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9.2 Modeling with Volume

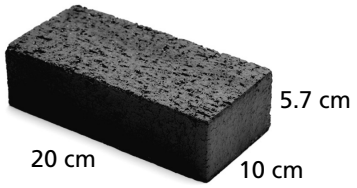
For use with Exploration 9.2

Essential Question How can you use the mass and volume of an object to describe the density of the object?

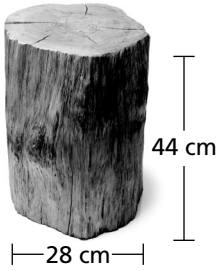
1 EXPLORATION: Finding Densities

Work with a partner. Approximate the volume of each object whose mass is given. Then find the mass per unit of volume, or *density*, of each object.

a. Brick: 2.3 kg



b. Log: 18.1 kg



c. Golf ball: 45.9 g

d. Cork: 2.6 g

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2 EXPLORATION: Analyzing Densities

Work with a partner. The objects in Exploration 1 with a density greater than 1 gram per cubic centimeter will sink in water. The objects with a density less than 1 gram per cubic centimeter will float in water. You place each object in Exploration 1 in a bucket of water.

a. Which object(s) sink? Justify your answer.

b. Would your answer in part (a) change when each object is cut in half and placed in water? Explain your reasoning.

c. You dissolve enough salt in a bucket of water to cause one of the sunken objects to float. Which object is it and why do you think this happens?

Communicate Your Answer

3. How can you use the mass and volume of an object to describe the density of the object?

4. Use the Internet or some other reference to research the densities of water, mineral oil, and beeswax. You combine these substances in a bucket. How do you think the liquids interact? Where would the beeswax settle?

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9.2 Practice

For use after Lesson 9.2

Notes:

Worked-Out Examples

Example #1

PROBLEM SOLVING The height of a tree trunk is 20 meters and the base diameter is 0.5 meter.

a. The wood has a density of 380 kilograms per cubic meter. Find the mass of the trunk to the nearest kilogram.

b. The trunk puts on a growth ring of 4 millimeters and its height increases by 0.2 meter this year. How many cubic meters of wood does the tree trunk produce? The tree grows at a constant rate for the next five years. Does the tree produce the same amount of wood each year? Explain.

a. To estimate the volume of the tree trunk, assume that the trunk is cylindrical. The radius is $0.5 \div 2 = 0.25$ meter. So, the volume of the trunk is

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi(0.25)^2(20) \\ &= 1.25\pi \approx 3.927 \text{ m}^3. \end{aligned}$$

Let x represent the mass in kilograms. Substitute the values for the volume and the density in the formula for density, and solve for x.

$$\begin{aligned} \text{Density} &= \frac{\text{Mass}}{\text{Volume}} \\ 380 &\approx \frac{x}{3.927} \\ 1492 &\approx x \end{aligned}$$

The mass of the trunk is about 1492 kilograms.

Name _____ Date _____

9.2 Practice (continued)

b. Make a table that shows the trunk dimensions and volume for five years.

Year	1	2	3	4	5
Height (meters)	20.2	20.4	20.6	20.8	30
Base radius (meters)	0.254	0.258	0.262	0.266	0.27
Volume (cubic meters)	$\pi(0.254)^2(20.2)$ ≈ 4.094	$\pi(0.258)^2(20.4)$ ≈ 4.266	$\pi(0.262)^2(20.6)$ ≈ 4.442	$\pi(0.266)^2(20.8)$ ≈ 4.624	$\pi(0.27)^2(30)$ ≈ 6.871
		$+ 0.172$	$+ 0.176$	$+ 0.182$	$+ 2.247$

The tree will produce about $4.094 - 3.927 = 0.167$ cubic meter of wood after one year. The tree will not produce the same amount of wood each year for five years because the differences between the volumes from year to year are increasing.

Example #2

MODELING WITH MATHEMATICS A pool in the shape of a rectangular prism is 6 meters long and 3 meters wide. The water in the pool is 1 meter deep.

a. The density of water is about 1 gram per cubic centimeter. Find the number of kilograms of water in the pool.

b. You add 6000 kilograms of water to the pool. What is the depth of the water in the pool?

a. Step 1 Convert the dimensions to centimeters using 1 meter = 100 centimeters.

Length $6 \text{ m} \cdot \frac{100 \text{ cm}}{1 \text{ m}} = 600 \text{ cm}$

Width $3 \text{ m} \cdot \frac{100 \text{ cm}}{1 \text{ m}} = 300 \text{ cm}$

Height $1 \text{ m} \cdot \frac{100 \text{ cm}}{1 \text{ m}} = 100 \text{ cm}$

9.2 Practice (continued)

Step 2 Find the volume.

$V = \ell wh$

$= (600)(300)(100)$

$= 18{,}000{,}000$

So, the volume of water in the pool is 18,000,000 cubic centimeters.

