3 Graphing Linear Functions

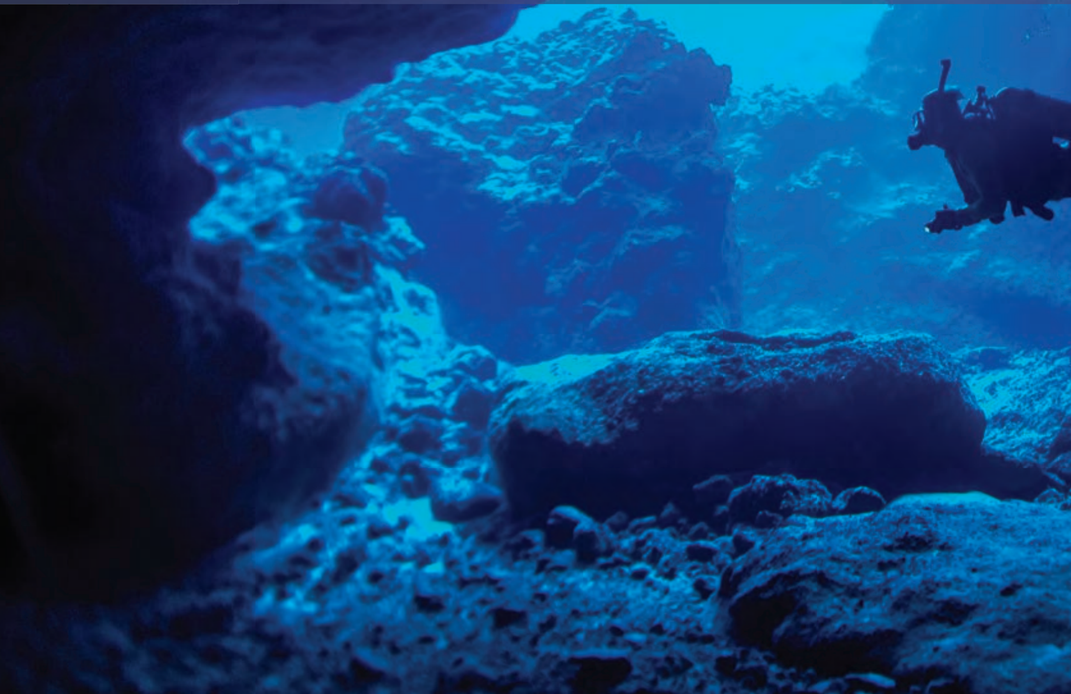
NATIONAL GEOGRAPHIC EXPLORER

Rhian G. Waller

WATCH

Dr. Rhian Waller is a professor of Marine Sciences at the University of Maine. She has led several scuba diving expeditions to some of the most remote locations in the world. Dr. Waller specializes in the ecology of cold-water organisms, with a passion for the conservation of deep-sea and polar ecosystems.

- What are some examples of deep-sea or cold-water organisms?
- What can scuba divers learn by studying some of the least accessible locations on the planet?
- How might scuba divers use mathematics when planning a diving expedition?



STEM

Divers must be sure that their scuba tank provides enough oxygen for their dive. In the Performance Task, you will plan a dive by selecting a tank size, the depth, and the amount of time you will spend underwater.

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Scuba Diving

Preparing for Chapter 3

Chapter Learning Target: Understand graphing linear functions.

Chapter Success Criteria:
- ◆ I can identify the graph of a linear function.
- ◆ I can graph linear functions written in different forms.
- ■ I can describe the characteristics of a function.
- ■ I can explain how a transformation affects the graph of a linear function.

