4

Ratios and Proportions

Chapter Learning Target:
Understand ratios and proportions.

Chapter Success Criteria:

- ◆ I can write and interpret ratios.
- ◆ I can describe ratio relationships and proportional relationships.
- ■ I can represent equivalent ratios.
- ■ I can model ratio relationships and proportional relationships to solve real-life problems.

◆ Surface ■ Deep

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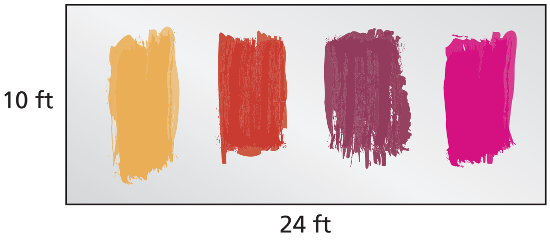
7 MTR *Mathematicians who apply mathematics to real-world contexts connect mathematical concepts to everyday experiences.*

STEAM Video Painting a Large Room

In the video, Alex and Enid use proportions to match paint colors.

1. Enid estimates that they need 2 gallons of paint to apply two coats to the wall shown. How many square feet does she expect $\frac{1}{2}$ gallon of paint will cover?



5 MTR **2.** **MAKE A PLAN** You record how many gallons of paint it takes to cover a wall using one coat of paint, and then measure the height of the wall. How can you estimate the length of the wall without measuring?

STEAM Performance Task Mixing Paint

After completing this chapter, you will be able to use the concepts you learned to answer the questions in the *STEAM Performance Task*. You will be given the amounts of each tint used to make different colors of paint. For example:

Plum Purple Paint

3 parts red tint per gallon

2 parts blue tint per gallon

1 part yellow tint per gallon

1 part white tint per gallon

You will be asked to solve various ratio problems about mixing paint.

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Getting Ready for Chapter 4 WITH CalcChat®

Chapter Exploration

1 MTR **Actively Participate in Effortful Learning Collectively**
Work with a partner to prepare for concepts in this chapter.

The Meaning of a Word ▶ Rate

When you rent snorkel gear at the beach, you should pay attention to the rental **rate**. The rental rate is in dollars per hour.

1. Complete each step.

- Match each description with a rate.
- Match each rate with a fraction.
- Give a reasonable value for each fraction. Then give an unreasonable value.

		Fraction	
Description	**Rate**	**Reasonable**	**Unreasonable**
Your speed in the 100-meter dash	Dollars per hour	$\frac{\square \text{ inches}}{\text{year}}$	$\frac{\square \text{ inches}}{\text{year}}$
The hourly wage of a worker at a fast-food restaurant	Inches per year	$\frac{\square \text{ pounds}}{\text{square foot}}$	$\frac{\square \text{ pounds}}{\text{square foot}}$
The average annual rainfall in a rain forest	Pounds per square foot	$\frac{\$\square}{\text{hour}}$	$\frac{\$\square}{\text{hour}}$
The amount of fertilizer spread on a lawn	Meters per second	$\frac{\square \text{ meters}}{\text{second}}$	$\frac{\square \text{ meters}}{\text{second}}$

3 MTR **2. ADAPT A PROCEDURE** Describe a situation to which the given fraction can apply. Show how to rewrite each expression as a division problem. Then simplify and interpret your result.

a. $\dfrac{\frac{1}{2}\text{ cup}}{4\text{ fluid ounces}}$ **b.** $\dfrac{2\text{ inches}}{\frac{3}{4}\text{ second}}$ **c.** $\dfrac{\frac{3}{8}\text{ cup sugar}}{\frac{3}{4}\text{ cup flour}}$ **d.** $\dfrac{\frac{5}{6}\text{ gallon}}{\frac{2}{3}\text{ second}}$

Vocabulary

The following terms are defined in this chapter. Think about what each might mean and record your thoughts.

proportional constant of proportionality scale drawing

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4.1 Ratios and Ratio Tables

Learning Target: Understand ratios of rational numbers and use ratio tables to represent equivalent ratios.

Success Criteria:
- I can write and interpret ratios involving rational numbers.
- I can use various operations to create tables of equivalent ratios.
- I can use ratio tables to solve ratio problems.

Algebraic Reasoning

MA.7.AR.4.4 Given any representation of a proportional relationship, translate the representation to a written description, table or equation.

MA.7.AR.4.5 Solve real-world problems involving proportional relationships.

Exploration 1 Describing Ratio Relationships

Work with a partner. Use the recipe shown.

Chicken Soup

stewed tomatoes	9 ounces	chopped spinach	9 ounces
chicken broth	15 ounces	grated parmesan	5 tablespoons
chopped chicken	1 cup		

a. Identify several ratios in the recipe.

b. You halve the recipe. Describe your ratio relationships in part (a) using the new quantities. Is the relationship between the ingredients the same as in part (a)? Explain.

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Exploration 2 Completing Ratio Tables

Work with a partner. Use the ratio tables shown.

x	5			
y	1			

x	$\frac{1}{4}$			
y	$\frac{1}{2}$			

4 MTR COMMUNICATE CLEARLY

How can you determine whether the ratios in each table are equivalent?

a. Complete the first ratio table using multiple operations. Use the same operations to complete the second ratio table.

b. Are the ratios in the first table equivalent? the second table? Explain.

c. Do the strategies for completing ratio tables of whole numbers work for completing ratio tables of fractions? Explain your reasoning.

4.1 Lesson

Key Vocabulary

ratio, *p. 273*
value of a ratio, *p. 273*
equivalent ratios, *p. 274*
ratio table, *p. 274*

Reading

Recall that phrases indicating ratios include *for each*, *for every*, and *per*.

Key Idea

Ratios

Words A **ratio** is a comparison of two quantities. The **value of the ratio** a to b is the number $\frac{a}{b}$, which describes the multiplicative relationship between the quantities in the ratio.

Examples 2 snails *to* 6 fish

$\frac{1}{2}$ cup of milk *for every* $\frac{1}{4}$ cup of cream

Algebra The ratio of a to b can be written as $a : b$.

Example 1 Writing and Interpreting Ratios

Flubber Ingredients

cold water	3/2 cups
hot water	4/3 cups
glue	2 cups
borax	3 teaspoons

You make *flubber* using the ingredients shown.

a. Write the ratio of cold water to glue.

The recipe uses $\frac{3}{2}$ cups of water per 2 cups of glue.

▶ So, the ratio of cold water to glue is $\frac{3}{2}$ to 2, or $\frac{3}{2} : 2$.

b. Find and interpret the value of the ratio in part (a).

The value of the ratio $\frac{3}{2} : 2$ is

$$\frac{\frac{3}{2}}{2} = \frac{3}{2} \div 2$$
$$= \frac{3}{2} \cdot \frac{1}{2}$$
$$= \frac{3}{4}.$$

So, the multiplicative relationship is $\frac{3}{4}$.

▶ The amount of cold water in the recipe is $\frac{3}{4}$ the amount of glue.

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Try It

1. You mix $\frac{2}{3}$ teaspoon of baking soda with 3 teaspoons of salt. Find and interpret the value of the ratio of baking soda to salt.

Two ratios that describe the same relationship are **equivalent ratios**. The values of equivalent ratios are equivalent. You can find and organize equivalent ratios in a **ratio table** by

- adding or subtracting quantities in equivalent ratios.
- multiplying or dividing each quantity in a ratio by the same number.

Example 2 Completing a Ratio Table

Find the missing values in the ratio table. Then write the equivalent ratios.

Cups	3	12	15	
Quarts	$\frac{3}{4}$			$\frac{5}{4}$

You can use a combination of operations to find the missing values.

Notice that you obtain the third column by adding the values in the first column to the values in the second column.

$3 + 12 = 15$

$\frac{3}{4} + 3 = \frac{15}{4}$

$\times 4$ $+ 3$ $\div 3$

Cups	3	12	15	5
Quarts	$\frac{3}{4}$	3	$\frac{15}{4}$	$\frac{5}{4}$

$\times 4$ $+ \frac{3}{4}$ $\div 3$

▶ The equivalent ratios are $3 : \frac{3}{4}$, $12 : 3$, $15 : \frac{15}{4}$, and $5 : \frac{5}{4}$.

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Try It **Find the missing values in the ratio table. Then write the equivalent ratios.**

2.

Kilometers	$\frac{5}{2}$		5
Hours	4	16	

3.

Gallons	0.4	1.2	1.6
Days	0.75		

In-Class Practice

1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

4. **WRITING AND INTERPRETING RATIOS** You include $\frac{1}{2}$ tablespoon of essential oils in a solution for every 12 tablespoons of jojoba oil. Find and interpret the value of the ratio of jojoba oil to essential oils.

5. **NUMBER SENSE** Find the missing values in the ratio table. Then write the equivalent ratios.

Pounds	$\frac{3}{2}$		$\frac{21}{2}$
Years	$\frac{1}{12}$	$\frac{2}{3}$	

Example 3 Modeling Real Life

7 MTR

You mix $\frac{1}{2}$ cup of yellow paint for every $\frac{3}{4}$ cup of blue paint to make 15 cups of green paint. How much yellow paint do you use?

Method 1: The ratio of yellow paint to blue paint is $\frac{1}{2}$ to $\frac{3}{4}$. Use a ratio table to find an equivalent ratio in which the total amount of yellow paint and blue paint is 15 cups.

	Yellow (cups)	Blue (cups)	Total (cups)	
	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{2}+\frac{3}{4}=\frac{5}{4}$	
× 4	2	3	5	× 4
× 3	6	9	15	× 3

▶ So, you use 6 cups of yellow paint.

Method 2: You can use the ratio of yellow paint to blue paint to find the fraction of the green paint that is made from yellow paint. You use $\frac{1}{2}$ cup of yellow paint for every $\frac{3}{4}$ cup of blue paint, so the fraction of the green paint that is made from yellow paint is

yellow

green

$$\frac{\frac{1}{2}}{\frac{1}{2}+\frac{3}{4}} = \frac{\frac{1}{2}}{\frac{5}{4}} = \frac{1}{2} \bullet \frac{4}{5} = \frac{2}{5}.$$

▶ So, you use $\frac{2}{5} \bullet 15 = 6$ cups of yellow paint.

In-Class Practice

1 I don't understand yet.
2 I can do it with help.
3 I can do it on my own.
4 I can teach someone else.

6. An engine runs on a mixture of 0.1 quart of oil for every 3.5 quarts of gasoline. You make 3 quarts of the mixture. How much oil and how much gasoline do you use?

7. **Dig Deeper** A satellite orbiting Earth travels $14\frac{1}{2}$ miles every 3 seconds. How far does the satellite travel in $\frac{3}{4}$ minute?

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4.1 Practice WITH CalcChat® AND CalcView®

Review & Refresh

Solve the inequality. Graph the solution.

1. $-3x \geq 18$

2. $\frac{2}{3}d > 8$

3. $2 \geq \frac{g}{-4}$

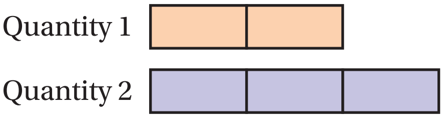
Find the quotient. Write fractions in simplest form.

4. $\frac{2}{9} \div \frac{4}{3}$

5. $10.08 \div 12$

6. $-\frac{5}{6} \div \frac{3}{10}$

7. Can the ratio 4 : 9 be represented by the tape diagram?



Concepts, Skills, & Problem Solving

OPEN-ENDED **Complete the ratio table using multiple operations. Are the ratios in the table equivalent? Explain.** (See Exploration 2.)

8.

x	4			
y	10			

9.

x	$\frac{4}{5}$			
y	$\frac{1}{2}$			

Fruit Punch Ingredients

chopped watermelon	3 cups
sugar	3/4 cup
mint leaves	1/2 cup
white grape juice	2 cups
lime juice	3/4 cup
club soda	4 cups

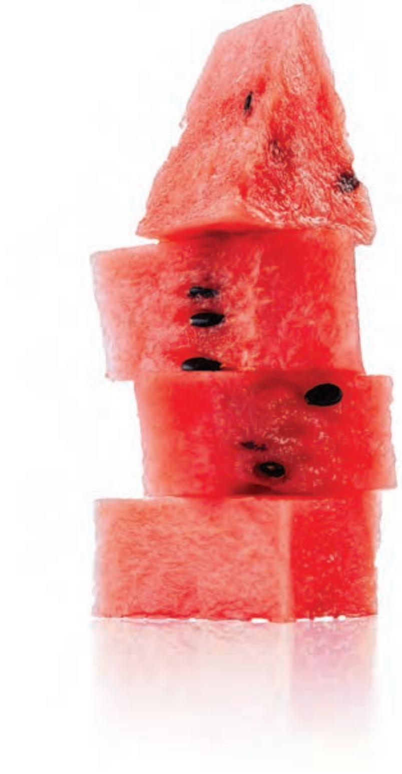
WRITING AND INTERPRETING RATIOS **Find the ratio. Then find and interpret the value of the ratio.** (See Example 1.)

10. club soda : white grape juice

▶**11.** mint leaves : chopped watermelon

12. white grape juice to sugar

13. lime juice to mint leaves



14. **YOU BE THE TEACHER** You have blue ribbon and red ribbon in the ratio $\frac{1}{2} : \frac{1}{5}$. Your friend finds the value of the ratio. Is your friend correct? Explain your reasoning.

The value of the ratio is

$$\frac{\frac{1}{2}}{\frac{1}{5}} = \frac{1}{2} \div \frac{1}{5} = \frac{1}{10}.$$

COMPLETING A RATIO TABLE **Find the missing values in the ratio table. Then write the equivalent ratios.** (See Example 2.)

▶ **15.**

Calories	20		10	90
Miles	$\frac{1}{6}$	$\frac{2}{3}$		

16.

Meters	8	4		
Minutes	$\frac{1}{3}$		$\frac{1}{4}$	$\frac{5}{12}$

17.

Feet	$\frac{1}{24}$		$\frac{1}{8}$	
Inches	$\frac{1}{2}$	1		$\frac{1}{4}$

18.

Tea (cups)	3.75			
Milk (cups)	1.5	1	3.5	2.5

19. **REASONING** Are the two statements equivalent? Explain your reasoning.

- The ratio of boys to girls is 2 to 3.
- The ratio of girls to boys is 3 to 2.

20. **MODELING REAL LIFE** A city dumps plastic *shade balls* into a reservoir to prevent water from evaporating during a drought. It costs $5760 for 16,000 shade balls. How much does it cost for 12,000 shade balls?

21. **MODELING REAL LIFE** An oil spill spreads 25 square meters every $\frac{1}{6}$ hour. What is the area of the oil spill after 2 hours?

7 MTR **22.** **MODELING REAL LIFE** You mix 0.25 cup of juice concentrate for 2 cups of water to make 18 cups of juice. How much juice concen do you use? How much water do you use? (See Example 3.)

7 MTR **23.** **MODELING REAL LIFE** A store sells $2\frac{1}{4}$ pounds of mulch for every $1\frac{1}{2}$ pounds of gravel sold. The store sells 180 pounds of mulch and gravel combined. How many pounds of each item does the store sell?

24. **Dig Deeper** You mix $\frac{1}{4}$ cup of red paint for every $\frac{1}{2}$ cup of blue paint to make 3 gallons of purple paint.

a. How much red paint do you use? How much blue paint do you use?

b. You make a new lighter purple paint by mixing $\frac{1}{4}$ cup of white paint for every $\frac{1}{4}$ cup of red paint and $\frac{1}{2}$ cup of blue paint. How much red paint, blue paint, and white paint do you use to make $1\frac{1}{2}$ gallons of the lighter purple paint?

4.2 Graphing Ratio Relationships

Learning Target: Represent ratio relationships in a coordinate plane.

Success Criteria:
- I can create and plot ordered pairs from a ratio relationship.
- I can create graphs to solve ratio problems.
- I can create graphs to compare ratios.

Algebraic Reasoning

MA.7.AR.4.3 Given a mathematical or real-world context, graph proportional relationships from a table, equation or a written description.

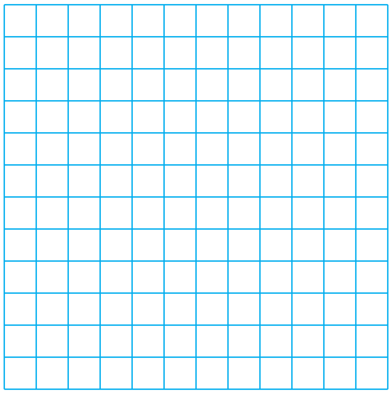
MA.7.AR.4.4 Given any representation of a proportional relationship, translate the representation to a written description, table or equation.

Also MA.7.AR.4.5

Exploration 1 Using a Coordinate Plane

Work with a partner. An airplane travels 300 miles per hour.

a. Represent the relationship between distance and time in a coordinate plane. Explain your choice for labeling and scaling the axes.



b. Write a question that can be answered using the graph. Exchange your question with another group. Answer their question, and discuss the solution with the other group.

1 MTR ANALYZE A PROBLEM

Could you have placed each quantity on the other axis? Explain why or why not.

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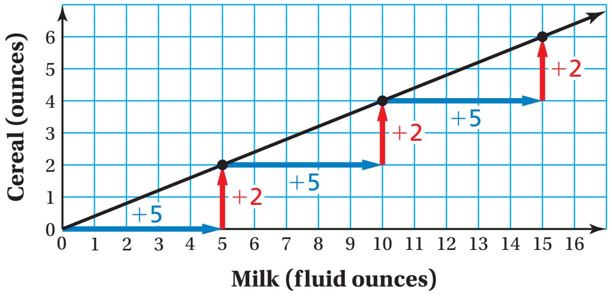
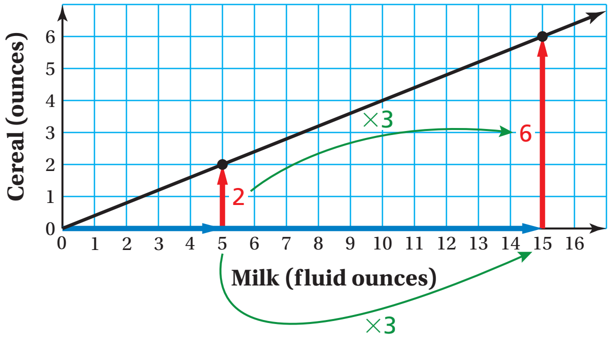
Exploration 2 Identifying Relationships in Graphs

Work with a partner. Use the graphs to make a ratio table. Explain how the blue, red, and green arrows correspond to the ratio table.





x						
y						

4.2 Lesson

For a ratio of two quantities, you can use equivalent ratios to create ordered pairs of the form (first quantity, second quantity). You can plot these ordered pairs in a coordinate plane and draw a line, starting at (0, 0), through the points.

