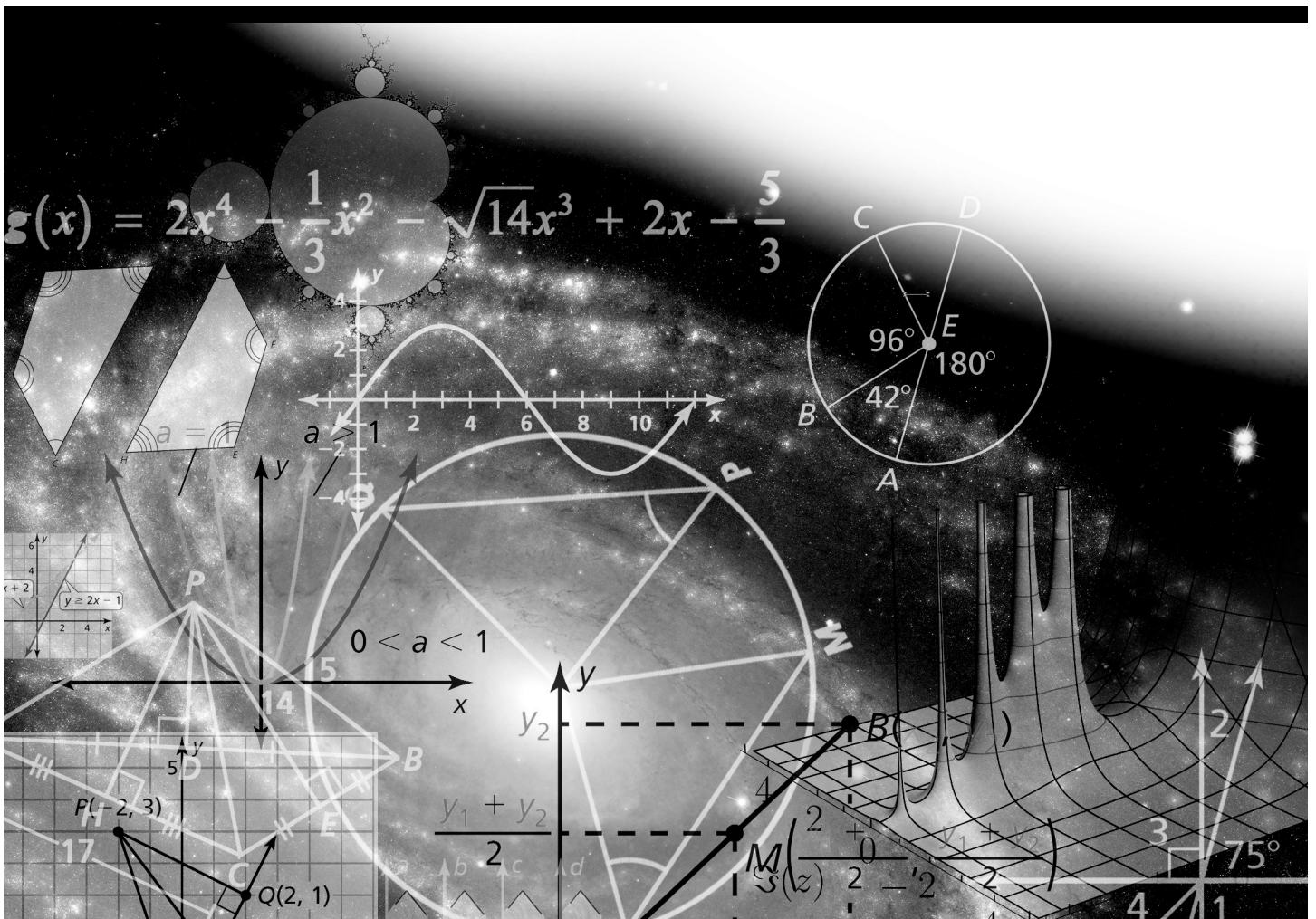
Maintaining Mathematical Proficiency

Find the area of the circle or regular polygon.

1.