◆ Surface
■ Deep

AZ VOCAB

Chapter Vocabulary

Work with a partner. Discuss each of the vocabulary terms.

relation	dependent variable	linear function
function	*x*-intercept	nonlinear function
independent variable	*y*-intercept	

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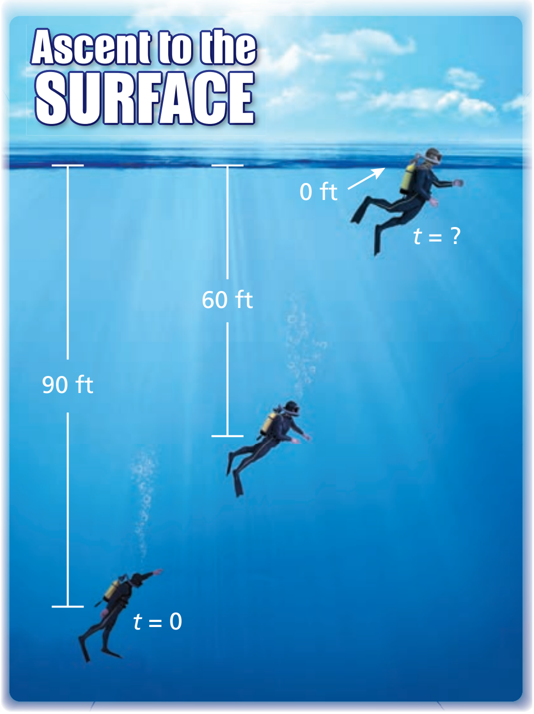
Mathematical Thinking and Reasoning

3 MTR

COMPLETE TASKS WITH MATHEMATICAL FLUENCY

Mathematicians who complete tasks with mathematical fluency select efficient and appropriate methods for solving problems within the given context.

Work with a partner. The equation $d = 90 - 15t$ represents the depth d (in feet) of a scuba diver t minutes after beginning an ascent to the surface. How long does it take the scuba diver to reach the surface?



1. Solve the problem using each tool.

a. pencil and paper

b. online or graphing calculator

c. spreadsheet

2. Describe the advantages and disadvantages of using each tool.

3. Why is it important to know how to choose appropriate tools to solve mathematical problems?

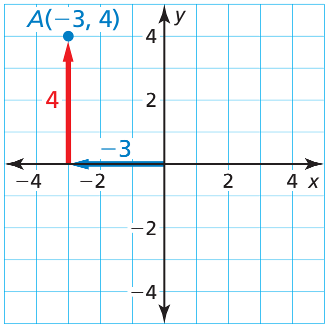
3 Prepare WITH CalcChat®

Plotting Points

WATCH

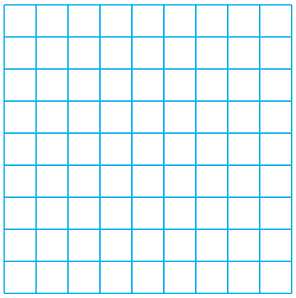
Example 1 **Plot the point $A(-3, 4)$ in a coordinate plane. Describe the location of the point.**

Start at the origin. Move 3 units left and 4 units up. Then plot the point.
The point is in Quadrant II.

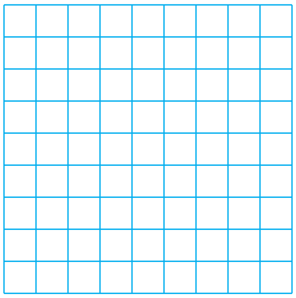


Plot the point in a coordinate plane. Describe the location of the point.