Example 1 Graphing Ratio Relationships

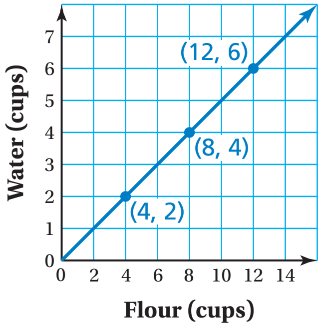
Represent each ratio relationship using a graph.

a.

Flour (cups)	4	8	12
Water (cups)	2	4	6

The ordered pairs (flour, water) are (4, 2), (8, 4), and (12, 6).

Plot the ordered pairs. Starting at (0, 0), draw a line through the points.



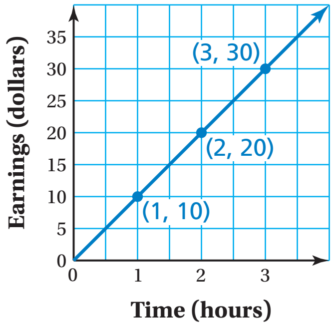
b.

Time (hours)	1	2	3
Earnings (dollars)	10	20	30

The ordered pairs (time, earnings) are (1, 10), (2, 20), and (3, 30).

Plot the ordered pairs. Starting at (0, 0), draw a line through the points.

Relationships involving time are often graphed with time on the horizontal axis.



Try It **Represent the ratio relationship using a graph.**

1.

Time (minutes)	Number of Words
1	50
2	100
3	150

2.

Number of 6th Graders	Number of 7th Graders
5	4
10	8
15	12

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Example 2 Using a Graph to Solve a Ratio Problem

You buy dark chocolate cashews for $12.50 per pound.

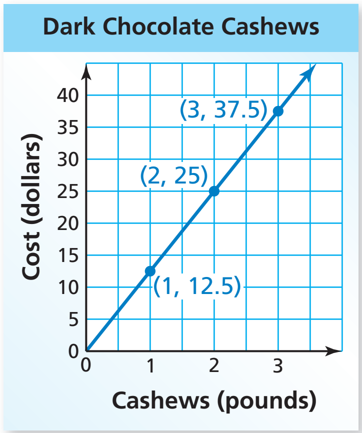
a. Represent the ratio relationship using a graph.

Create a ratio table.

Cashews (pounds)	1	2	3
Cost (dollars)	12.5	25	37.5

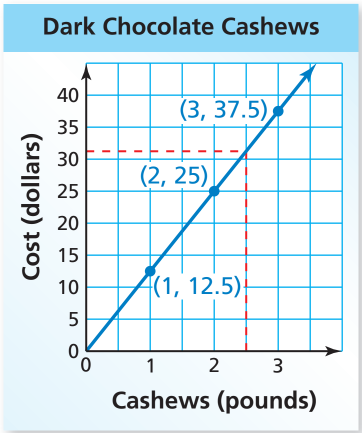
Relationships involving cost are often graphed with cost on the vertical axis.

The ordered pairs (cashews, cost) are (1, 12.5), (2, 25), and (3, 37.5). Plot the ordered pairs and draw a line, starting at (0, 0), through the points.



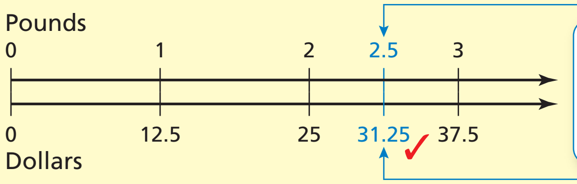
b. How much does 2.5 pounds of dark chocolate cashews cost?

Using the graph, you can see that the cost of 2.5 pounds is halfway between $25 and $37.50.



▶ So, 2.5 pounds of dark chocolate cashews cost $31.25.

Another Method Use a double number line to find the cost.



2.5 is halfway between 2 and 3. So, find the value halfway between 25 and 37.5.

Try It

3. **WHAT IF?** Repeat Example 2 when the cost of the dark chocolate cashews is $15 per pound.

In-Class Practice

1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

Rain (inches)	Snow (inches)
3	5
6	10
9	15

4. **GRAPHING A RATIO RELATIONSHIP** Represent the ratio relationship using a graph.

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In-Class Practice

1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

1 MTR **5.** **HELP A CLASSMATE** Use what you know about equivalent ratios to help a classmate understand why the graph of a ratio relationship passes through (0, 0).

4 MTR **6.** **WHICH ONE DOESN'T BELONG?** Which ordered pair does *not* belong with the other three? Explain your reasoning.

(4, 1) (12, 3) (8, 2) (24, 4)

Example 3 Modeling Real Life

7 MTR

A hot-air balloon rises 9 meters every 3 seconds. A blimp rises 7 meters every 2 seconds. Graph each ratio relationship in the same coordinate plane. Which rises faster?

Create ratio tables for each rising object. Then plot the ordered pairs (time, height) from the table and use the graph to determine which rises faster.

Balloon	
Time (seconds)	Height (meters)
3	9
6	18
9	27

Blimp	
Time (seconds)	Height (meters)
2	7
4	14
6	21

Check
From the ratio tables, you can see that every 6 seconds, the balloon rises 18 meters and the blimp rises 21 meters. So, the blimp rises faster. ✓

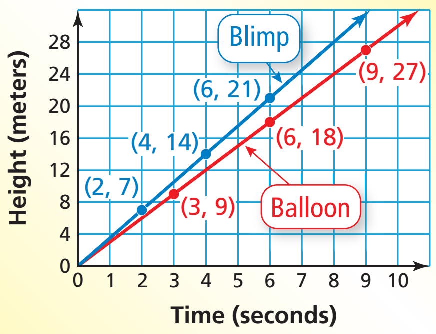
Balloon: (3, 9), (6, 18), (9, 27)

Blimp: (2, 7), (4, 14), (6, 21)

Plot and label each set of ordered pairs. Then draw a line, starting at (0, 0), through each set of points.

▶ Both graphs begin at (0, 0). The graph for the blimp is steeper, so the blimp rises faster than the hot-air balloon.

Rising Objects



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In-Class Practice

1 I don't understand yet.
2 I can do it with help.
3 I can do it on my own.
4 I can teach someone else.

7. You are skateboarding at a pace of 30 meters every 5 seconds. Your friend is in-line skating at a pace of 9 meters every 2 seconds. Graph each ratio relationship in the same coordinate plane. Who is faster?

8. You buy 2.5 pounds of pumpkin seeds and 2.5 pounds of sunflower seeds. Use a graph to find your total cost. Then use the graph to determine how much more you pay for pumpkin seeds than for sunflower seeds.

4.2 Practice WITH CalcChat® AND CalcView®

Review & Refresh

Find the missing values in the ratio table. Then write the equivalent ratios.

1.

Flour (cups)	$\frac{3}{4}$		3	1
Oats (cups)	$\frac{1}{3}$	$\frac{2}{3}$		

2.

Pages	$\frac{1}{4}$	$\frac{3}{4}$		5
Minutes	$\frac{1}{2}$		3	

Write the name of the decimal number.

3. 7.1

4. 3.54

5. 13.6

6. 8.132

Write two equivalent ratios that describe the relationship.

7. baseballs to gloves

8. ladybugs to bees

Concepts, Skills, & Problem Solving

USING A COORDINATE PLANE **Represent the relationship between distance and time in a coordinate plane.** (See Exploration 1.)

9. A train travels 45 miles per hour.

10. A motorcycle travels 70 kilometers per hour.

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11. A snail travels 80 centimeters per minute.

12. A whale travels 800 yards per minute.

GRAPHING RATIO RELATIONSHIPS **Represent the ratio relationship using a graph.** (See Example 1.)

13.

Height (inches)	20	40	60
Weight (pounds)	30	60	90

14.

Students	9	18	27
Computers	4	8	12

▶ **15.**

Ribbon (inches)	1	2	3
String (inches)	3	6	9

16.

Water (gallons)	30	60	90
Soda (gallons)	5	10	15

17.

Cherries	5	10	15
Limes	8	16	24

18.

Jog (miles)	2	4	6
Sprint (meters)	400	800	1200

19. **MODELING REAL LIFE** A radio station collects donations for a new broadcast tower. The cost to construct the tower is $25.50 per inch. (See Example 2.)

a. Represent the ratio relationship using a graph.

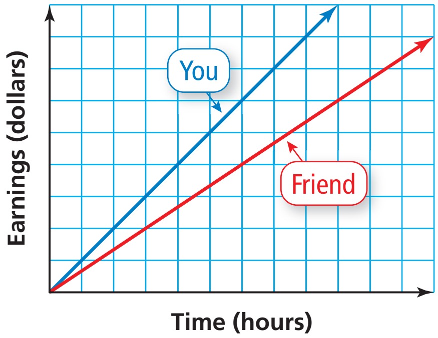
b. How much does it cost to fund 4.5 inches of the construction?

20. **MODELING REAL LIFE** Your school organizes a clothing drive as a fundraiser for a class trip. The school earns $100 for every 400 pounds of donated clothing.

a. Represent the ratio relationship using a graph.

b. How much money does your school earn for donating 2200 pounds of clothing?

21. **NUMBER SENSE** Just by looking at the graph, determine who earns a greater hourly wage. Explain.



7 MTR

22. **MODELING REAL LIFE** An airplane traveling from Orlando to Chicago travels 15 miles every 2 minutes. On the return trip, the plane travels 25 miles every 3 minutes. Graph each ratio relationship in the same coordinate plane. Does the plane fly faster when traveling to Chicago or to Orlando? (See Example 3.)

7 MTR

23. **MODELING REAL LIFE** Your freezer produces 8 ice cubes every 2 hours. Your friend's freezer produces 24 ice cubes every 5 hours. Graph each ratio relationship in the same coordinate plane. Whose freezer produces ice faster?

2 MTR

24. **MODEL A PROBLEM** A chemist prepares two acid solutions.

a. Use a ratio table to determine which solution is more acidic.

x					
y					

b. Use a graph to determine which solution is more acidic.

c. Which method do you prefer? Explain.

25. **Dig Deeper** A company offers a nut mixture with 7 peanuts for every 3 almonds. The company changes the mixture to have 9 peanuts for every 5 almonds, but the number of nuts per container does not change.

a. How many nuts are in the smallest possible container?

b. Graph each ratio relationship. What can you conclude?

c. Almonds cost more than peanuts. Should the company change the price of the mixture? Explain your reasoning.

5 MTR

26. **STRUCTURE** The point (p, q) is on the graph of values from a ratio table. What are two additional points on the graph?

4.3 Rates and Unit Rates

Learning Target: Understand rates involving fractions and use unit rates to solve problems.

Success Criteria:
- I can find unit rates for rates involving fractions.
- I can use unit rates to solve rate problems.

Algebraic Reasoning

MA.7.AR.4.4 Given any representation of a proportional relationship, translate the representation to a written description, table or equation.

MA.7.AR.4.5 Solve real-world problems involving proportional relationships.

Exploration 1 Writing Rates

Work with a partner.

a. How many degrees does the minute hand on a clock move every 15 minutes? Write a rate that compares the number of degrees moved by the minute hand to the number of hours elapsed.

b. Can you use the rate in part (a) to determine how many degrees the minute hand moves in $\frac{1}{2}$ hour? Explain your reasoning.

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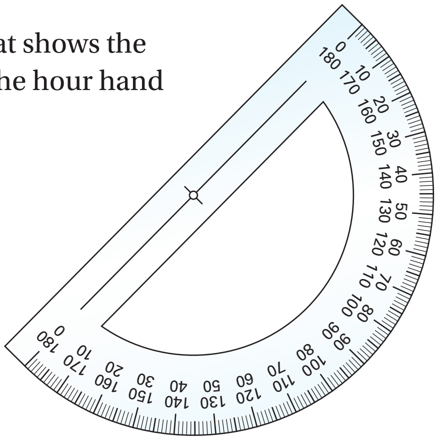
c. Write a rate that represents the number of degrees moved by the minute hand every hour.

How can you use this rate to find the number of degrees moved by the minute hand in $2\frac{1}{2}$ hours?

d. Draw a clock with hour and minute hands. Draw another clock that shows the time after the minute hand moves 900°. How many degrees does the hour hand move in this time? in one hour? Explain your reasoning.



4 MTR **COMPARE METHODS**

When could you use a protractor to find the number of degrees the minute hand moves? When would you need to use a rate?

4.3 Lesson

Key Vocabulary
rate, *p. 293*
unit rate, *p. 293*
equivalent rates, *p. 293*

Key Ideas

Rates and Unit Rates

Words A **rate** is a ratio of two quantities using different units. A **unit rate** compares a quantity to one unit of another quantity. **Equivalent rates** have the same unit rate.

Numbers You pay \$350 for every $\frac{1}{4}$ ounce of gold.

\$350	\$350	\$350	\$350
$\frac{1}{4}$ oz	$\frac{1}{4}$ oz	$\frac{1}{4}$ oz	$\frac{1}{4}$ oz

Rate: \$350 : $\frac{1}{4}$ oz

Unit Rate: \$1400 : 1 oz

Algebra Rate: a units : b units Unit rate: $\frac{a}{b}$ units : 1 unit

Example 1 Finding Unit Rates

A nutrition label shows that every $\frac{1}{4}$ cup of tuna has $\frac{1}{2}$ gram of fat.

$\frac{1}{4}$ c	$\frac{1}{4}$ c	$\frac{1}{4}$ c	$\frac{1}{4}$ c
$\frac{1}{2}$ g	$\frac{1}{2}$ g	$\frac{1}{2}$ g	$\frac{1}{2}$ g

a. How many grams of fat are there for every cup of tuna?

There is $\frac{1}{2}$ gram of fat for every $\frac{1}{4}$ cup of tuna. Find the unit rate.

▶ There are $\frac{\frac{1}{2}}{\frac{1}{4}} = 2$ grams of fat for every cup of tuna.

b. How many cups of tuna are there for every gram of fat?

There is $\frac{1}{4}$ cup of tuna for every $\frac{1}{2}$ gram of fat. Find the unit rate.

▶ There is $\frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$ cup of tuna per gram of fat.

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Try It

1. There is $\frac{1}{4}$ gram of fat for every $\frac{1}{3}$ tablespoon of powdered peanut butter. How many grams of fat are there for every tablespoon of the powder?

Example 2 Using a Unit Rate to Solve a Rate Problem

A scientist estimates that a jet of liquid iron in the Earth's core travels 9 feet every $\frac{1}{2}$ hour. How far does the liquid iron travel in 1 day?

The ratio of feet to hours is $9 : \frac{1}{2}$. Using a ratio table, divide the quantity by $\frac{1}{2}$ to find the unit rate in feet per hour. Then multiply each quantity by 24 to find the distance traveled in 24 hours, or 1 day.

× 2 × 24

Distance (feet)	9	18	432
Time (hours)	$\frac{1}{2}$	1	24

× 2 × 24

▶ So, the liquid iron travels about 432 feet in 1 day.

Try It

2. **WHAT IF?** The scientist later says that the iron travels 3 feet every 10 minutes. Does this change your answer in Example 2? Explain.

GO DIGITAL

In-Class Practice

1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

3. **VOCABULARY** How can you tell when a rate is a unit rate?

4. **WRITING** Explain why rates are usually written as unit rates.

Find the unit rate.

5. \$1.32 for 12 ounces

6. $\frac{1}{4}$ gallon for every $\frac{3}{10}$ mile

7. **FINDING UNIT RATES** Find the missing values in the ratio table. Then write the unit rate of grams per cup and the unit rate of cups per gram.

Grams	$\frac{5}{2}$		1	$\frac{15}{4}$	
Cups	$\frac{2}{3}$	$\frac{1}{6}$			4

Example 3 Modeling Real Life

7 MTR

You hike up a mountain trail at a rate of $\frac{1}{4}$ mile every 10 minutes. You hike 5 miles every 2 hours on the way down the trail. How much farther do you hike in 3 hours on the way down than in 3 hours on the way up?

Because 10 minutes is $\frac{1}{6}$ of an hour, the ratio of miles to hours on the way up is $\frac{1}{4} : \frac{1}{6}$. On the way down, the ratio is 5 : 2. Use ratio tables to find how far you hike in 3 hours at each rate.

Hiking Up	
Distance (miles)	Time (hours)
$\frac{1}{4}$	$\frac{1}{6}$
$\frac{3}{2}$	1
$\frac{9}{2}$	3

Hiking Down	
Distance (miles)	Time (hours)
5	2
$\frac{5}{2}$	1
$\frac{15}{2}$	3

Find the unit rate for each part of the hike.

Find the distance you hike in 3 hours on each part of the hike.

▶ So, you hike $\frac{15}{2} - \frac{9}{2} = \frac{6}{2} = 3$ miles farther in 3 hours on the way down than you hike in 3 hours on the way up.

Check Your rate on the way down is $\frac{5}{2} - \frac{3}{2} = \frac{2}{2} = 1$ mile per hour faster than your rate on the way up. So, you hike 3 miles farther in 3 hours on the way down than you hike in 3 hours on the way up. ✓

In-Class Practice

1 I don't understand yet.
2 I can do it with help.
3 I can do it on my own.
4 I can teach someone else.

8. Two people compete in a five-mile go-kart race. Person A travels $\frac{1}{10}$ mile every 15 seconds. Person B travels $\frac{3}{8}$ mile every 48 seconds. Who wins the race? What is the difference of the finish times of the competitors?

9. **Dig Deeper** A bus travels 0.8 mile east every 45 seconds. A second bus travels 0.55 mile west every 30 seconds. The buses start at the same location. Use two methods to determine how far apart the buses are after 15 minutes. Explain your reasoning.