4.7 in.
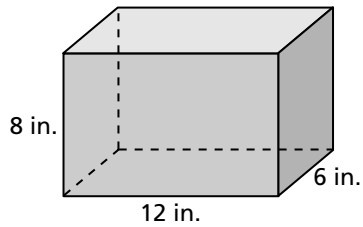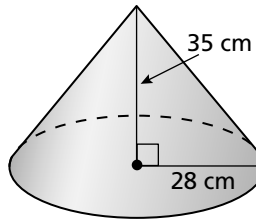
2.

3.7 ft

5 ft
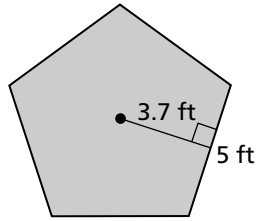
3. a circle with a diameter of 74.6 centimeters

4. a regular hexagon with a perimeter of 42 yards and an apothem of 4.25 yards

5. a circle with a circumference of 24π meters

Find the surface area and volume of the solid.

6.

8 in.

6 in.

12 in.

7.

35 cm

28 cm

8.

58 yd

9.

37 m

23 m
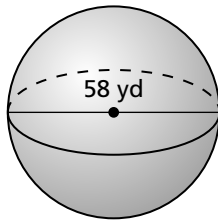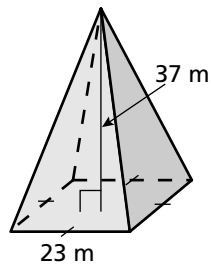
9.1 Modeling with Area
For use with Exploration 9.1

Essential Question   How can you use the population and area of a region to describe how densely the region is populated?

1   EXPLORATION: Exploring Population and Area

Work with a partner. Use the Internet to find the population and land area of each county in California. Then find the number of people per square mile for each county.

a. Mendocino County

b. Lake County

c. Yolo County

d. Napa County

e. Sonoma County

f. Marin County

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| **9.1** | **Modeling with Area** (continued) |

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2 **EXPLORATION:** Analyzing Population and Area

Work with a partner. The six counties in Exploration 1 appear on a map as shown.
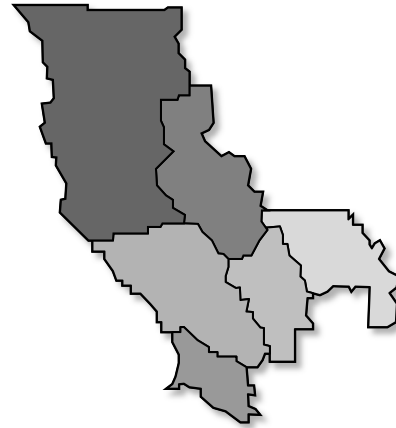
a. Without calculating, how would you expect the number of people per square mile in the entire 6-county region to compare to the values for each individual county in Exploration 1?

b. Use the populations and land areas in Exploration 1 to justify your answer in part (a).

Communicate Your Answer

3. How can you use the population and area of a region to describe how densely the region is populated?

4. Find the population and land area of the county in which you live. How densely populated is your county compared to the counties in Exploration 1?

5. In Exploration 1, the two northern counties are less densely populated than the other four. What factors do you think might influence how densely a region is populated?

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Notes:

Worked-Out Examples

Example #1

Find the indicated measure.

About 650, 000 people live in a circular region with a 6-mile radius. Find the population density in people per square mile.

The area of the region is $\pi 6^2 = 36\pi \approx 113.1$ square miles.

The population density is:

$$\text{Population density} = \frac{\text{number of people}}{\text{area of land}} = \frac{650,000}{113.1}$$

$$\approx 5747.1$$

The population density is about 5747 people per square mile.

Example #2

MODELING WITH MATHEMATICS   A soccer field of length ℓ and width w has a perimeter of 320 yards.

a. Write an expression that represents the area of the scoccer field in terms of ℓ.

b. Use your expression from part (a) to determine the dimensions of the field that maximize the area. What do you notice?

  a. Use what you know about the area and perimeter of the soccer field to find an expression that represents the area of the field.

  The perimeter P of a field of length ℓ and width w is $P = 2\ell + 2w$. So, $320 = 2\ell + 2w$.

$$320 = 2\ell + 2w$$
$$320 - 2\ell = 2w$$
$$\frac{320 - 2\ell}{2} = \frac{2w}{2}$$
$$160 - \ell = w$$

  Solving for w gives $w = 160 - \ell$.

  So, an expression that represents the area of the soccer field is $A = \ell w = \ell(160 - \ell)$.