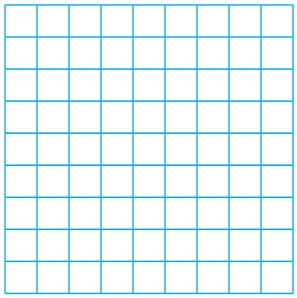
1. $A(3, 2)$



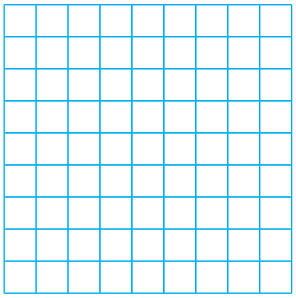
2. $B(-5, 1)$



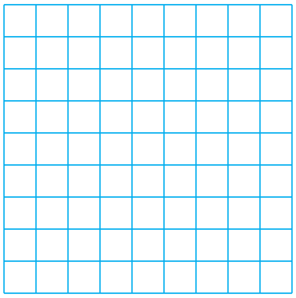
3. $C(0, 3)$



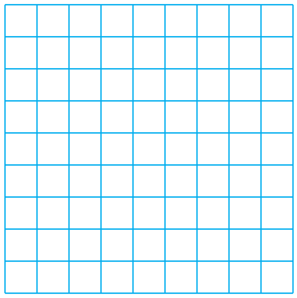
4. $D\left(-2\frac{1}{2}, -4\right)$



5. $E(-6.5, 0)$



6. $F\left(\frac{5}{2}, -\frac{3}{2}\right)$



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Evaluating Expressions

WATCH

Example 2 **Evaluate $4x - 5$ when $x = 3$.**

$$4x - 5 = 4(3) - 5 \quad \text{Substitute 3 for } x.$$
$$= 12 - 5 \quad \text{Multiply.}$$
$$= 7 \quad \text{Subtract.}$$

WATCH

Example 3 **Evaluate $-2x + 9$ when $x = -8.5$.**

$$-2x + 9 = -2(-8.5) + 9 \quad \text{Substitute } -8.5 \text{ for } x.$$
$$= 17 + 9 \quad \text{Multiply.}$$
$$= 26 \quad \text{Add.}$$

Evaluate the expression for the given value of *x*.

7. $3x - 4;\ x = 7$

8. $-5x + 8;\ x = 3$

9. $10x + 18;\ x = \frac{1}{5}$

10. $-9x - 2;\ x = -4$

11. $24 - 8x;\ x = -2.25$

12. $15x + 9;\ x = -\frac{1}{10}$

5 MTR **13.** **STRUCTURE** Let a and b be positive real numbers. Describe how to plot (a, b), $(-a, b)$, $(a, -b)$, and $(-a, -b)$.

3.1 Functions

Learning Target: Understand the concept of a function.

Success Criteria:
- I can determine whether a relation is a function.
- I can find the domain and range of a function.
- I can distinguish between independent and dependent variables.

Algebraic Reasoning
MA.912.AR.2.5 Solve and graph mathematical and real-world problems that are modeled with linear functions. Interpret key features and determine constraints in terms of the context.

WORDS AND MATH
Think about the meanings of the compound words *input* and *output* by analyzing their prefixes and the root word *put*.

A **relation** pairs inputs with outputs. For example, the number of hours you work *and* the pay you receive form a relation. The inputs are the hours worked, and the outputs are the pay amounts. Can you think of other relations?

EXPLORE IT! Describing Relations

Work with a partner. You buy an item from the vending machine.

a. Describe two possible relations associated with the vending machine.

b. Think about each relation in part (a).

- What are the inputs?

- What are the outputs?

- Does each input pair with *exactly* one output? Explain.

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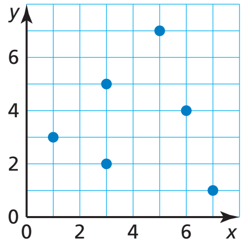
In mathematics, a **function** is a relation that pairs each input with exactly one output.

COMMUNICATE CLEARLY 4 MTR

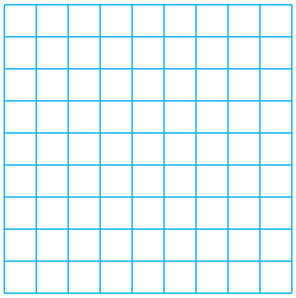
Can you think of any mathematical relations? Are any of these relations functions?

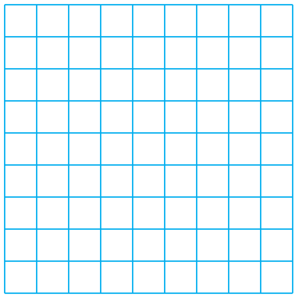
c. How can you use a coordinate plane to represent a relation? What are the inputs? What are the outputs?

d. The graph shows a relation. Is the relation a function? Explain your reasoning.



e. Draw two more graphs that represent relations, one that is a function and one that is not a function. Compare your graphs with other students in your class.