GO DIGITAL

4.3 Practice WITH CalcChat® AND CalcView®

Review & Refresh

Represent the ratio relationship using a graph.

1.

Push-Ups	5	10	15
Sit-Ups	10	20	30

2.

Texts Sent	4	8	12
Texts Received	3	6	9

3.

Seeds	15	30	45
Plants	12	24	36

4.

Run (minutes)	6	12	18
Walk (minutes)	2	4	6

Complete the statement using <, >, or =.

5. $\frac{9}{2} \;\square\; \frac{8}{3}$

6. $-\frac{8}{15} \;\square\; \frac{10}{18}$

7. $\frac{-6}{24} \;\square\; \frac{-2}{8}$

GO DIGITAL

Concepts, Skills, & Problem Solving

WRITING RATES Find the number of degrees moved by the minute hand of a clock in the given amount of time. Explain your reasoning. (See Exploration 1.)

8. $\frac{2}{3}$ hour

9. $\frac{7}{12}$ hour

10. $1\frac{1}{4}$ hours

FINDING UNIT RATES Find the unit rate. (See Example 1.)

11. 180 miles in 3 hours

12. 256 miles per 8 gallons

▶**13.** $\frac{1}{2}$ pound : 5 days

14. 4 grams for every $\frac{3}{4}$ serving

15. $9.60 for 4 pounds

16. $4.80 for 6 cans

17. 297 words in 5.5 minutes

18. $\frac{1}{3}$ kilogram : $\frac{2}{3}$ foot

19. $\frac{5}{8}$ ounce per $\frac{1}{4}$ pint

20. $21\frac{3}{4}$ meters in $2\frac{1}{2}$ hours

USING TOOLS Find the missing values in the ratio table. Then write the equivalent ratios.

21.

Calories	25	50		
Servings	$\frac{1}{3}$		1	$\frac{4}{3}$

22.

Oxygen (liters)	4	$\frac{4}{3}$		16
Time (minute)	$\frac{3}{4}$		1	

23. **MODELING REAL LIFE** You can sand $\frac{4}{9}$ square yard of wood in $\frac{1}{2}$ hour. How many square yards can you sand in 3.2 hours? Justify your answer. (See Example 2.)

24. **PROBLEM SOLVING** In January 2012, the U.S. population was about 313 million people. In January 2017, it was about 324 million. What was the average rate of population change per year?

REASONING **Tell whether the rates are equivalent. Justify your answer.**

25. 75 pounds per 1.5 years

38.4 ounces per 0.75 year

26. $7\frac{1}{2}$ miles for every $\frac{3}{4}$ hour

$\frac{1}{2}$ mile for every 3 minutes

27. **PROBLEM SOLVING** The table shows nutritional information for three beverages.

Beverage	Serving Size	Calories	Sodium
Whole milk	1 c	146	98 mg
Orange juice	1 pt	210	10 mg
Apple juice	24 fl oz	351	21 mg

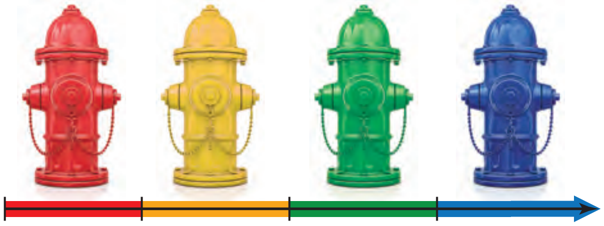
a. Which has the most calories per fluid ounce?

b. Which has the least sodium per fluid ounce?

28. **MODELING REAL LIFE** A shuttle leaving Earth's atmosphere travels 15 miles every 2 seconds. When entering Earth's atmosphere, the shuttle travels $2\frac{3}{8}$ miles per $\frac{1}{2}$ second. Find the difference in the distances traveled after 15 seconds when leaving and entering the atmosphere. (See Example 3.)

7 MTR

29. **INVESTIGATE** Fire hydrants are one of four different colors to indicate the rate at which water comes from the hydrant.



a. Use the Internet to find the ranges of rates indicated by each color.

b. Research why a firefighter needs to know the rate at which water comes out of a hydrant.

30. **Dig Deeper** You and a friend start riding bikes toward each other from opposite ends of a 24-mile biking route. You ride $2\frac{1}{6}$ miles every $\frac{1}{4}$ hour. Your friend rides $7\frac{1}{3}$ miles per hour.

a. After how many hours do you meet?

b. When you meet, who has traveled farther? How much farther?

4.4 Converting Measures between Systems

Learning Target: Use ratio reasoning to convert units of measure between systems.

Success Criteria:
- I can convert units of measure between systems using ratio tables.
- I can convert units of measure between systems using conversion factors.
- I can convert rates using conversion factors.
- I can convert currencies using exchange rates.

Algebraic Reasoning

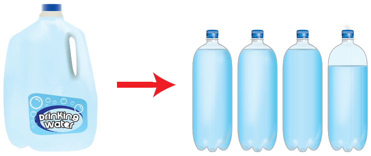
MA.7.AR.3.3 Solve mathematical and real-world problems involving the conversion of units across different measurement systems.

MA.7.AR.4.5 Solve real-world problems involving proportional relationships.

Exploration 1 Estimating Unit Conversions

Work with a partner. You are given 4 one-liter containers and a one-gallon container.

a. A full one-gallon container can be used to fill the one-liter containers, as shown below. Write a unit rate that estimates the number of liters per gallon.



b. A full one-liter container can be used to partially fill the one-gallon container, as shown below. Write a unit rate that estimates the number of gallons per liter.

c. Estimate the number of liters in 5.5 gallons and the number of gallons in 12 liters. What method(s) did you use? What other methods could you have used?

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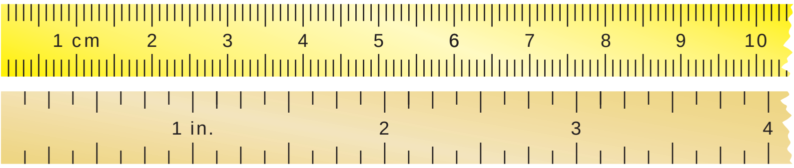
Exploration 2 Converting Units in a Rate

Work with a partner. The rate that a caterpillar moves is given in inches per minute. Using the rulers below, how can you convert the rate to centimeters per second? Justify your answer.



3 MTR **SELECT METHODS**

Explain when a ruler is useful for converting between centimeters and inches.

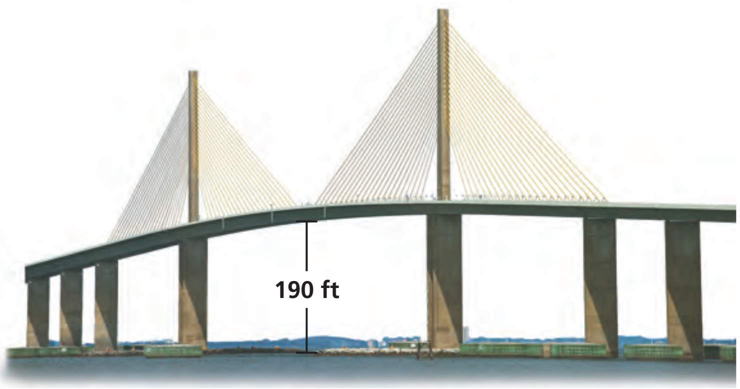
4.4 Lesson

For a list of conversion facts, see the Mathematics Reference Sheet in the back of this book.

Recall that the U.S. customary system is a system of measurement that contains units for length, capacity, and weight and the metric system is a decimal system of measurement, based on powers of 10, that contains units for length, capacity, and mass.

You can use unit rates and ratio tables to convert measures between systems.

Example 1 Converting Measures between Systems



The Sunshine Skyway Bridge connects St. Petersburg and Bradenton across Tampa Bay. How high above the water is the roadway in meters?

Because 1 foot ≈ 0.3 meter, there is about 0.3 meter per foot. Because 1 meter ≈ 3.28 feet, there are about 3.28 feet per meter. You can use either of these unit rates to find an equivalent rate with 190 feet.

6 MTR ASSESS REASONABLENESS

Does it make sense that Example 1 has two slightly different answers?

Method 1: Create a ratio table using the unit rate 0.3 meter per foot.

× 190

Meters	0.3	57
Feet	1	190

× 190

▶ So, the roadway is about 57 meters above the water.

Method 2: Create a ratio table using the unit rate 3.28 feet per meter.

÷ 3.28 × 190

Feet	3.28	1	190
Meters	1	$\frac{1}{3.28}$	$\frac{190}{3.28}$

÷ 3.28 × 190

▶ So, the roadway is about $\frac{190}{3.28} \approx 57.93$ meters above the water.

Try It **Complete the statement. Round to the nearest hundredth if necessary.**

1. 32 kg ≈ ☐ lb

2. 10 ft ≈ ☐ m

3. 1.5 mi ≈ ☐ km

4. 5 qt ≈ ☐ L

5. 42 cm ≈ ☐ in.

6. 8 m ≈ ☐ ft

GO DIGITAL

Recall that another way to convert units of measure is to multiply by one or more *conversion factors*. You can use unit analysis to decide which conversion factor will produce the appropriate units.

Example 2 Using Conversion Factors

a. Convert 4 pounds to kilograms.

Use a conversion factor that relates pounds and kilograms.

You can "cross out" a unit that appears in both a numerator and a denominator of a product.

1 lb ≈ 0.45 kg

$$4\text{ lb} \approx 4\,\cancel{\text{lb}} \times \frac{0.45\text{ kg}}{1\,\cancel{\text{lb}}} = 1.8\text{ kg}$$

▶ So, 4 pounds is about 1.8 kilograms.

b. Convert 8 quarts per day to liters per week.

Write 8 quarts per day as a fraction. Then use conversion factors.

1 wk = 7 days

1 qt ≈ 0.95 L

$$8\text{ quarts per day} \approx \frac{8\,\cancel{\text{qt}}}{1\,\cancel{\text{day}}} \times \frac{7\,\cancel{\text{days}}}{1\text{ wk}} \times \frac{0.95\text{ L}}{1\,\cancel{\text{qt}}} = \frac{53.2\text{ L}}{1\text{ wk}}$$

▶ So, 8 quarts per day is about 53.2 liters per week.

Try It

7. Convert 20 quarts to liters. Round to the nearest hundredth if necessary.

8. Convert 30 meters per second to miles per hour. Round to the nearest hundredth if necessary.

GO DIGITAL

In-Class Practice

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

4 MTR

9. DIFFERENT WORDS, SAME QUESTION Which is different? Find "both" answers.

Convert 5 inches to centimeters.	Find the number of inches in 5 centimeters.
How many centimeters are in 5 inches?	Five inches equals how many centimeters?

CONVERTING MEASURES **Complete the statement. Round to the nearest hundredth if necessary.**

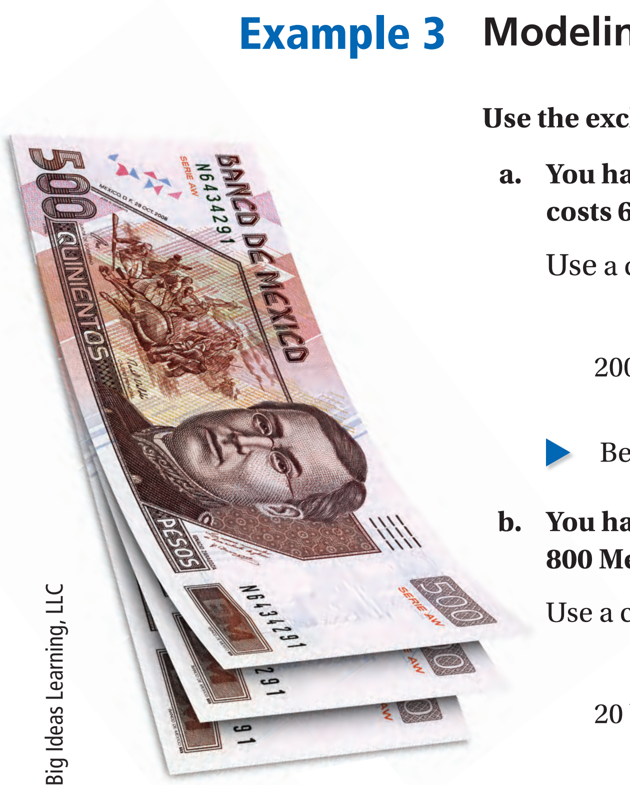
10. $\dfrac{12 \text{ m}}{\text{min}} \approx \dfrac{\square \text{ ft}}{\text{min}}$

11. $\dfrac{5 \text{ gal}}{\text{sec}} \approx \dfrac{\square \text{ L}}{\text{min}}$

An *exchange rate* represents the value of one currency relative to another. You can use an exchange rate as a conversion factor between currencies. Exchange rates between two currencies change over time.

Example 3 Modeling Real Life

7 MTR



Use the exchange rate 1 U.S. dollar (USD) ≈ 22.65 Mexican pesos (MXN).

a. You have 2000 Mexican pesos and want to buy a video game that costs 60 U.S. dollars. Do you have enough money?

Use a conversion factor.

1 USD ≈ 22.65 MXN

$$2000 \text{ MXN} \approx 2000 \cancel{\text{MXN}} \times \frac{1 \text{ USD}}{22.65 \cancel{\text{MXN}}} \approx 88.30 \text{ USD}$$

▶ Because 88.30 > 60, you have enough money to buy the video game.

b. You have 20 U.S. dollars and want to buy a video game that costs 800 Mexican pesos. Do you have enough money?

Use a conversion factor.

1 USD ≈ 22.65 MXN

$$20 \text{ USD} \approx 20 \cancel{\text{USD}} \times \frac{22.65 \text{ MXN}}{1 \cancel{\text{USD}}} = 453 \text{ MXN}$$

▶ Because 453 < 800, you do not have enough money to buy the video game.

GO DIGITAL

In-Class Practice

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

12. Use the exchange rate 1 U.S. dollar (USD) ≈ 1.36 Canadian dollars (CAD).

a. You have 475 Canadian dollars and want to buy a laptop that costs 300 U.S. dollars. Do you have enough money? Explain.

b. You have 400 U.S. dollars and want to buy a laptop that costs 600 Canadian dollars. Do you have enough money? Explain.

13. The table shows the currencies of four countries.

Country	Currency	Value in Dollars
United States	Dollar	$1
Japan	Yen	
Spain	Euro	
Great Britain	Pound	

7 MTR **a.** **INVESTIGATE** Use the Internet to find exchange rates for the currencies listed in the table.

b. How much of each currency would you receive in exchange for $20?

4.4 Practice WITH CalcChat® AND CalcView®

Review & Refresh

Find the unit rate.

1. 30 inches per 5 years

2. 486 games every 3 seasons

3. 8750 steps every 1.25 hours

4. 3.75 pints out of every 5 gallons

Find the sum.

5. $(4.3w - 7.3) + (-2.1w + 6.7)$

6. $\left(\frac{5}{2}x + \frac{1}{3}\right) + \left(-\frac{7}{2}x - \frac{5}{3}\right)$

Evaluate the expression.

7. $\left(\frac{3}{2}\right)^3$

8. -0.2^4

9. $-\left(\frac{4}{7}\right)^2$

Concepts, Skills, & Problem Solving

COMPARING MEASURES **Answer the question. Explain your answer.** (See Explorations 1 and 2.)

10. Which juice container is larger: 2 L or 1 gal?

11. Which is longer: 1 in. or 2 cm?

GO DIGITAL

CONVERTING MEASURES **Complete the statement. Round to the nearest hundredth if necessary.** (See Example 1.)

12. 12 L ≈ ☐ qt

▶ **13.** 14 m ≈ ☐ ft

14. 4 ft ≈ ☐ m

15. 64 lb ≈ ☐ kg

16. 0.3 km ≈ ☐ mi

17. 75.2 in. ≈ ☐ cm

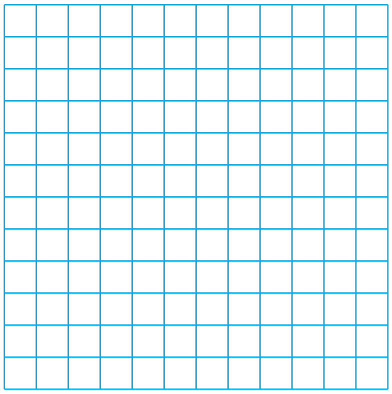
18. 17 kg ≈ ☐ lb

19. 15 cm ≈ ☐ in.

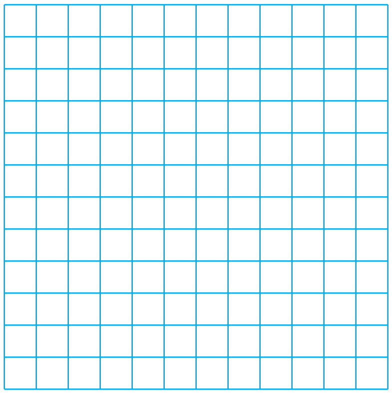
20. 9 mi ≈ ☐ km

21. **GRAPHING RELATIONSHIPS** Represent the relationship between each pair of units in a coordinate plane.

a. feet and meters



b. pounds and kilograms



7 MTR **22.** **MODELING REAL LIFE** Earth travels 30 kilometers each second as it revolves around the Sun. How many miles does Earth travel in 1 second?

USING CONVERSION FACTORS **Complete the statement. Round to the nearest hundredth if necessary.** (See Example 2.)

▶ **23.** 6 qt ≈ ☐ L

24. 12 cu ft ≈ ☐ gal

25. 5 L ≈ ☐ gal

26. $\frac{13\text{ km}}{\text{h}} \approx \frac{\square\text{ mi}}{\text{h}}$

27. $\frac{22\text{ L}}{\text{min}} \approx \frac{\square\text{ gal}}{\text{h}}$

28. $\frac{63\text{ mi}}{\text{h}} \approx \frac{\square\text{ km}}{\text{sec}}$

4 MTR

29. **YOU BE THE TEACHER** Your friend converts 8 liters to quarts. Is your friend correct? Explain your reasoning.

$$8\text{ L} \approx 8\text{ L} \cdot \frac{1.06\text{ qt}}{1\text{ L}}$$
$$= 8.48\text{ qt}$$

30. **B.E.S.T. Test Prep** Which quantity is greater than 3 meters?