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9.1 **Practice (continued)**

b. Use the *table* feature of a graphing calculator to create
a table of values to find the length ℓ that maximizes the
value of $\ell(160 - \ell)$.

X	Y₁
65	6175
70	6300
75	6375
80	6400
85	6375
90	6300
95	6175

X=80

The length that maximizes the value of $\ell(160 - \ell)$
is 80 yards. So, the width of the field is
$w = 160 - \ell = 160 - 80 = 80$ yards. Because the width
and the length are both 80 yards, the area of the field is
maximized when it is a square.

Practice A

1. About 70,000 people live in a circular region with a 30-mile radius. Find the population
density in people per square mile.

2. About 370,000 people live in a circular region with a 5-mile radius. Find the population
density in people per square mile.

3. A map of the state of Montana is approximately rectangular with a length of 590 miles
and a width of 250 miles.

   a. Montana has a population of about 990,000. Find the population density in people
   per square mile.

   b. The table shows the estimated populations for domestic sheep and cattle in Montana.
   Find the population density in animals per square mile for each animal.

 | **Animal** | Sheep | Cattle |
   |------------|---------|-------------|
   | **Population** | 225,000 | 2.6 million |

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4. About 860,000 people live in a circular region with a population density of about 6480 people per square mile. Find the radius of the region.

5. You have 350 yards of fencing to build a rectangular corral of length ℓ and width w.

   a. Write an expression that represents the area of the corral in terms of ℓ.

   b. Use your expression from part (a) to determine the dimensions of the corral that maximize the area.

In Exercises 6 and 7, describe how the change affects the surface area of the right prism or cylinder.

6. doubling all linear dimensions
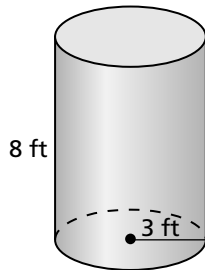
7. multiplying the height by 4
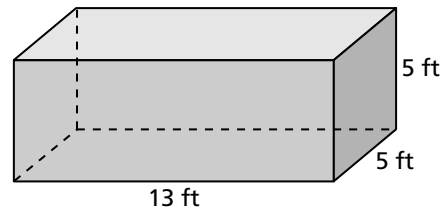
8. You are wrapping a birthday present that is a rectangular prism. The present is 26 inches long, 12 inches tall, and 20 inches wide.

   a. What is the minimum area of wrapping paper required to cover the box?

   b. What is the minimum area of wrapping paper required if you place an identical box on top of the original and wrap them together?

   c. Should you cut your wrapping paper to the minimum area you found in parts (a) and (b)? Explain.

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Practice B

**In Exercises 1–4, find the indicated measure.**

1. A state park has an area of 112 acres. The table shows the estimated park populations for several animals. Find the population density in animals per acre for each animal.

| Animal	Otter	Raccoon	Fox	Bobcat
Population	35	186	9	3

2. A city park is triangular with a base length of 4 blocks and a height of 7 blocks. During an evening concert, its population density is about 54 people per square block. Find the number of people in the park that evening.

3. About 150,000 people live in a circular region with a population density of about 1578 people per square mile. Find the radius of the region.

4. About 1.75 million people live in a circular region with a population density of about 5050 people per square mile. Find the radius of the region.

In Exercises 5 and 6, describe how the change affects the surface area of the right prism or right cylinder.

5. doubling the diameter

3 cm

5 cm
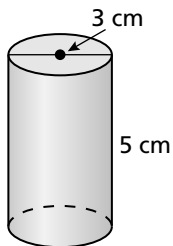
6. multiplying the base edge by 2 and the height by $\frac{1}{3}$

15 yd

8 yd
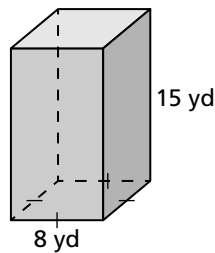
7. A baseball with a 2.9-inch diameter has a layer of leather on its surface.

   a. Does a softball with a diameter that is $\frac{4}{3}$ times the diameter of the baseball need $\frac{4}{3}$ times the amount of leather? Explain.

   b. What is the radius of a softball that uses four-thirds of the amount of leather used to cover the 2.9-inch baseball?