Determining Whether Relations Are Functions

Vocabulary

relation, *p. 175*
function, *p. 175*
domain, *p. 177*
range, *p. 177*
independent variable, *p. 179*
dependent variable, *p. 179*

A **relation** pairs inputs with outputs. When a relation is given as ordered pairs, the *x*-coordinates are inputs and the *y*-coordinates are outputs. A relation that pairs each input with *exactly one* output is a **function.**

EXAMPLE 1 Determining Whether Relations Are Functions

Determine whether each relation is a function. Explain.

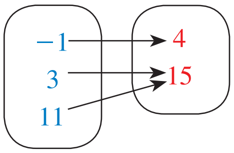
a. $(-2, 2), (-1, 2), (0, 2), (1, 0), (2, 0)$

b. $(4, 0), (8, 7), (6, 4), (4, 3), (5, 2)$

c.

Input, *x*	−2	−1	0	0	1	2
Output, *y*	3	4	5	6	7	8

d. Input, *x* Output, *y*



REMEMBER
A relation can be represented by a mapping diagram.

SOLUTION

a. Every input has exactly one output.

▶ So, the relation is a function.

b. The input 4 has two outputs, 0 and 3.

▶ So, the relation is *not* a function.

c. The input 0 has two outputs, 5 and 6.

▶ So, the relation is *not* a function.

d. Every input has exactly one output.

▶ So, the relation is a function.

SELF-ASSESSMENT

1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

Determine whether the relation is a function. Explain.

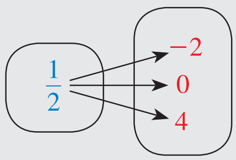
1. $(-5, 0), (0, 0), (5, 0), (5, 10)$

2. $(-4, 8), (-1, 2), (2, -4), (5, -10)$

3.

Input, *x*	Output, *y*
2	2.6
4	5.2
6	7.8

4. Input, *x* Output, *y*



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SELF-ASSESSMENT

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5. **OPEN-ENDED** Write a relation that is not a function. How can you change the relation so that it is a function?

6. **REASONING** Two quantities *x* and *y* are in a proportional relationship. Do the ordered pairs (x, y) represent a function? Explain your reasoning.

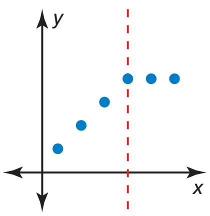
KEY IDEA

Vertical Line Test

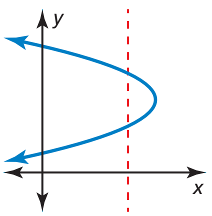
Words A graph represents a function when no vertical line passes through more than one point on the graph.

Examples

Function



Not a function



EXAMPLE 2 Using the Vertical Line Test

WATCH

Determine whether each graph represents a function. Explain.

SOLUTION

a. You can draw a vertical line through (2, 2) and (2, 5).

▶ So, the graph does *not* represent a function.

b. No vertical line can be drawn through more than one point on the graph.

▶ So, the graph represents a function.

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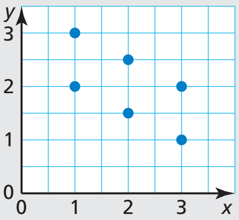
SELF-ASSESSMENT

1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

Determine whether the graph represents a function. Explain.

7.