Ⓐ 9 ft Ⓑ 120 in. Ⓒ 3.2 yd Ⓓ 0.002 km

COMPARING MEASURES **Complete the statement using < or >.**

31. 30 oz ☐ 8 kg

32. 6 ft ☐ 300 cm

33. 3 gal ☐ 6 L

34. 10 in. ☐ 200 mm

35. 5 lb ☐ 1200 g

36. 1500 m ☐ 3000 ft

37. **PROBLEM SOLVING** Thunder is the sound caused by lightning. You hear thunder 5 seconds after a lightning strike. The speed of sound is about 1225 kilometers per hour. About how many miles away was the lightning?

CONVERTING CURRENCY **Use the exchange rate 1 U.S. dollar (USD) ≈ 0.79 British pound (GBP) to convert between currencies.**

38. 30 USD ≈ ☐ GBP

39. 650 GBP ≈ ☐ USD

40. 765 GBP ≈ ☐ USD

7 MTR

▶ **41.** **MODELING REAL LIFE** Use the exchange rate 1 U.S. dollar (USD) ≈ 0.89 Euro (EUR). (See Example 3.)

a. You want to rent a hotel room that costs 150 euros per night. How much does the room cost per night in U.S. dollars?

b. You want to buy a plane ticket that costs 326 euros. How much does the plane ticket cost in U.S. dollars?

42. **REASONING** Hard Rock Stadium in Miami contains rectangular video boards that are 112 feet long and 49 feet wide.

a. Find the area of one of the video boards in square feet. Then find the area in square meters.

b. Compare the ratio of the areas to the conversion factor for feet to meters. What do you notice?

5 MTR

43. **STRUCTURE** Use your results from Exercise 42 to convert 50 square miles to square kilometers.

5 MTR

44. **STRUCTURE** Consider the conversion facts 1 inch $=$ 2.54 centimeters and 1 centimeter $\approx$ 0.39 inch.

a. Write an expression for the exact number of inches in 1 centimeter.

b. Use a calculator to evaluate your expression in part (a). Explain why measurement conversions may be slightly different when converting between metric units and U.S. customary units using the conversion facts in the back of this book.

45. **Dig Deeper** One liter of interior paint covers 100 square feet. How many gallons of paint does it take to cover 800 square meters of walls?

4.5 Identifying Proportional Relationships

Learning Target: Determine whether two quantities are in a proportional relationship.

Success Criteria:
- I can determine whether ratios form a proportion.
- I can explain how to determine whether quantities are proportional.
- I can distinguish between proportional and nonproportional situations.

Algebraic Reasoning

MA.7.AR.4.1 Determine whether two quantities have a proportional relationship by examining a table, graph or written description.

MA.7.AR.4.5 Solve real-world problems involving proportional relationships.

Exploration 1 Determining Proportional Relationships

Work with a partner.

a. You can paint 50 square feet of a surface every 40 minutes. How long does it take you to paint the mural shown? Explain how you found your answer.

b. The number of square feet you paint is *proportional* to the number of minutes it takes you. What do you think it means for a quantity to be *proportional* to another quantity?

c. Assume your friends paint at the same rate as you. The table shows how long it takes you and different numbers of friends to paint a fence. Is x proportional to y in the table? Explain.

Painters, x	1	2	3	4
Hours, y	4	2	$\frac{4}{3}$	1

d. How long will it take you and four friends to paint the fence? Explain how you found your answer.

5 MTR **USE A PATTERN**

How can the table in part (c) help you answer the question in part (d)?

GO DIGITAL

4.5 Lesson

Key Vocabulary
proportion, *p. 312*
cross products, *p. 313*
proportional, *p. 315*

Key Idea

Proportions

Words A **proportion** is an equation stating that the values of two ratios are equivalent.

Numbers Equivalent ratios: 2 : 3 and 4 : 6

Proportion: $\frac{2}{3} = \frac{4}{6}$

Example 1 Determining Whether Ratios Form a Proportion

Tell whether the ratios form a proportion.

a. 6 : 4 and 8 : 12

Compare the values of the ratios.

When you are determining whether ratios form a proportion, you are checking whether the ratios are equivalent.

$$\frac{6}{4} = \frac{6 \div 2}{4 \div 2} = \frac{3}{2}$$

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

The values of the ratios are *not* equivalent.

▶ Because $\frac{3}{2} \neq \frac{2}{3}$, the ratios 6 : 4 and 8 : 12 do *not* form a proportion.

b. 10 : 40 and 2.5 : 10

Compare the values of the ratios.

$$\frac{10}{40} = \frac{10 \div 10}{40 \div 10} = \frac{1}{4}$$

The values of the ratios are equivalent.

$$\frac{2.5}{10} = \frac{2.5 \times 10}{10 \times 10} = \frac{25}{100} = \frac{25 \div 25}{100 \div 25} = \frac{1}{4}$$

▶ Because $\frac{1}{4} = \frac{1}{4}$, the ratios 10 : 40 and 2.5 : 10 form a proportion.

GO DIGITAL

Try It **Tell whether the ratios form a proportion.**

1. 1 : 2 and 5 : 10

2. 4 : 6 and 18 : 24

3. 4.5 to 3 and 6 to 9

4. $\frac{1}{2}$ to $\frac{1}{4}$ and 8 to 4

Key Ideas

Cross Products

In the proportion $\frac{a}{b} = \frac{c}{d}$, the products $a \cdot d$ and $b \cdot c$ are called **cross products.**

Cross Products Property

Words The cross products of a proportion are equal.

Numbers

$$\frac{2}{3} = \frac{4}{6}$$

$$2 \cdot 6 = 3 \cdot 4$$

Algebra

$$\frac{a}{b} = \frac{c}{d}$$

$ad = bc$,
where $b \neq 0$ and $d \neq 0$

You can use the Multiplication Property of Equality to show that the cross products are equal.

$$\frac{a}{b} = \frac{c}{d}$$

$$\cancel{b}d \cdot \frac{a}{\cancel{b}} = b\cancel{d} \cdot \frac{c}{\cancel{d}}$$

$$ad = bc$$

Example 2 Using Cross Products

Tell whether the ratios form a proportion.

a. 6 : 9 and 12 : 18

Use the Cross Products Property to determine whether the ratios form a proportion.

$\frac{6}{9} \overset{?}{=} \frac{12}{18}$ Determine whether the values of the ratios are equivalent.

$6 \cdot 18 \overset{?}{=} 9 \cdot 12$ Find the cross products.

$108 = 108$ The cross products are equal.

▶ So, the ratios 6 : 9 and 12 : 18 form a proportion.

b. 2 : 3 and 4 : 5

Use the Cross Products Property to determine whether the ratios form a proportion.

$\frac{2}{3} \overset{?}{=} \frac{4}{5}$ Determine whether the values of the ratios are equivalent.

$2 \cdot 5 \overset{?}{=} 3 \cdot 4$ Find the cross products.

$10 \neq 12$ The cross products are *not* equal.

▶ So, the ratios 2 : 3 and 4 : 5 do *not* form a proportion.

Try It **Tell whether the ratios form a proportion.**

5. 6 : 2 and 12 : 1

6. 8 : 12 and $\frac{2}{3}$: 1

GO DIGITAL

Two quantities are **proportional** when all the ratios relating the quantities are equivalent. These quantities are said to be in a *proportional relationship*.

Example 3 Determining Whether Two Quantities Are Proportional

2 MTR USE ANOTHER METHOD

Can you use the values of the ratios y to x in Example 3? Explain.

Tell whether x and y are proportional.

x	y
$\frac{1}{2}$	3
1	6
$\frac{3}{2}$	9
2	12

Compare the values of the ratios x to y.

$$\frac{\frac{1}{2}}{3} = \frac{1}{6} \qquad \frac{1}{6} \qquad \frac{\frac{3}{2}}{9} = \frac{1}{6} \qquad \frac{2}{12} = \frac{1}{6}$$

The values of the ratios are equivalent.

▶ So, x and y are proportional.

Try It **Tell whether x and y are proportional.**

7.

x	1	2	3	4
y	2	4	6	8

8.

x	2	4	6	8	10
y	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$

GO DIGITAL

In-Class Practice

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

PROPORTIONS **Tell whether the ratios form a proportion.**

9. 4 : 14 and 12 : 40

10. 9 : 3 and 45 : 15

11. **VOCABULARY** Explain how to determine whether two quantities are proportional.

4 MTR **12.** **WHICH ONE DOESN'T BELONG?** Which ratio does *not* belong with the other three? Explain your reasoning.

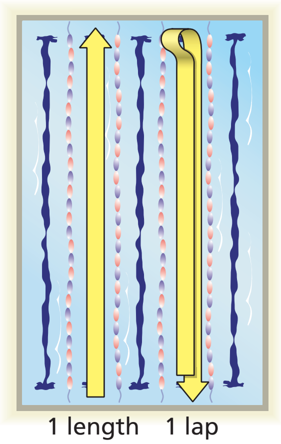
4 : 10

2 : 5

3 : 5

6 : 15

Example 4 Modeling Real Life



You swam for 16 minutes and completed 20 laps. You swam your first 4 laps in 2.4 minutes. How long does it take you to swim 10 laps?

Compare unit rates to determine whether the number of laps is proportional to your time. If it is, then you can use ratio reasoning to find the time it takes you to swim 10 laps.

2.4 minutes for every 4 laps: $\frac{2.4}{4} = 0.6$ minute per lap

16 minutes for every 20 laps: $\frac{16}{20} = 0.8$ minute per lap

The number of laps is *not* proportional to the time. So, you *cannot* use ratio reasoning to determine the time it takes you to swim 10 laps.

Because you slowed down after your first 4 laps, you can estimate that you swim 10 laps in more than

$$\frac{0.6\text{ minute}}{1\ \cancel{\text{lap}}} \cdot 10\ \cancel{\text{laps}} = 6\text{ minutes,}$$

but less than

$$\frac{0.8\text{ minute}}{1\ \cancel{\text{lap}}} \cdot 10\ \cancel{\text{laps}} = 8\text{ minutes.}$$

▶ So, you can estimate that it takes you about 7 minutes to swim 10 laps.

GO DIGITAL

In-Class Practice

1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

13. After making 20 servings of pasta, a chef has used 30 cloves of garlic. The chef used 6 cloves to make the first 4 servings. How many cloves of garlic are used to make 10 servings? Justify your answer.

14. A runner completed a 25-mile race in 5 hours. The runner completed the first 7.5 miles in 1.5 hours.

a. Do these rates form a proportion? Justify your answer.

b. Can you determine, with certainty, the time it took the runner to complete 10 miles? Explain your reasoning.

4.5 Practice WITH CalcChat® AND CalcView®

Review & Refresh

Complete the statement. Round to the nearest hundredth if necessary.

1. 62 in. ≈ m

2. 589 cu ≈ L

3. 2725 km ≈ mi

4. 48 cm ≈ ft

Add or subtract.

5. $-28 + 15$

6. $-6 + (-11)$

7. $-10 - 8$

8. $-17 - (-14)$

Solve the equation.

9. $\frac{x}{6} = 25$

10. $8x = 72$

11. $150 = 2x$

12. $35 = \frac{x}{4}$

Concepts, Skills, & Problem Solving

REASONING You can paint 75 square feet of a surface every 45 minutes. Determine how long it takes you to paint a wall with the given dimensions. (See Exploration 1.)

13. 8 ft × 5 ft

14. 7 ft × 6 ft

15. 9 ft × 9 ft

GO DIGITAL

PROPORTIONS **Tell whether the ratios form a proportion.** (See Examples 1 and 2.)

16. 1 to 3 and 7 to 21

17. 1 : 5 and 6 : 30

18. 3 to 4 and 24 to 18

▶**19.** 3.5 : 2 and 14 : 8

20. 24 : 30 and $3 : \frac{7}{2}$

21. $\frac{21}{2} : 3$ and 16 : 6

22. 0.6 : 0.5 and 12 : 10

▶**23.** 2 to 4 and 11 to $\frac{11}{2}$

24. $\frac{5}{8} : \frac{2}{3}$ and $\frac{1}{4} : \frac{1}{3}$

IDENTIFYING PROPORTIONAL RELATIONSHIPS **Tell whether *x* and *y* are proportional.** (See Example 3.)

▶**25.**

x	1	2	3
y	7	8	9

26.

x	2	4	6
y	5	10	15

27.

x	0.25	0.5	0.75
y	4	8	12

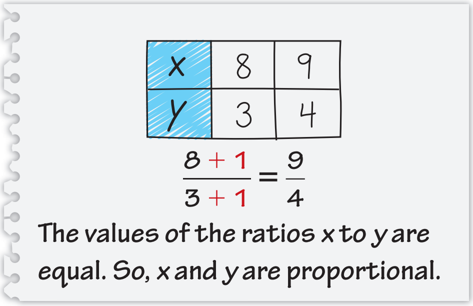
28.

x	$\frac{2}{3}$	1	$\frac{4}{3}$
y	$\frac{7}{10}$	$\frac{3}{5}$	$\frac{1}{2}$

4 MTR

YOU BE THE TEACHER **Your friend determines whether *x* and *y* are proportional. Is your friend correct? Explain your reasoning.**

29.

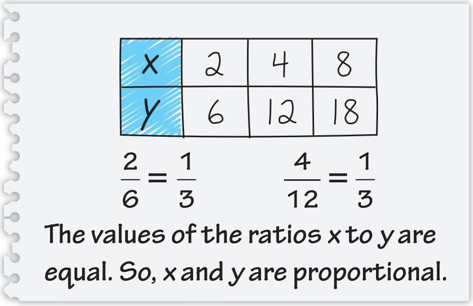


x	8	9
y	3	4

$\frac{8+1}{3+1} = \frac{9}{4}$

The values of the ratios x to y are equal. So, x and y are proportional.

30.



x	2	4	8
y	6	12	18

$\frac{2}{6} = \frac{1}{3}$ $\frac{4}{12} = \frac{1}{3}$

The values of the ratios x to y are equal. So, x and y are proportional.

PROPORTIONS **Tell whether the rates form a proportion.**

31. 7 inches in 9 hours;
42 inches in 54 hours

32. 12 players from 21 teams;
15 players from 24 teams

33. 385 calories in 3.5 servings; 300 calories in 3 servings

34. 4.8 laps every 8 minutes; 3.6 laps every 6 minutes

35. $\frac{3}{4}$ pound for every 5 gallons; $\frac{4}{5}$ pound for every $5\frac{1}{3}$ gallons

7 MTR

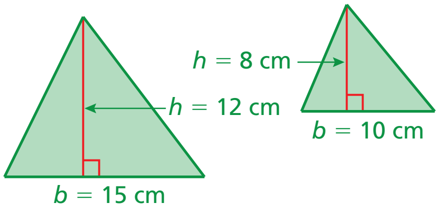
36. **MODELING REAL LIFE** You do 90 sit-ups in 2 minutes. Your friend does 126 sit-ups in 2.8 minutes. Do these rates form a proportion? Explain.

37. **MODELING REAL LIFE** Find the heart rates of yourself and your friend. Do these rates form a proportion? Explain.

	Heartbeats	Seconds
You	22	20
Friend	18	15

38. **PROBLEM SOLVING** You earn \$56 walking your neighbor's dog for 8 hours. Your friend earns \$36 painting your neighbor's fence for 4 hours. Are the pay rates equivalent? Explain.

39. **GEOMETRY** Are the heights and bases of the two triangles proportional? Explain.



40. **REASONING** A pitcher coming back from an injury limits the number of pitches thrown in bullpen sessions as shown.

Session Number, x	Pitches, y	Curveballs, z
1	10	4
2	20	8
3	30	12
4	40	16

a. Which quantities are proportional?

b. How many pitches that are *not* curveballs will the pitcher likely throw in Session 5?

5 MTR

41. **STRUCTURE** You add the same numbers of pennies and dimes to the coins shown. Is the new ratio of pennies to dimes proportional to the original ratio of pennies to dimes? If so, illustrate your answer with an example. If not, show why with a counterexample.

a.

b.

42. **REASONING** You are 13 years old, and your cousin is 19 years old. As you grow older, is your age proportional to your cousin's age? Explain your reasoning.

7 MTR

▶ **43.** **MODELING REAL LIFE** The shadow of the moon during a solar eclipse traveled 2300 miles in 1 hour. In the first 20 minutes, the shadow traveled $766\frac{2}{3}$ miles. How long did it take for the shadow to travel 1150 miles? Justify your answer. (See Example 4.)

7 MTR

44. **MODELING REAL LIFE** In 60 seconds, a car in a parade traveled 0.2 mile. The car traveled the last 0.05 mile in 12 seconds. How long did it take for the car to travel 0.1 mile? Justify your answer.

45. **OPEN-ENDED** Describe (a) a real-life situation where you expect two quantities to be proportional and (b) a real-life situation where you do *not* expect two quantities to be proportional. Explain your reasoning.

46. **PROBLEM SOLVING** A specific shade of red nail polish requires 7 parts red to 2 parts yellow. A mixture contains 35 quarts of red and 8 quarts of yellow. Is the mixture the correct shade? If so, justify your answer. If not, explain how you can fix the mixture to make the correct shade of red.

47. **NUMBER SENSE** The quantities x and y are proportional. Use each of the integers 1–5 to complete the table. Justify your answer.

x	10		6	
y				0.5

5 MTR

48. **STRUCTURE** Ratio A and Ratio B form a proportion. Ratio B and Ratio C also form a proportion. Do Ratio A and Ratio C form a proportion? Justify your answer.

4.6 Writing and Solving Proportions

Learning Target: Use proportions to solve ratio problems.

Success Criteria:
- I can solve proportions using various methods.
- I can find a missing value that makes two ratios equivalent.
- I can use proportions to represent and solve real-life problems.

Algebraic Reasoning

MA.7.AR.3.2 Apply previous understanding of ratios to solve real-world problems involving proportions.

MA.7.AR.4.4 Given any representation of a proportional relationship, translate the representation to a written description, table or equation.