Step 3 Let x represent the mass in grams. Substitute the values for the volume and the density in the formula for density, and solve for x.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$1 \approx \frac{x}{18{,}000{,}000}$$

$18{,}000{,}000 \approx x$

The mass of the water in the pool is about

18,000,000 grams, or about

$18{,}000{,}000 \text{ g} \cdot \frac{1 \text{ kg}}{1000 \text{ g}} = 18{,}000$ kilograms.

b. The mass of the water in the pool is now about

$18{,}000 + 6000 = 24{,}000$ kilograms, or

$24{,}000 \text{ kg} \cdot \frac{1000 \text{ g}}{1 \text{ kg}} = 24{,}000{,}000$ grams.

Let V represent the volume of the water in the pool. Substitute the values for the mass and the density in the formula for density, and solve for V.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$1 \approx \frac{24{,}000{,}000}{V}$$

$$V \approx 24{,}000{,}000$$

Let h represent the height of the water in the pool. Substitute the values for the volume, the length, and the width in the formula for the volume of a rectangular prism, and solve for h.

$$V = \ell wh$$

$24{,}000{,}000 = (600)(300)h$

$24{,}000{,}000 = 180{,}000h$

$133\frac{1}{3} = h$

So, the water is about $133\frac{1}{3}$ centimeters, or about $133\frac{1}{3}$ cm $\cdot \frac{1 \text{ m}}{100 \text{ cm}} = 1\frac{1}{3}$ meters deep.

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9.2 Practice (continued)

Practice A

1. The density of a feather is about 0.0025 gram per cubic centimeter and the density of a brick is about 1.84 grams per cubic centimeter. You have 1000 grams of feathers and 1000 grams of bricks. What volume of each do you have?

2. Before 1965, dimes and quarters were made of silver. The density of silver is about 10.5 grams per cubic centimeter. Find the mass of the coins given their dimensions.

   a. Dime: 1.79 centimeters diameter, 0.135 centimeter thickness

   b. Quarter: 2.43 centimeters diameter, 0.175 centimeter thickness

3. A tree harvester estimates the height of a white pine trunk to have a height of about 22 meters and a base diameter of about 0.75 meter.

   a. The wood has a density of 425 kilograms per cubic meter. Find the mass of the trunk to the nearest kilogram.

   b. The tree puts on a growth ring of 8 millimeters and its height increases by 1 meter this year. How many cubic meters of wood does the tree trunk produce after one year?

   c. The tree grows at a constant rate. Compare the amount of growth this year to the amount of growth 3 years from now.

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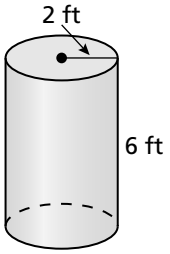
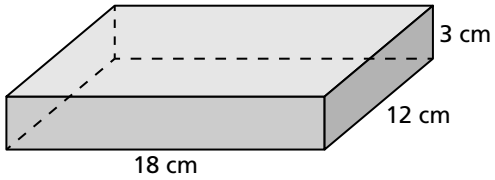
9.2 Practice (continued)

In Exercises 4 and 5, describe how the change affects the volume of the prism or cylinder.

4. multiplying the radius by 5



5. multiplying the length and width by $\frac{1}{6}$



6. A conical pile of sand has a diameter of 175 feet and a slant height of 100 feet. After one day, the linear dimensions are $\frac{2}{3}$ of the original dimensions.

a. How does this change affect the volume of the pile?

b. It takes about 200 pounds of sand to fill a traditional sand box. How many sand boxes can be filled with the remaining sand? A cubic foot of sand weighs about 125 pounds.

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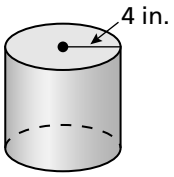
Practice B

1. A cylindrical container with a radius of 8 centimeters is filled to a height of 10 centimeters with sulfuric acid. The density of sulfuric acid is 1.84 grams per cubic centimeter. What is the mass of the sulfuric acid to the nearest gram?
2. A block of ice is in the shape of a square with side lengths of 0.4 meter.
   a. The ice has a density of 917 kilograms per cubic meter. Find the mass of the block of ice to the nearest kilogram.
   b. The side length of the block of ice decreases by 0.05 meter this week. How many cubic centimeters of ice does the block lose? The side length decreases at a constant rate for the next 5 weeks. Does the block of ice lose the same amount of ice each week? Explain.

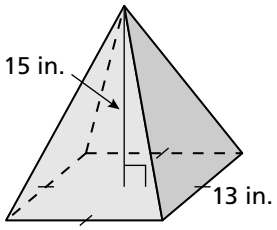
In Exercises 3 and 4, describe how the change affects the volume of the prism or pyramid.

3. multiplying the radius by $\frac{3}{2}$



$V = 112\pi$ in.^{3}

4. tripling all the linear dimensions



5. A cone has height h and a base with radius r. You want to change the cone so its volume is halved. What is the new height if you only change the height? What is the new radius if you only change the radius? Explain.
6. A 10 karat gold paper weight has a volume of 125 cubic centimeters. The paper weight consists of 41.7% gold and 58.3% other metal. The density of gold is 19,300 kilograms per cubic meter. The density of the other metal is 7140 kilograms per cubic meter.
   a. What is the mass of the gold in the paper weight to the nearest tenth of a kilogram?
   b. What is the mass of the other metal in the paper weight to the nearest tenth of a kilogram?
   c. What is the total mass of the paper weight?
   d. What is the total density of the paper weight in kilograms per cubic meter?