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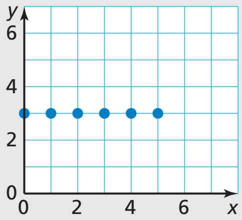

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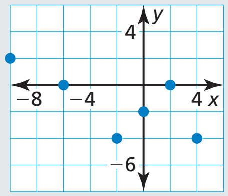

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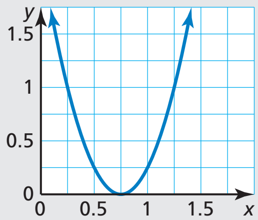

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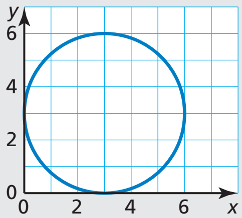

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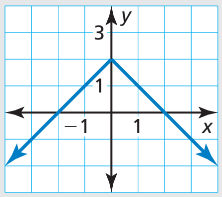
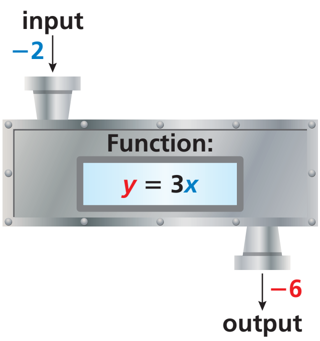


13. WRITING Explain why you can use vertical lines to determine whether a graph represents a function.



Finding the Domain and Range of a Function

KEY IDEAS

The Domain and Range of a Function

The **domain** of a function is the set of all possible input values.

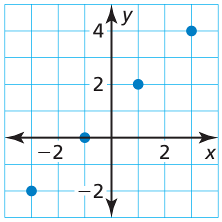
The **range** of a function is the set of all possible output values.

EXAMPLE 3 Finding the Domain and Range from a Graph

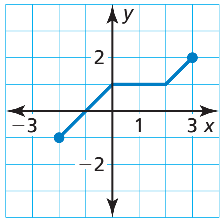
WATCH

Find the domain and range of the function represented by the graph.

a.

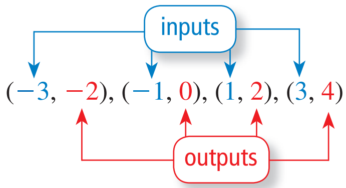


b.

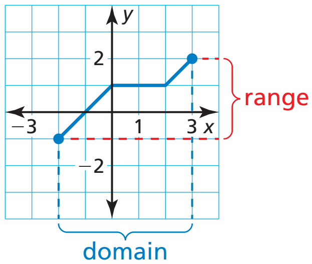


SOLUTION

a. Write the ordered pairs. Identify the inputs and outputs.



b. Identify the x- and y-values represented by the graph.



STUDY TIP

You can also use set-builder notation to write the domain and range of a function. In part (b), the domain is $\{x \mid -2 \le x \le 3\}$ and the range is $\{y \mid -1 \le y \le 2\}$.

▶ The domain is −3, −1, 1, and 3. The range is −2, 0, 2, and 4.

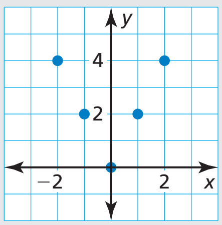
▶ The domain is $-2 \le x \le 3$. The range is $-1 \le y \le 2$.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Find the domain and range of the function represented by the graph.

14.



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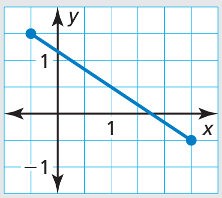

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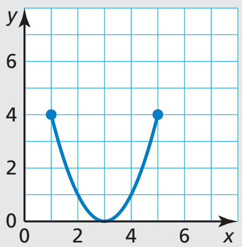

4 MTR **17. DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

Find the range of the function represented by the table.

Find the x-values of the function represented by $(-1, 7)$, $(0, 5)$, and $(1, -1)$.

Find the inputs of the function represented by the table.