Also MA.7.AR.4.5

Exploration 1 Solving a Ratio Problem

Work with a partner. A train travels 50 miles every 40 minutes. To determine the number of miles the train travels in 90 minutes, your friend creates the following table.

Miles	50	x
Minutes	40	90

a. Explain how you can find the value of x.

b. Can you use the information in the table to write a proportion? If so, explain how you can use the proportion to find the value of x. If not, explain why not.

1 MTR ANALYZE A PROBLEM

What equation can you use to find the answer in part (c)?

c. How far does the train below travel in 2 hours?

d. Share your results in part (c) with other groups. Compare and contrast methods used to solve the problem.

GO DIGITAL

4.6 Lesson

You can solve proportions using various methods.

Example 1 Solving a Proportion Using Mental Math

Solve $\frac{3}{2} = \frac{x}{8}$.

Step 1: Think: The product of 2 and what number is 8?

$$\underset{2 \times ? = 8}{\frac{3}{2} = \frac{x}{8}}$$

Step 2: Because the product of 2 and 4 is 8, multiply the numerator by 4 to find x.

$$\overset{3 \times 4 = 12}{\underset{2 \times 4 = 8}{\frac{3}{2} = \frac{x}{8}}}$$

▶ The solution is $x = 12$.

Try It **Solve the proportion.**

1. $\frac{5}{8} = \frac{20}{d}$

2. $\frac{7}{z} = \frac{14}{10}$

3. $\frac{21}{24} = \frac{x}{8}$

Example 2 Solving a Proportion Using Multiplication

Solve $\frac{5}{7} = \frac{x}{21}$.

$\frac{5}{7} = \frac{x}{21}$ Write the proportion.

$21 \cdot \frac{5}{7} = 21 \cdot \frac{x}{21}$ Multiplication Property of Equality

$15 = x$ Simplify.

▶ The solution is $x = 15$.

Try It **Solve the proportion.**

4. $\frac{w}{6} = \frac{6}{9}$

5. $\frac{12}{10} = \frac{a}{15}$

6. $\frac{y}{10} = \frac{3}{5}$

Example 3 Solving a Proportion Using Cross Products

Solve each proportion.

a. $\frac{x}{8} = \frac{7}{10}$

$x \cdot 10 = 8 \cdot 7$ Cross Products Property

$10x = 56$ Multiply.

$x = 5.6$ Divide each side by 10.

▶ The solution is $x = 5.6$.

b. $\frac{9}{y} = \frac{3}{17}$

$9 \cdot 17 = y \cdot 3$ Cross Products Property

$153 = 3y$ Multiply.

$51 = y$ Divide each side by 3.

▶ The solution is $y = 51$.

Try It **Solve the proportion.**

7. $\frac{2}{7} = \frac{x}{28}$

8. $\frac{12}{5} = \frac{6}{y}$

9. $\frac{40}{z+1} = \frac{15}{6}$

GO DIGITAL

Example 4 Writing and Solving a Proportion

Find the value of x so that the ratios 3 : 8 and x : 20 are equivalent.

For the ratios to be equivalent, the values of the ratios must be equal. So, find the value of x for which $\frac{3}{8}$ and $\frac{x}{20}$ are equal by solving a proportion.

$$\frac{3}{8} = \frac{x}{20} \qquad \text{Write a proportion.}$$

$$20 \cdot \frac{3}{8} = 20 \cdot \frac{x}{20} \qquad \text{Multiplication Property of Equality}$$

$$7.5 = x \qquad \text{Simplify.}$$

▶ So, 3 : 8 and x : 20 are equivalent when $x = 7.5$.

Try It **Find the value of x so that the ratios are equivalent.**

10. 2 : 4 and x : 6

11. x : 5 and 8 : 2

12. 4 to 3 and 10 to x

Example 5 B.E.S.T. Test Prep: Writing a Proportion

Black Bean Soup

1.5 cups black beans
0.5 cup salsa
2 cups water
1 tomato
2 teaspoons seasoning

A chef increases the amounts of ingredients in a recipe to make a proportional recipe. The new recipe has 6 cups of black beans. Which proportion can be used to find the number x of cups of water in the new recipe?

Ⓐ $\frac{2}{1.5} = \frac{6}{x}$

Ⓑ $\frac{1.5}{6} = \frac{x}{2}$

Ⓒ $\frac{1.5}{2} = \frac{x}{6}$

Ⓓ $\frac{1.5}{2} = \frac{6}{x}$

In the original recipe, the ratio of cups of black beans to cups of water is 1.5 : 2. In the new recipe, the ratio is 6 : x.

For the new recipe to be proportional to the original recipe, these ratios must be equivalent. So, the values of the ratios must be equal, $\frac{1.5}{2} = \frac{6}{x}$.

▶ The correct answer is Ⓓ.

GO DIGITAL

Try It

13. Write a proportion that can be used to find the number of tomatoes in the new recipe.

In-Class Practice

1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

SOLVING A PROPORTION **Solve the proportion.**

14. $\frac{5}{12} = \frac{b}{36}$

15. $\frac{6}{p} = \frac{42}{35}$

16. **WRITING AND SOLVING A PROPORTION** Find the value of x so that the ratios $x : 9$ and $5 : 6$ are equivalent.

4 MTR

17. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

Solve $\frac{3}{x} = \frac{12}{8}$.

Find x so that $3 : x$ and $12 : 8$ are equivalent.

Find x so that $3 : 12$ and $x : 8$ are equivalent.

Solve $\frac{12}{x} = \frac{3}{8}$.

Example 6 Modeling Real Life 7 MTR

A titanosaur's heart pumped 50 gallons of blood for every 2 heartbeats. How many heartbeats did it take to pump 1000 gallons of blood?

Understand the problem.

You are given the rate at which a titanosaur's heart pumped blood. Because all the rates you can write using this relationship are equivalent, the amount of blood pumped is proportional to the number of heartbeats. You are asked to find how many heartbeats it took to pump 1000 gallons of blood.

Make a plan.

The ratio of heartbeats to gallons of blood is 2 : 50. The number x of heartbeats for every 1000 gallons of blood can be represented by the ratio x : 1000. Use a proportion to find the value of x for which $\frac{2}{50}$ and $\frac{x}{1000}$ are equal.

Solve and check.

$\frac{2}{50} = \frac{x}{1000}$ Write a proportion.

$40 = x$ Multiply each side by 1000.

▶ So, it took 40 heartbeats to pump 1000 gallons of blood.

Another Method

You can use a ratio table to solve the problem.

× 20

Heartbeats	2	40
Blood (gallons)	50	1000

✓

× 20

In-Class Practice

1 I don't understand yet.

2 I can do it with help.

3 I can do it on my own.

4 I can teach someone else.

18. You burn 35 calories every 3 minutes running on a treadmill. You want to run for at least 15 minutes, but no more than 30 minutes. What are the possible numbers of calories that you will burn? Justify your answer.

19. **Dig Deeper** Two airboats travel at the same speed to different destinations. Airboat A reaches its destination in 12 minutes. Airboat B reaches its destination in 18 minutes. Airboat B travels 3 miles farther than Airboat A. How fast do the airboats travel? Justify your answer.

GO DIGITAL

4.6 Practice WITH CalcChat® AND CalcView®

Review & Refresh

Tell whether *x* and *y* are proportional.

1.

x	4	6	8
y	6	8	10

2.

x	$\frac{2}{5}$	$\frac{4}{5}$	4
y	3	6	30

Plot the ordered pair in a coordinate plane.

3. $A(-5, -2)$

4. $B(-3, 0)$

5. $C(-1, 2)$

6. $D(1, 4)$

7. Simplify $(3w - 8) - 4(2w + 3)$.

Concepts, Skills, & Problem Solving

SOLVING A RATIO PROBLEM **Determine how far the vehicle travels in 3 hours.** (See Exploration 1.)

8. A helicopter travels 240 miles every 2 hours.

9. A motorcycle travels 25 miles every 0.5 hour.

10. A train travels 10 miles every $\frac{1}{4}$ hour.

11. A ferry travels 45 miles every $1\frac{1}{2}$ hours.

SOLVING A PROPORTION **Solve the proportion. Explain your choice of method.**
(See Examples 1, 2, and 3.)

12. $\frac{1}{4} = \frac{z}{20}$

▶ **13.** $\frac{3}{4} = \frac{12}{y}$

14. $\frac{35}{k} = \frac{7}{3}$

15. $\frac{b}{36} = \frac{5}{9}$

16. $\frac{x}{8} = \frac{3}{12}$

▶ **17.** $\frac{3}{4} = \frac{v}{14}$

18. $\frac{15}{8} = \frac{45}{c}$

19. $\frac{35}{28} = \frac{n}{12}$

20. $\frac{a}{6} = \frac{15}{2}$

21. $\frac{y}{9} = \frac{44}{54}$

22. $\frac{4}{24} = \frac{c}{36}$

23. $\frac{20}{16} = \frac{d}{12}$

24. $\frac{10}{7} = \frac{8}{k}$

▶ **25.** $\frac{5}{n} = \frac{16}{32}$

26. $\frac{9}{10} = \frac{d}{6.4}$

27. $\frac{2.4}{1.8} = \frac{7.2}{k}$

28. **YOU BE THE TEACHER** Your friend solves the proportion $\frac{m}{8} = \frac{15}{24}$. Is your friend correct? Explain your reasoning.

$$\frac{m}{8} = \frac{15}{24}$$

$$m \cdot 24 = 8 \cdot 15$$

$$m = 5$$

29. **NUMBER SENSE** Without solving, determine whether $\frac{x}{4} = \frac{15}{3}$ and $\frac{x}{15} = \frac{4}{3}$ have the same solution. Explain your reasoning.

WRITING A PROPORTION **Use the table to write a proportion.**

30.

	Game 1	**Game 2**
Points	12	18
Shots	14	w

31.

	May	**June**
Winners	n	34
Entries	85	170

32.

	Today	**Yesterday**
Miles	15	m
Hours	2.5	4

33.

	Race 1	**Race 2**
Meters	100	200
Seconds	x	22.4

WRITING AND SOLVING A PROPORTION **Find the value of x so that the ratios are equivalent.** (See Example 4.)

34. 1 : 8 and 4 : x

▶ **35.** 4 to 5 and x to 20

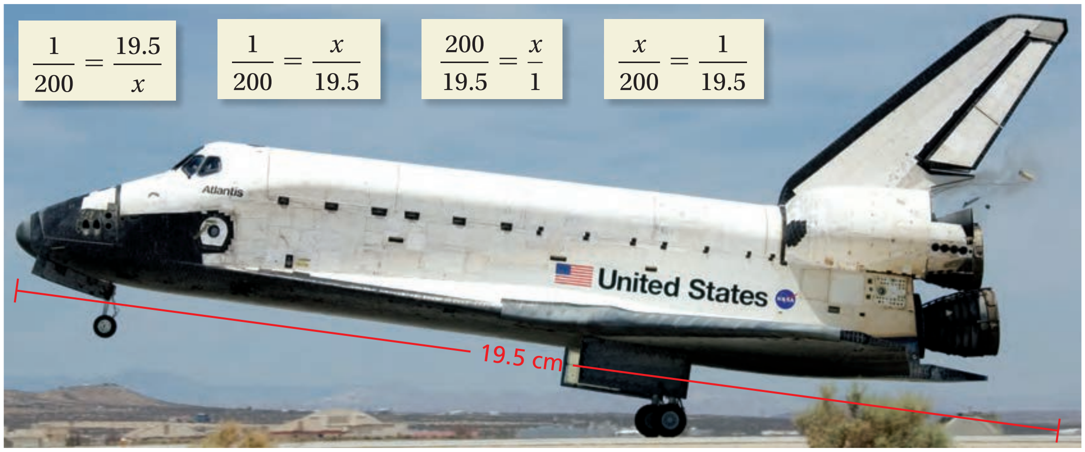
36. 3 : x and 12 : 40

37. x to 0.25 and 6 to 1.5

38. $x : \frac{5}{2}$ and 8 : 10

39. $\frac{7}{4}$ to 14 and x to 32

40. **WRITING A PROPORTION** Your science teacher has a photograph of the space shuttle *Atlantis*. Every 1 centimeter in the photograph represents 200 centimeters on the actual shuttle. Which of the proportions can you use to find the actual length x of *Atlantis*? Explain. (See Example 5.)



$\frac{1}{200} = \frac{19.5}{x}$ $\frac{1}{200} = \frac{x}{19.5}$ $\frac{200}{19.5} = \frac{x}{1}$ $\frac{x}{200} = \frac{1}{19.5}$

7 MTR **41.** **MODELING REAL LIFE** In an orchestra, the ratio of trombones to violas is 1 to 3. There are 9 violas. How many trombones are in the orchestra? (See Example 6.)

7 MTR **42.** **MODELING REAL LIFE** A dance team has 80 dancers. The ratio of seventh-grade dancers to all dancers is 5 : 16. Find the number of seventh-grade dancers on the team.

7 MTR **43.** **MODELING REAL LIFE** There are 144 people in an audience. The ratio of adults to children is 5 to 3. How many are adults?

44. **PROBLEM SOLVING** You have $50 to buy T-shirts. You can buy 3 T-shirts for $24. Do you have enough money to buy 7 T-shirts? Justify your answer.

45. **PROBLEM SOLVING** You buy 10 vegetarian pizzas and pay with $100. How much change do you receive?

46. **MODELING REAL LIFE** A person who weighs 120 pounds on Earth weighs 20 pounds on the Moon. How much does a 93-pound person weigh on the Moon?

47. **PROBLEM SOLVING** Three pounds of grass seed covers 1800 square feet. How many bags are needed to cover 8400 square feet?

48. **MODELING REAL LIFE** There are 180 white lockers in a school. There are 3 white lockers for every 5 blue lockers. How many lockers are in the school?

CONVERTING MEASURES **Use a proportion to complete the statement. Round to the nearest hundredth if necessary.**

49. 6 km ≈ ____ mi

50. 2.5 L ≈ ____ gal

51. 90 lb ≈ ____ kg

SOLVING A PROPORTION **Solve the proportion.**

52. $\frac{2x}{5} = \frac{9}{15}$

53. $\frac{5}{2} = \frac{d-2}{4}$

54. $\frac{4}{k+3} = \frac{8}{14}$

6 MTR

55. **ASSESS REASONABLENESS** It takes 6 hours for 2 people to build a swing set. Your friend uses the proportion $\frac{2}{6} = \frac{5}{h}$ to determine the number of hours h it will take 5 people to build the swing set. Will your friend's method result in a reasonable answer? Explain.

5 MTR

56. **STRUCTURE** The ratios $a : b$ and $c : d$ are equivalent. Which of the following equations are proportions? Explain your reasoning.

$\frac{b}{a} = \frac{d}{c}$

$\frac{a}{c} = \frac{b}{d}$

$\frac{a}{d} = \frac{c}{b}$

$\frac{c}{a} = \frac{d}{b}$

5 MTR

57. **STRUCTURE** Consider the proportions $\frac{m}{n} = \frac{1}{2}$ and $\frac{n}{k} = \frac{2}{5}$. What is $\frac{m}{k}$? Explain your reasoning.

4.7 Using Graphs of Proportional Relationships

Learning Target: Represent proportional relationships using graphs and equations.

Success Criteria:
- I can graph an equation in two variables.
- I can determine whether quantities are proportional using a graph.
- I can find the unit rate of a proportional relationship using a graph.
- I can create equations to represent proportional relationships.

Algebraic Reasoning

MA.7.AR.4.1 Determine whether two quantities have a proportional relationship by examining a table, graph or written description.

MA.7.AR.4.2 Determine the constant of proportionality within a mathematical or real-world context given a table, graph or written description of a proportional relationship.

Also MA.7.AR.4.3, MA.7.AR.4.4, MA.7.AR.4.5

Exploration 1 Representing Relationships Graphically

Work with a partner. The tables represent two different ways that red and blue food coloring are mixed.

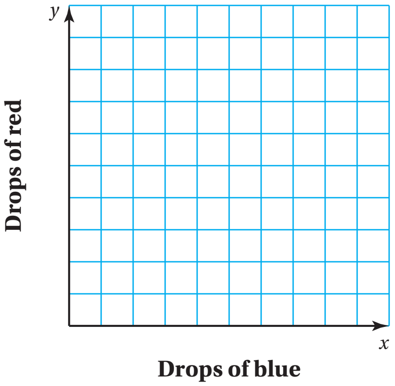
Mixture 1

Drops of Blue, x	Drops of Red, y
1	2
2	4
3	6
4	8

Mixture 2

Drops of Blue, x	Drops of Red, y
0	2
2	4
4	6
6	8

a. Represent each table in the same coordinate plane. Which graph represents a proportional relationship? How do you know?



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2 MTR **MAKE A CONNECTION**

How is the graph of the proportional relationship different from the other graph?

b. Find the unit rate of the proportional relationship. How is the unit rate shown on the graph?

c. What is the multiplicative relationship between x and y for the proportional relationship? How can you use this value to write an equation that relates y and x?

4.7 Lesson

Key Vocabulary

equation in two variables, *p. 339*
solution of an equation in two variables, *p. 339*
independent variable, *p. 339*
dependent variable, *p. 339*
constant of proportionality, *p. 340*

An **equation in two variables** represents two quantities that change in relationship to one another. A **solution of an equation in two variables** is an ordered pair that makes the equation true. The variable representing the quantity that can change freely is the **independent variable**. The other variable is called the **dependent variable** because its value *depends* on the independent variable.

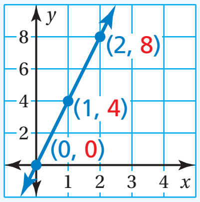
Key Idea

Tables, Graphs, and Equations

You can use tables and graphs to represent equations in two variables. The independent variable is graphed on the horizontal axis, and the dependent variable is graphed on the vertical axis. The table and graph below represent the equation $y = 4x$.

When you draw a line through the points, you graph *all* the solutions of the equation.

Independent Variable, *x*	Dependent Variable, *y*	Ordered Pair, (*x*, *y*)
0	0	(0, 0)
1	4	(1, 4)
2	8	(2, 8)



Example 1 Graphing an Equation in Two Variables

Graph $y = 10x$.

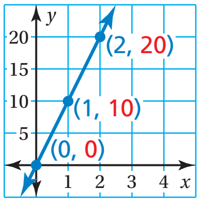
To graph the equation, first make a table.

Independent Variable, *x*	$y = 10x$	Dependent Variable, *y*	Ordered Pair, (*x*, *y*)
0	$y = 10(0)$	0	(0, 0)
1	$y = 10(1)$	10	(1, 10)
2	$y = 10(2)$	20	(2, 20)

Remember

In a coordinate plane, the horizontal axis is often called the *x-axis*. The vertical axis is often called the *y-axis*. In real-life problems, other variables can be used.

Then plot the ordered pairs and draw a line through the points.



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Try It **Graph the equation.**

1. $y = 12x$

2. $y = \frac{1}{2}x$

3. $y = x + 1$

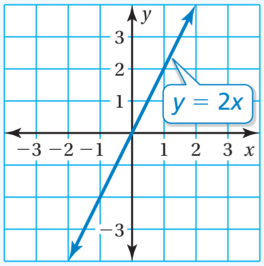
The equation $y = kx$ can also be written as $\frac{y}{x} = k$. So, k is equal to the value of the ratio $y : x$.

Key Idea

Graphs of Proportional Relationships

Words Two quantities x and y are proportional when $y = kx$, where k is a number and $k \neq 0$. The number k represents the multiplicative relationship between the quantities and is called the **constant of proportionality**.

Graph The graph of $y = kx$ is a line that passes through the origin.



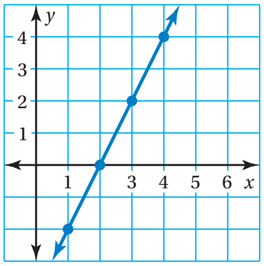
Example 2 Determining Whether Two Quantities Are Proportional

Tell whether x and y are proportional. If so, find the constant of proportionality. Explain your reasoning.

a.

x	1	2	3	4
y	−2	0	2	4

Plot the points. Draw a line through the points.

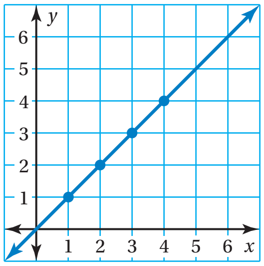


▶ The line does *not* pass through the origin. So, x and y are not proportional.

b.

x	1	2	3	4
y	1	2	3	4

Plot the points. Draw a line through the points.



▶ The line passes through the origin. So, x and y are proportional. The constant of proportionality is $k = 1$ because the value of the ratio $\frac{y}{x}$ is equal to 1.

Try It **Tell whether x and y are proportional. If so, find the constant of proportionality. Explain your reasoning.**

4.

x	1	2	3	4
y	1	4	7	10

5.

x	1	2	3	4
y	4	8	12	16

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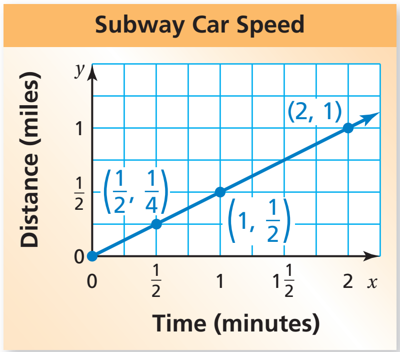
Example 3 Finding a Unit Rate from a Graph



The graph shows the speed of a subway car. Find the speed in miles per minute.

The graph is a line through the origin, so time and distance are proportional. To find the speed in miles per minute, use a point on the graph to find the unit rate.

One Way: Use the point (2, 1) to find the speed.

The point (2, 1) indicates that the subway car travels 1 mile every 2 minutes. So, the unit rate is

$\frac{1}{2}$ mile per minute.

▶ The speed of the subway car is $\frac{1}{2}$ mile per minute.

On the graph of a proportional relationship, the point (1, *k*) indicates the unit rate, *k* : 1, and the constant of proportionality, *k*. This value is a measure of the steepness, or slope, of the line.

Another Way: Use the point $\left(1, \frac{1}{2}\right)$ to find the speed.

The point $\left(1, \frac{1}{2}\right)$ indicates that the subway car travels $\frac{1}{2}$ mile every 1 minute. This is the unit rate.

▶ The speed of the subway car is $\frac{1}{2}$ mile per minute.

Try It

6. **WHAT IF?** Does your answer change when you use the point $\left(\frac{1}{2}, \frac{1}{4}\right)$ to find the speed of the subway car? Explain your reasoning.

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In-Class Practice

1 I don't understand yet.
2 I can do it with help.
3 I can do it on my own.
4 I can teach someone else.

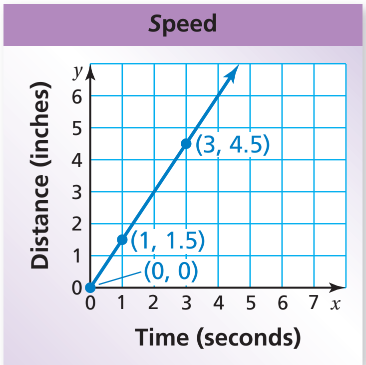
GRAPHING EQUATIONS **Graph the equation.**

7. $y = 7x$

8. $y = \frac{1}{4}x$

9. $y = 2x + 3$

10. **IDENTIFYING A PROPORTIONAL RELATIONSHIP** Use the graph shown to tell whether x and y are proportional. Explain your reasoning.



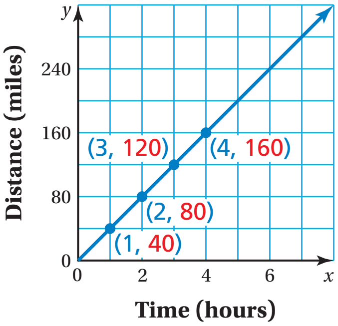
11. **FINDING A UNIT RATE** Interpret each plotted point in the graph. Then identify the unit rate, if possible.

Example 4 Modeling Real Life

7 MTR

A train averages 40 miles per hour between two cities. Write and graph an equation that represents the relationship between the time and the distance traveled. How long does it take the train to travel 220 miles?

The rate is 40 miles per hour. Because all the rates you can write using this relationship are equivalent, the distance traveled is proportional to the time spent traveling. The constant of proportionality is 40, so an equation for the distance traveled y (in miles) after x hours is $y = 40x$.



Make a table and graph the equation.

Time (hours), x	1	2	3	4
Distance (miles), y	40	80	120	160

Use the equation to find the value of x when $y = 220$.

$y = 40x$ Write the equation.

$220 = 40x$ Substitute 220 for y.

$5.5 = x$ Divide each side by 40.

▶ The train travels 220 miles in 5.5 hours.

Another Method

Use a ratio table.

÷ 2 × 11

Time (hours), x	1	0.5	5.5
Distance (miles), y	40	20	220

÷ 2 × 11

✓

You can also tell from the graph that the distance is about 220 miles after 5.5 hours.

In-Class Practice

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

12. A sky lantern rises at an average speed of 8 feet per second. Write and graph an equation that represents the relationship between the time and the distance risen. How long does it take the lantern to rise 100 feet?

13. You and a friend start biking in opposite directions from the same point. You travel 108 feet every 8 seconds. Your friend travels 63 feet every 6 seconds. How far apart are you and your friend after 15 minutes?

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4.7 Practice WITH CalcChat® AND CalcView®

Review & Refresh

Find the value of x so that the ratios are equivalent.

1. $2 : 7$ and $8 : x$

2. 3 to 2 and x to 18

3. $9 : x$ and $54 : 8$

Find the quotient, if possible.

4. $36 \div 4$

5. $42 \div (-6)$

6. $-39 \div 3$

7. $-44 \div (-4)$

Solve the inequality. Graph the solution.

8. $-\frac{x}{3} < 2$

9. $\frac{1}{3}p \geq 4$

10. $-8 < \frac{2}{3}n$

11. $-2w \leq 10$

Concepts, Skills, & Problem Solving

REPRESENTING RELATIONSHIPS GRAPHICALLY **Represent the table graphically. Does the graph represent a proportional relationship? How do you know?** (See Exploration 1.)

12.

Hours, x	Miles, y
0	50
1	100
2	150

13.

Cucumbers, x	Tomatoes, y
2	4
3	6
4	8

GRAPHING EQUATIONS **Graph the equation.** (See Example 1.)

14. $y = 2x$

15. $y = 5x$

16. $y = 6x$

17. $y = x + 2$

18. $y = x + 0.5$

▶ **19.** $y = x + 4$

20. $y = x + 10$

21. $y = 3x + 2$

22. $y = 2x + 4$

23. $y = \frac{2}{3}x + 8$

24. $y = \frac{1}{4}x + 6$

25. $y = 2.5x + 12$

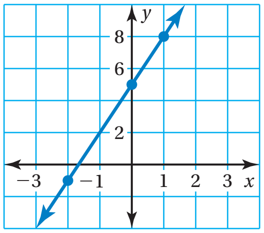
26. **B.E.S.T. Test Prep** Which equation is shown in the graph?

Ⓐ $y = 3x - 5$

Ⓑ $y = 3x + 5$

Ⓒ $y = 4x - 2$

Ⓓ $y = 4x + 2$



27. **MODELING REAL LIFE** The number of people y entering the Dalí Museum in St. Petersburg each hour x can be representedby $y = 20x$. Graph the equation. After how many hours do 120 people enter the Dalí Museum?

IDENTIFYING A PROPORTIONAL RELATIONSHIP **Tell whether x and y are proportional. If so, find the constant of proportionality. Explain your reasoning.** (See Example 2.)

28.

x	1	2	3	4
y	2	4	6	8

▶ **29.**

x	−2	−1	0	1
y	0	2	4	6

30.

x	−1	0	1	2
y	−2	−1	0	1

31.

x	3	6	9	12
y	2	4	6	8

32.

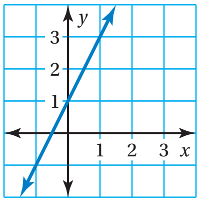
x	1	2	3	4
y	3	4	5	6

33.

x	1	3	5	7
y	0.5	1.5	2.5	3.5

4 MTR

34. **YOU BE THE TEACHER** Your friend uses the graph to determine whether x and y are proportional. Is your friend correct? Explain your reasoning.



> The graph is a line, so x and y are proportional.

CONSTANT OF PROPORTIONALITY **Identify the constant of proportionality in the situation.**

35. A car travels 75 miles in 1 hour.

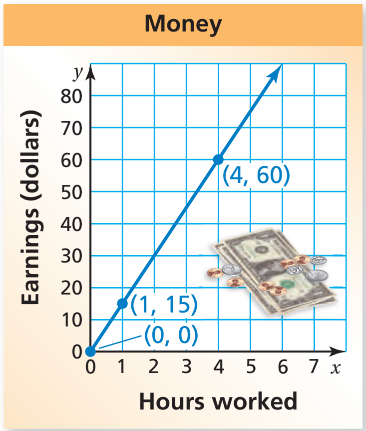
36. In 2019, there were about 80 alligators per 4 square miles in Florida.

37. A restaurant prepares 15 *pan con bistec* in 30 minutes.

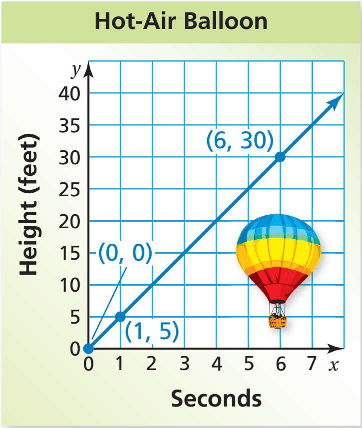
38. A veterinarian sees 48 pets in 8 hours.

FINDING A UNIT RATE **Interpret each plotted point in the graph. Then identify the unit rate.** (See Example 3.)

▶**39.**



40.



IDENTIFYING A PROPORTIONAL RELATIONSHIP **Tell whether x and y are proportional. If so, identify the constant of proportionality. Explain your reasoning.**

41. $x - y = 0$

42. $\frac{x}{y} = 2$

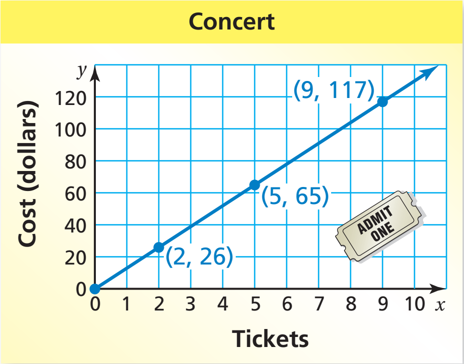
43. $8 = xy$

44. $x^2 = y$

45. **MODELING REAL LIFE** The table shows the profit y for recycling x pounds of aluminum. Find the profit for recycling 75 pounds of aluminum.

Aluminum (lb), x	10	20	30	40
Profit, y	\$4.50	\$9.00	\$13.50	\$18.00

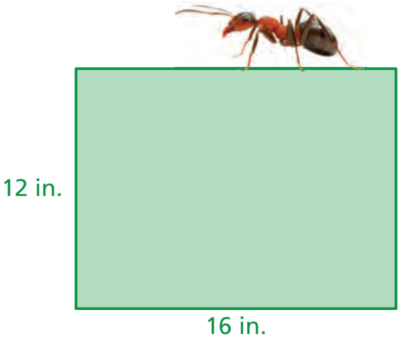
46. **MODELING REAL LIFE** The graph shows the cost of buying concert tickets. Tell whether x and y are proportional. If so, find and interpret the constant of proportionality. Then find the cost of 14 tickets.



47. **MODELING REAL LIFE** You charge \$10 to mow a lawn. Write and graph an equation that represents the amount you earn (in dollars) for mowing lawns in your neighborhood. How much do you earn when you mow 17 lawns? (See Example 4.)

48. **MODELING REAL LIFE** It costs \$35 a month for membership at a wholesale store. Write and graph an equation that represents the monthly cost (in dollars) of a membership. What is the cost of a membership for an entire year?

49. **GEOMETRY** How fast should the ant walk to go around the rectangle in 4 minutes?



50. **REASONING** The graph of a proportional relationship passes through (12, 16) and (1, y). Find y.

8000 gallons

51. **PROBLEM SOLVING** The amount of chlorine in a swimming pool is proportional to the volume of water. The pool has 2.5 milligrams of chlorine per liter of water. How much chlorine is in the pool?

7 MTR

52. **MODELING REAL LIFE** To estimate how far you are from lightning (in miles), count the number of seconds between a lightning flash and the thunder that follows. Then divide the number of seconds by 5. Use two different methods to find the number of seconds between a lightning flash and the thunder that follows when a storm is 2.4 miles away.

53. **Dig Deeper** A vehicle travels 250 feet every 3 seconds. Find the value of the ratio, the unit rate, and the constant of proportionality. How are they related?

4.8 Scale Drawings

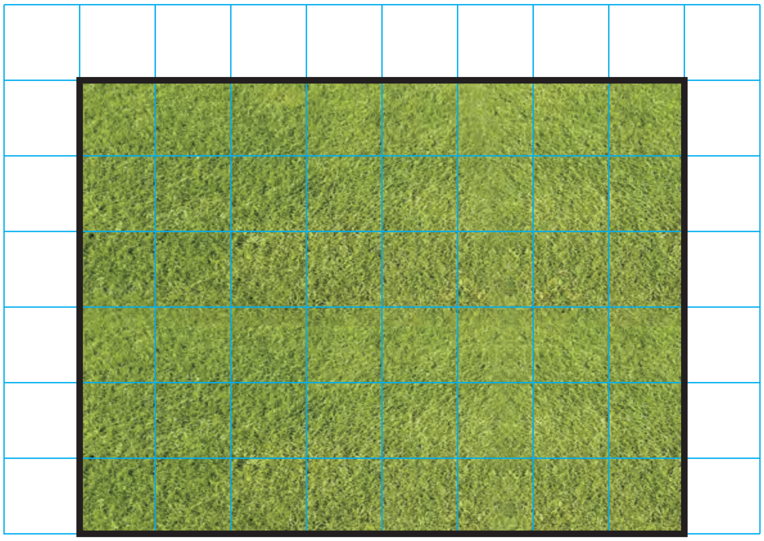
Learning Target: Solve problems involving scale drawings.

Success Criteria:
- I can find an actual distance in a scale drawing.
- I can explain the meaning of scale and scale factor.
- I can use a scale drawing to find the actual lengths and areas of real-life objects.

Algebraic Reasoning
MA.7.AR.4.5 Solve real-world problems involving proportional relationships.
Geometric Reasoning
MA.7.GR.1.5 Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.

Exploration 1 Creating a Scale Drawing

Work with a partner. An enclosure in a zoo is drawn on 1-centimeter grid paper as shown. Each centimeter in the drawing represents 4 meters.



a. Describe the relationship between the side lengths of the fence in the drawing and the actual side lengths of the fence.

b. Describe the relationship between the area of the enclosure in the drawing and the actual area of the enclosure.

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c. Are the relationships in parts (a) and (b) the same? Explain your reasoning.

d. Choose a different distance to represent each centimeter on a piece of 1-centimeter grid paper. Then create a new drawing of the enclosure using the distance you chose. Describe any similarities or differences in the drawings.

1 MTR

ANALYZE A PROBLEM

How does the information given about the drawing shown help you create an accurate drawing in part (d)?

4.8 Lesson

Key Vocabulary

scale drawing, *p. 353*
scale model, *p. 353*
scale, *p. 353*
scale factor, *p. 354*

Recall that a ratio $a : b$ is equivalent to $1 : \frac{b}{a}$. A scale is usually written as a ratio where the first quantity is 1 unit.

Key Ideas

Scale Drawings and Models

A **scale drawing** is a proportional, two-dimensional drawing of an object.

A **scale model** is a proportional, three-dimensional model of an object.

Scale

The measurements in scale drawings and models are proportional to the measurements of the actual object. The **scale** gives the ratio that compares the measurements of the drawing or model with the actual measurements.

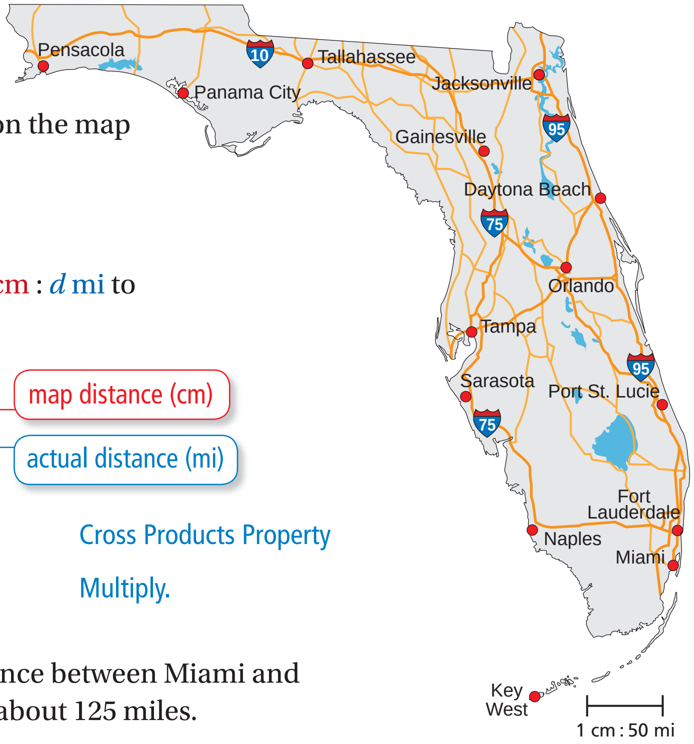
1 in. : 10 mi

drawing

actual

Example 1 Finding an Actual Distance

What is the actual distance *d* between Miami and Key West?



Step 1: Use a centimeter ruler to find the distance on the map between Miami and Key West.

The map distance is about 2.5 centimeters.

Step 2: Use the scale 1 cm : 50 mi and the ratio 2.5 cm : *d* mi to write and solve a proportion.

$$\frac{1}{50} = \frac{2.5}{d}$$

map distance (cm)

actual distance (mi)

$d = 50 \cdot 2.5$ Cross Products Property

$d = 125$ Multiply.

▶ So, the distance between Miami and Key West is about 125 miles.

Another Method

You can use a ratio table.

× 2.5

Centimeters	1	2.5
Miles	50	125

✓

× 2.5

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Try It

1. What is the actual distance between Pensacola and Jacksonville?

A scale can be written without units when the units are the same. The value of this ratio is called the **scale factor**. The scale factor is the constant of proportionality between the dimensions of a scale drawing or scale model and the dimensions of the actual object.

Example 2 Finding a Scale Factor

A scale model of the Sergeant Floyd Monument in Sioux City, Iowa, is 10 inches tall. The actual monument is 100 feet tall.

a. What does 1 inch represent in the model? What is the scale?

The ratio of the model height to the actual height is 10 in. : 100 ft. Divide each quantity by 10 to determine the number of feet represented by 1 inch in the model.

$$\begin{tikzcd} 10 \text{ in.} : 100 \text{ ft} \arrow[d, bend right, "\div 10"'] \arrow[d, bend left, "\div 10"] \\ 1 \text{ in.} : 10 \text{ ft} \end{tikzcd}$$

▶ In the model, 1 inch represents 10 feet. So, the scale is 1 in. : 10 ft.

b. What is the scale factor of the model?

Write the scale with the same units. Use the fact that 1 ft = 12 in.

$$10 \text{ ft} = 10 \cancel{\text{ft}} \times \frac{12 \text{ in.}}{1 \cancel{\text{ft}}} = 120 \text{ in.}$$

▶ The scale is 1 in. : 120 in., or 1 : 120. So, the scale factor is $\frac{1}{120}$.

Try It

2. A drawing has a scale of 1 mm : 20 cm. What is the scale factor of the drawing?

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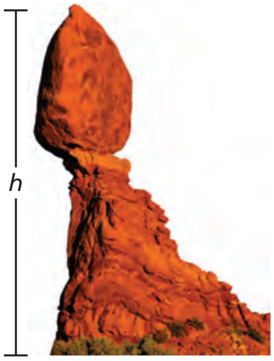
In-Class Practice

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

3. **VOCABULARY** In your own words, explain the meaning of the scale and scale factor of a drawing or model.

4. **FINDING AN ACTUAL DISTANCE** Consider the scale drawing of Balanced Rock in Arches National Park in Utah. What is the actual height of the structure?



1 cm : 32 ft

5. **FINDING A SCALE FACTOR** A drawing has a scale of 3 in. : 2 ft. What is the scale factor of the drawing?

6. **REASONING** Describe the scale factor of a model that is (a) larger than the actual object and (b) smaller than the actual object.

For a scale factor k, the constant of proportionality between corresponding perimeters is k and the constant of proportionality between corresponding areas is k^2.

$$\frac{\text{drawing perimeter}}{\text{actual perimeter}} = k \qquad \frac{\text{drawing area}}{\text{actual area}} = k^2$$

Example 3 Modeling Real Life

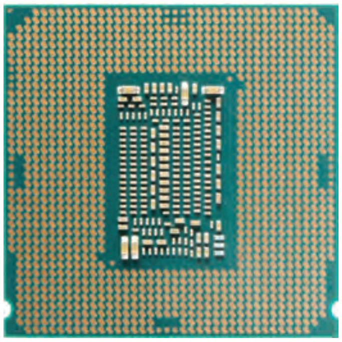
7 MTR

The scale drawing of a square computer chip helps you see the individual components on the chip.

a. **Find the perimeter and the area of the computer chip in the scale drawing.**

When measured using a centimeter ruler, the scale drawing of the computer chip has a side length of 4 centimeters, or 40 millimeters.

▶ So, the perimeter of the computer chip in the scale drawing is $4(40) = 160$ millimeters, and the area is $40^2 = 1600$ square millimeters.



5 mm : 1 mm

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b. Find the actual perimeter and area of the computer chip.

The scale is 5 mm : 1 mm, or 5 : 1. So, the scale factor is $k = 5$.

$$\frac{\text{drawing perimeter}}{\text{actual perimeter}} = k \qquad \text{Write equation.}$$

$$\frac{160}{\text{actual perimeter}} = 5 \qquad \text{Substitute.}$$

$$32 = \text{actual perimeter} \qquad \text{Solve for actual perimeter.}$$

$$\frac{\text{drawing area}}{\text{actual area}} = k^2 \qquad \text{Write equation.}$$

$$\frac{1600}{\text{actual area}} = 5^2 \qquad \text{Substitute.}$$

$$64 = \text{actual area} \qquad \text{Solve for actual area.}$$

▶ So, the actual perimeter of the computer chip is 32 millimeters, and the actual area is 64 square millimeters.

In-Class Practice

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

7. A scale drawing of the Parthenon is shown. Find the actual perimeter and area of the rectangular face of the Parthenon. Then recreate the scale drawing with a scale factor of 0.2. Find the perimeter and area of the rectangular face in your drawing.

Scale: 1 ft : 11.2 ft

8. **Dig Deeper** You are in charge of creating a billboard advertisement that is 16 feet long and 8 feet tall. Choose a product. Create a scale drawing of the billboard using words and a picture. What is the scale factor of your design?

4.8 Practice WITH CalcChat® AND CalcView®

Review & Refresh

Tell whether x and y are proportional. Explain your reasoning.

1.

x	10	9	8	7
y	5	4	3	2

2.

x	6	12	18	24
y	7	14	21	28

Simplify the expression.

3. $7p + 6p$

4. $8 + 3d - 17$

5. $-2 + \frac{2}{5}b - \frac{1}{4}b + 6$

Write the word sentence as an inequality.

6. A number c is less than -3.

7. 7 plus a number z is more than 5.

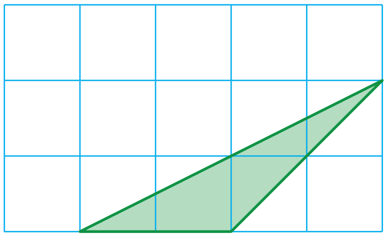
8. The product of a number m and 6 is no less than 30.

GO DIGITAL

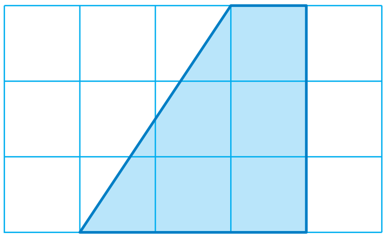
Concepts, Skills, & Problem Solving

CREATING A SCALE DRAWING **Each centimeter on the 1-centimeter grid paper represents 8 inches. Create a proportional drawing of the figure that is larger or smaller than the figure shown.** (See Exploration 1.)

9.



10.



FINDING AN ACTUAL DISTANCE **Use the map in Example 1 to find the actual distance between the cities.** (See Example 1.)

▶ **11.** Tallahassee and Gainesville

12. Naples and Daytona Beach

13. Fort Lauderdale and Panama City

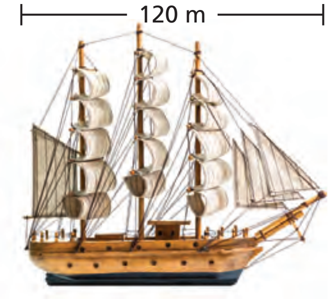
14. Tampa and Jacksonville

USING A SCALE **Find the missing dimension. Use the scale 1 : 12.**

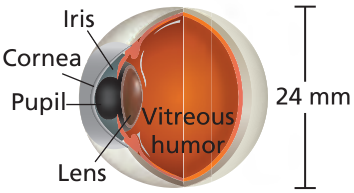
	Item	Model	Actual
15.	Mattress	Length: 6.25 in.	Length: ☐ in.
16.	Corvette	Length: ☐ in.	Length: 15 ft
17.	Water tower	Depth: 32 cm	Depth: ☐ m
18.	Wingspan	Width: 5.4 ft	Width: ☐ yd
19.	Football helmet	Diameter: ☐ mm	Diameter: 21 cm

FINDING A SCALE FACTOR **Use a centimeter ruler to find the scale and the scale factor of the drawing.** (See Example 2.)

20.

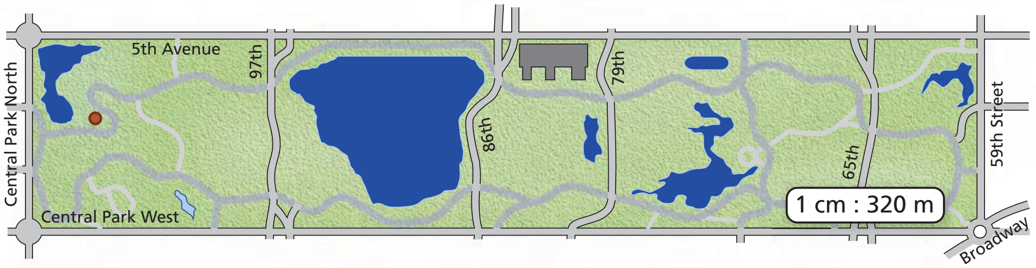


▶ **21.**



22. **REASONING** You know the length and the width of a scale model. What additional information do you need to know to find the scale of the model? Explain.

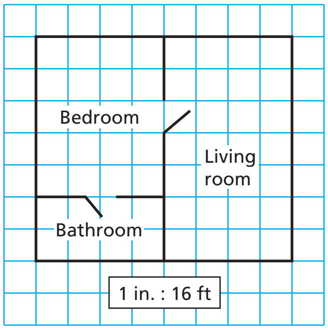
7 MTR ▶ **23.** **MODELING REAL LIFE** Central Park is a rectangular park in New York City. (See Example 3.)



a. Find the perimeter and the area of the scale drawing of Central Park.

b. Find the actual perimeter and area of Central Park.

24. PROBLEM SOLVING In a blueprint, each square has a side length of $\frac{1}{4}$ inch.

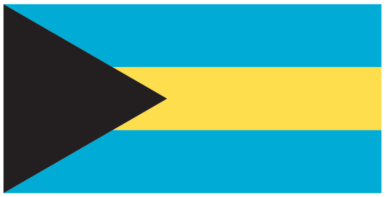


a. Ceramic tile costs \$5 per square foot. How much does it cost to tile the bathroom?

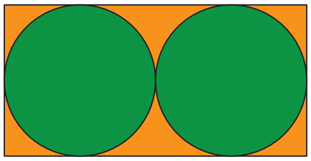
b. Carpet costs \$18 per square yard. How much does it cost to carpet the bedroom and living room?

REPRODUCING A SCALE DRAWING **Recreate the scale drawing so that it has a scale of 1 cm : 4 m.**

25.



26.



27. **Dig Deeper** Make a conjecture about the relationship between the scale factor of a model and the quotient $\frac{\text{model volume}}{\text{actual volume}}$. Explain your reasoning.

4 Connecting Concepts

5 MTR *Mathematicians who use patterns and structure to help understand and connect mathematical concepts relate previously learned concepts to new concepts.*

1 MTR **1.** **ANALYZE A PROBLEM** The table shows the toll y (in dollars) for traveling x miles on a turnpike. You have \$8.25 to pay your toll. How far can you travel on the turnpike?

Distance, x (miles)	25	30	35	40
Toll, y (dollars)	3.75	4.50	5.25	6.00

Using the Problem-Solving Plan

Understand the problem.

The table shows the tolls for traveling several different distances on a turnpike. You have \$8.25 to pay the toll. You are asked to find how far you can travel on the turnpike with \$8.25 for tolls.

Make a plan.

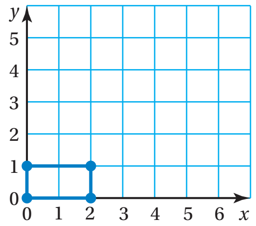
First, determine the relationship between x and y and write an equation to represent the relationship. Then use the equation to determine the distance you can travel.

Solve and check.

Use the plan to solve the problem. Then check your solution.

2. A company uses a silo in the shape of a rectangular prism to store birdseed. The base of the silo is a square with side lengths of 20 feet. Are the height and the volume of the silo proportional? Justify your answer.

3. A rectangle is drawn in a coordinate plane as shown. In the same coordinate plane, create a scale drawing of the rectangle that has a vertex at (0, 0) and a scale factor of 3.



STEAM Performance Task

Mixing Paint

At the beginning of this chapter, you watched a STEAM Video called "Painting a Large Room." You are now ready to complete the STEAM Performance Task, where you will solve ratio problems about different colors of paint. Be sure to use mathematical thinking and reasoning, and the problem-solving plan as you work through the performance task.

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4

Chapter Review WITH CalcChat®

Review Vocabulary

Write the definition and give an example of each vocabulary term.

ratio, *p. 273*

value of a ratio, *p. 273*

equivalent ratios, *p. 274*

ratio table, *p. 274*

rate, *p. 293*

unit rate, *p. 293*

equivalent rates, *p. 293*

proportion, *p. 312*

cross products, *p. 313*

proportional, *p. 315*

equation in two variables, *p. 339*

solution of an equation in two variables, *p. 339*

independent variable, *p. 339*

dependent variable, *p. 339*

constant of proportionality, *p. 340*

scale drawing, *p. 353*

scale model, *p. 353*

scale, *p. 353*

scale factor, *p. 354*

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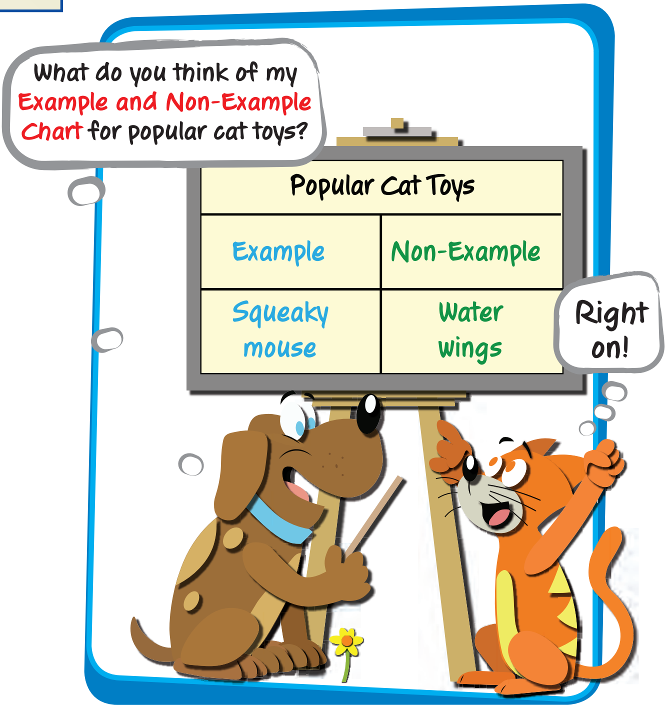
Graphic Organizers

You can use an **Example and Non-Example Chart** to list examples and non-examples of a concept. Here is an Example and Non-Example Chart for ***scale factor***.

Scale factor

Examples	Non-Examples
$\frac{5}{1}$	1 cm : 2 mm
$\frac{1}{200}$	1 mm : 20 cm
$\frac{1}{1}$	12 in. : 1 ft
$\frac{3}{2}$	3 mi : 2 in.



Choose and complete a graphic organizer to help you study the concept.

1. ratio
2. equivalent ratios
3. rate
4. unit rate
5. equivalent rates
6. proportion
7. cross products
8. proportional
9. scale

Chapter Learning Target: Understand ratios and proportions.

Chapter Success Criteria:
- ◆ I can write and interpret ratios.
- ◆ I can describe ratio relationships and proportional relationships.
- ■ I can represent equivalent ratios.
- ■ I can model ratio relationships and proportional relationships to solve real-life problems.

◆ Surface
■ Deep

Rate your understanding after each section.

1 I don't understand yet.
2 I can do it with help.
3 I can do it on my own.
4 I can teach someone else.

4.1 Ratios and Ratio Tables *(pp. 271–280)*

Learning Target: Understand ratios of rational numbers and use ratio tables to represent equivalent ratios.

Write the ratio. Then find and interpret the value of the ratio.

1. salt: flour

2. water to flour

3. salt to water

Modeling Clay

Ingredients:
2 cups flour $\frac{1}{2}$ cup salt $\frac{3}{4}$ cup water

Find the missing values in the ratio table. Then write the equivalent ratios.

4.

Flour (cups)	$\frac{3}{2}$	3		
Milk (cups)	$\frac{1}{2}$		$\frac{3}{2}$	2

5.

Miles	45	135		90
Hours	0.75		3	

6. The cost for 16 ounces of cheese is \$3.20. What is the cost for 20 ounces of cheese?

7. You mix 3 fluid ounces of food coloring for every 2 cups of oil to make 10 homemade lava lamps. Each lava lamp has a volume of $9\frac{1}{2}$ cups.

a. How many ounces of food coloring do you use?

b. How many cups of oil do you use?

8. A poison-dart frog can lay up to 12 eggs about 4 times a year. Use a ratio to determine the minimum number of years it will take for a poison-dart frog to lay 288 eggs.

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4.2 Graphing Ratio Relationships *(pp. 281–290)*

Learning Target: Represent ratio relationships in a coordinate plane.

Represent the ratio relationship using a graph.

9.

Time (years)	1	2	3
Penguins	6	12	18

10.

Televisions	12	24	36
Houses	4	8	12

11. You buy magnesium sulfate for $1.50 per pound.

a. Represent the ratio relationship using a graph.

b. How much does 3.5 pounds of magnesium sulfate cost?

12. A 5-ounce can of tuna costs $0.90. A 12-ounce can of tuna costs $2.40. Graph each ratio relationship in the same coordinate plane. Which is the better buy?

4.3 Rates and Unit Rates *(pp. 291–300)*

Learning Target: Understand rates involving fractions and use unit rates to solve problems.

Find the unit rate.

13. 289 miles on 10 gallons

14. $6\frac{2}{5}$ revolutions in $2\frac{2}{3}$ seconds

15. \$6.56 for 8.2 grams

16. $\frac{11}{2}$ miles in $\frac{2}{7}$ hour

17. You can mow 23,760 square feet in $\frac{1}{2}$ hour. How many square feet can you mow in 2 hours? Justify your answer.

Tell whether the rates are equivalent. Justify your answer.

18. 60 centimeters every 2.5 years

30 centimeters every 15 months

19. \$2.56 per $\frac{1}{2}$ pound

\$0.48 per 6 ounces

4.4 Converting Measures between Systems *(pp. 301–310)*

Learning Target: Use ratio reasoning to convert units of measure between systems.

Complete the statement. Round to the nearest hundredth if necessary.

20. 9.5 c ≈ ☐ L

21. 24 ft ≈ ☐ m

22. 2000 g ≈ ☐ lb

23. 3 L ≈ ☐ qt

24. 9.2 in. ≈ ☐ cm

25. 15 lb ≈ ☐ kg

26. $\frac{240\text{ m}}{\text{min}} \approx \frac{\square\text{ in.}}{\text{sec}}$

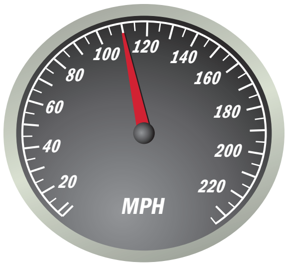
27. $\frac{17\text{ gal}}{\text{h}} \approx \frac{\square\text{ L}}{\text{h}}$

28. $\frac{8\text{ ft}}{\text{h}} \approx \frac{\square\text{ m}}{\text{min}}$

29. Explain how to use conversion factors to find the number of ounces in any given number of kilograms.

30. Water flows through a pipe at a rate of 10 gallons per minute. How many liters of water flow through the pipe in an hour?

31. Germany suggests a speed limit of 130 kilometers per hour on highways. Is the speed shown greater than the suggested limit?



32. You have 1180 Indian rupees and want to buy a T-shirt that costs 16 U.S. dollars. Use the exchange rate 1 U.S. dollar ≈ 74.74 Indian rupees to determine whether you have enough money to buy the T-shirt.

4.5 Identifying Proportional Relationships *(pp. 311–324)*

Learning Target: Determine whether two quantities are in a proportional relationship.

Tell whether the ratios form a proportion.

33. 4 to 9 and 2 to 3

34. 12 : 22 and 18 : 33

35. $\frac{1}{2} : 2$ and $\frac{1}{4} : \frac{1}{10}$

36. 3.2 to 8 and 1.2 to 3

37. Tell whether *x* and *y* are proportional.

x	1	3	6	8
y	4	12	24	32

38. You can type 250 characters in 60 seconds. Your friend can type 375 characters in 90 seconds. Do these rates form a proportion? Explain.

4.6 Writing and Solving Proportions *(pp. 325–336)*

Learning Target: Use proportions to solve ratio problems.

Solve the proportion. Explain your choice of method.

39. $\frac{3}{8} = \frac{9}{x}$

40. $\frac{3}{4} = \frac{7.2}{q}$

41. $\frac{3}{h-3} = \frac{6}{8}$

42. $\frac{8}{6} = \frac{4w}{9}$

Find the value of *x* so that the ratios are equivalent.

43. 4 : x and 12 : 9

44. 3.4 : x and 8.5 : 2.5

45. $\frac{1}{6}$ to 6 and x to 48

Use the table to write a proportion.

46.

	Game 1	**Game 2**
Penalties	6	8
Minutes	12	m

47.

	Concert 1	**Concert 2**
Songs	15	18
Hours	2.5	h

48. Swamp gas consists primarily of methane, a chemical compound consisting of a 1 : 4 ratio of carbon to hydrogen atoms. If a sample of methane contains 1564 hydrogen atoms, how many carbon atoms are present in the sample?

Use a proportion to complete the statement. Round to the nearest hundredth if necessary.

49. 16 mi ≈ ☐ km

50. 54.2 oz ≈ ☐ qt

51. 152 g ≈ ☐ lb

4.7 Using Graphs of Proportional Relationships *(pp. 337–350)*

Learning Target: Represent proportional relationships using graphs and equations.

Graph the equation.

52. $y = 7x$

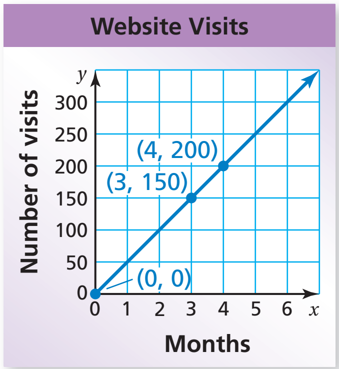
53. $y = 16x$

54. $y = \frac{1}{4}x$

55. Tell whether x and y are proportional. If so, find the constant of proportionality. Explain your reasoning.

x	-3	-1	1	3
y	6	2	-2	-6

56. The graph shows the number of visits your website received over the past 6 months. Interpret each plotted point in the graph. Then identify the unit rate.



Tell whether x and y are proportional. If so, identify the constant of proportionality. Explain your reasoning.

57. $x + y = 6$

58. $y - x = 0$

59. $\frac{x}{y} = 20$

60. $x = y + 2$

61. The variables x and y are proportional. When $y = 4$, $x = \frac{1}{2}$. Find the constant of proportionality. Then write an equation that relates x and y.

4.8 Scale Drawings *(pp. 351–360)*

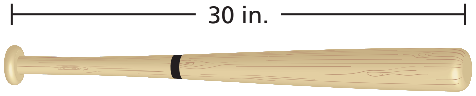
Learning Target: Solve problems involving scale drawings.

Find the missing dimension. Use the scale factor 1 : 20.

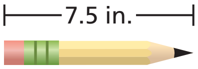
	Item	Model	Actual
62.	Basketball player	Height: ▭ in.	Height: 90 in.
63.	Dinosaur	Length: 3.75 ft	Length: ▭ ft

Use a centimeter ruler to find the scale and the scale factor of the drawing.

64.

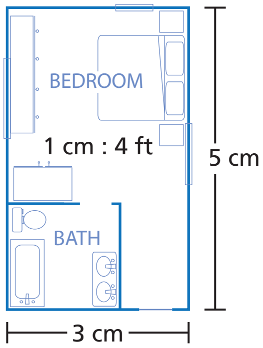


65.



66. A scale model of a lighthouse has a scale of 1 in. : 8 ft. The scale model is 20 inches tall. How tall is the lighthouse?

67. The floor plan for a rectangular room is shown.



a. Find the perimeter and area of the scale drawing.

b. Find the actual perimeter and area of the room.

4 Practice Test WITH CalcChat®

Find the unit rate.

1. 84 miles in 12 days

2. $2\frac{2}{5}$ kilometers in $3\frac{3}{4}$ minutes

Complete the statement. Round to the nearest hundredth if necessary.

3. 128 fl oz ≈ ▭ L

4. 14 ft ≈ ▭ m

Tell whether the ratios form a proportion.

5. 1 to 0.4 and 9 to 3.6

6. $2 : \frac{8}{3}$ and $\frac{2}{3} : 6$

Tell whether *x* and *y* are proportional. Explain your reasoning.

7.

x	2	4	6	8
y	10	20	30	40

8.

x	1	3	5	7
y	3	7	11	15

Solve the proportion. Explain your choice of method.

9. $\frac{x}{8} = \frac{9}{4}$

10. $\frac{17}{4} = \frac{y}{6}$

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Tell whether x and y are proportional. If so, identify the constant of proportionality. Explain your reasoning.

11. $x + y = 5$

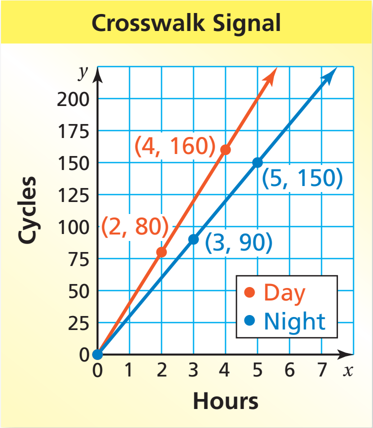
12. $\frac{y}{x} = 8$

13. A recipe calls for $\frac{2}{3}$ cup flour for every $\frac{1}{2}$ cup sugar. Write the ratio of sugar to flour. Then find and interpret the value of the ratio.

14. The graph shows the number of cycles of a crosswalk signal during the day and during the night.

a. Write equations that relate x and y for both the day and night periods.

b. Find how many more cycles occur during the day than during the night for a six-hour period.



15. An engineer is using computer-aided design (CAD) software to design a component for a space shuttle. The scale of the drawing is 1 cm : 60 in. The actual length of the component is 12.75 feet. What is the length of the component in the drawing?

16. A specific shade of green glaze is made of 5 parts blue glaze to 3 parts yellow glaze. A glaze mixture contains 25 quarts of blue glaze and 9 quarts of yellow glaze. How can you fix the mixture to make the specific shade of green glaze?

4 Review & Refresh WITH CalcChat®

while Building Fluency

Write the product using exponents.

1. $5 \times 5 \times 5 \times 5 \times 5 \times 5$

2. $\left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right)$

3. Your teacher says that if the average typing speed of a seventh-grade class is greater than 20 words per minute, the class will receive a reward. Write and graph an inequality that represents the average words per minute the class can type to receive the reward.

4. In 2019, a research institute reported the discovery of 71 new species, including 17 fish, 15 geckos, and 6 sea slugs. What percentage of the new species reported were *not* fish, geckos, or sea slugs? Round to the nearest tenth of a percent.

Write the fraction or mixed number as a decimal.

5. $\frac{7}{8}$

6. $-3\frac{2}{3}$

7. $-\frac{12}{5}$

8. $6\frac{1}{4}$

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Multiply. Write fractions in simplest form.

9. $\frac{1}{8} \times \frac{2}{5}$

10. $-\frac{6}{5} \cdot \frac{7}{2}$

11. $5\frac{1}{2} \cdot -\frac{1}{3}$

12. $2\frac{6}{7} \times 1\frac{4}{7}$

13. You are helping to build a wishing well that requires two different colors of bricks. You use *b* brown bricks and twice as many red bricks as brown bricks to construct the well.

a. Write and simplify an expression that represents the total number of bricks you use.

b. How many brown bricks do you use when you use 68 red bricks?

Order the numbers from least to greatest.

14. 1.2, −3.7, 1.1, 0.8, −3.5, −0.5

15. $\frac{1}{3}, -\frac{1}{5}, -\frac{2}{7}, \frac{3}{8}, -\frac{3}{8}, \frac{2}{3}$

Weekly Hours	
17	24
30	19
35	15
34	35
22	26

16. The table shows the number of hours ten employees work each week. On average, how many hours does an employee work each week?

17. A fundraising event charged $8 per person for admission and received a donation of $85. How many people visited the fundraiser when the total revenue was $581?

4 B.E.S.T. Test Prep WITH CalcChat®

Cumulative Practice

1. What is the simplified form of the expression?

$$3x - (2x - 5)$$

Ⓐ $x - 5$

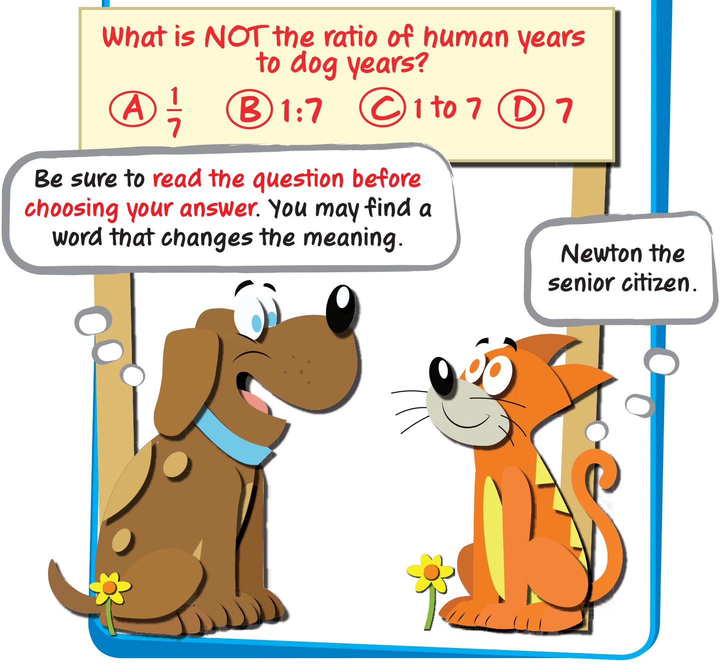
Ⓑ $x + 5$

Ⓒ $5x - 5$

Ⓓ $-x - 5$

Test-Taking Strategy

Read Question before Answering



2. Which fraction is equivalent to -1.25?

Ⓐ $-12\frac{1}{2}$

Ⓑ $-1\frac{1}{4}$

Ⓒ $-\frac{125}{1000}$

Ⓓ $1\frac{1}{4}$

3. What is the value of x for the proportion?

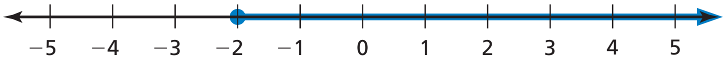
GRIDDED RESPONSE

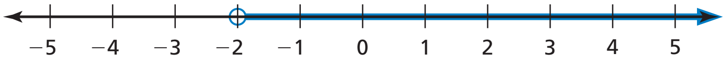
$$\frac{8}{12} = \frac{x}{18}?$$

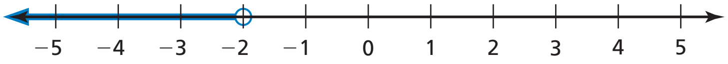
⊖	⊖	⊖	⊖	⊖	⊖	⊖
	⊘	⊘	⊘	⊘	⊘	
⊙	⊙	⊙	⊙	⊙	⊙	⊙
⓪	⓪	⓪	⓪	⓪	⓪	⓪
①	①	①	①	①	①	①
②	②	②	②	②	②	②
③	③	③	③	③	③	③
④	④	④	④	④	④	④
⑤	⑤	⑤	⑤	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	⑦	⑦	⑦	⑦
⑧	⑧	⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨	⑨	⑨

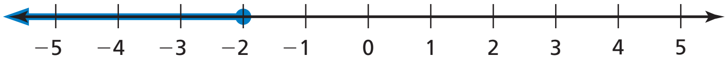
GO DIGITAL

4. Which graph represents a number that is at most -2?

Ⓐ 

Ⓑ 

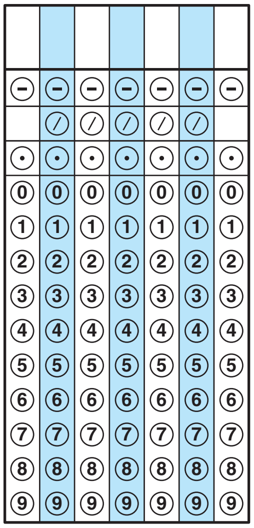
Ⓒ 

Ⓓ 

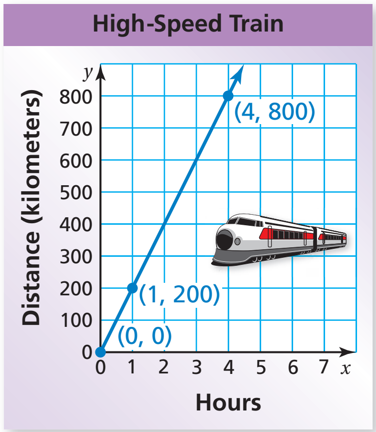
5. What is the value of the expression?

GRIDDED RESPONSE

$$\frac{8^5 \cdot 8^0}{8^5} \cdot 8^2$$



6. The distance traveled by a high-speed train is proportional to the number of hours traveled. Which of the following is *not* a valid interpretation of the graph?



Ⓐ The train travels 0 kilometers in 0 hours.

Ⓑ The unit rate is 200 kilometers per hour.

Ⓒ After 4 hours, the train is traveling 800 kilometers per hour.

Ⓓ The train travels 800 kilometers in 4 hours.

7. Which value of t makes the equation true?

$$3^8 \cdot 3^2 = 3^t$$

Ⓐ $t = 4$ Ⓑ $t = 6$ Ⓒ $t = 10$ Ⓓ $t = 16$

8. The quantities x and y are proportional. What is the missing value in the table?

Ⓐ 38 Ⓑ 42 Ⓒ 46 Ⓓ 56

x	y
$\frac{5}{7}$	10
$\frac{9}{7}$	18
$\frac{15}{7}$	30
4