Find the domain of the function represented by $(-1, 7)$, $(0, 5)$, and $(1, -1)$.

x	y
−1	7
0	5
1	−1

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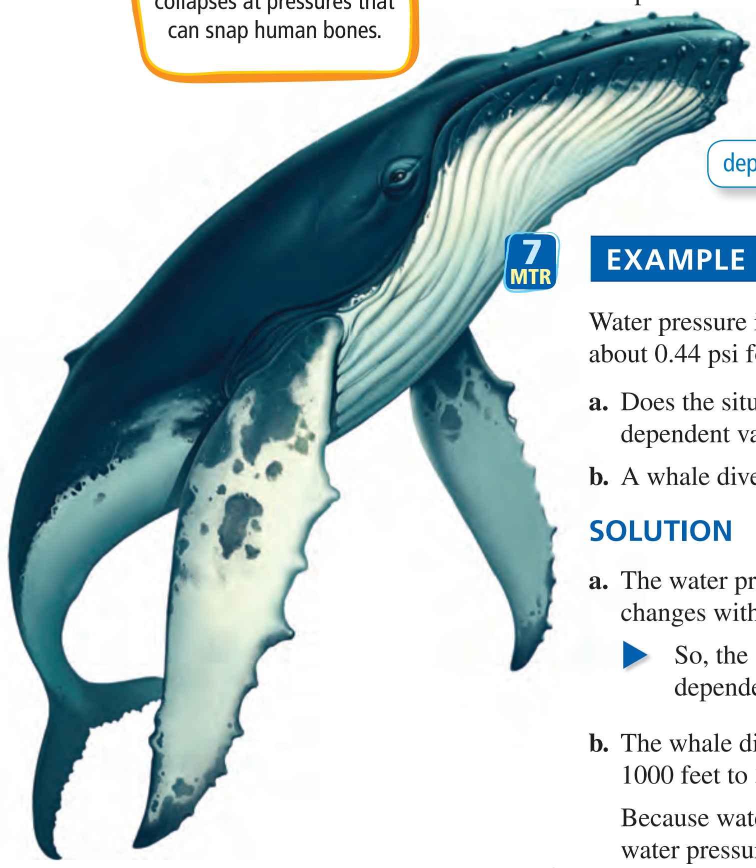
Identifying Independent and Dependent Variables

A whale's rib cage safely collapses at pressures that can snap human bones.



The variable that represents the input values of a function is the **independent variable** because it can be *any* value in the domain. The variable that represents the output values of a function is the **dependent variable** because it *depends* on the value of the independent variable. When an equation represents a function, the dependent variable is defined in terms of the independent variable. The statement "y is a function of x" means that y varies depending on the value of x.

$$y = -x + 10$$

dependent variable, y — independent variable, x

7 MTR

EXAMPLE 4 Modeling Real Life

WATCH

Water pressure is 0 psi (pounds per square inch) at sea level and increases by about 0.44 psi for every foot an object descends in water.

a. Does the situation represent a function? If so, identify the independent and dependent variables.

b. A whale dives from 1000 feet to 3500 feet. Find the domain and range.

SOLUTION

a. The water pressure depends on the depth of the water. Because the water pressure changes with any change in depth, each input has exactly one output.

▶ So, the situation represents a function in which the water pressure is the dependent variable and depth is the independent variable.

b. The whale dives from 1000 feet to 3500 feet, so the domain is any depth from 1000 feet to 3500 feet.

Because water pressure is 0 psi at sea level and increases at a constant rate, water pressure is proportional to depth. So, the water pressure y (in psi) at a depth of x feet can be represented by $y = 0.44x$. To find the range, find y when $x = 1000$ and when $x = 3500$.

REMEMBER

A proportional relationship can be represented by $y = kx$, where k is the constant of proportionality.

Input, x	$0.44x$	Output, y
1000	0.44(1000)	440
3500	0.44(3500)	1540

▶ So, the domain is $\{x \mid 1000 \le x \le 3500\}$ and the range is $\{y \mid 440 \le y \le 1540\}$.

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SELF-ASSESSMENT

1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

18. You arrange coins in stacks so that each stack has twice as many coins as the previous stack. The first stack has 2 coins.

a. Does the situation represent a function? If so, identify the independent and dependent variables.

b. You have 6 stacks of coins. Find the domain and range.

19. The total pressure exerted on an object in water is the sum of air pressure at sea level and water pressure. You are scuba diving at a depth where the total pressure is three times the air pressure at sea level. Use the information in Example 4 to find your depth. Explain how you found your answer.

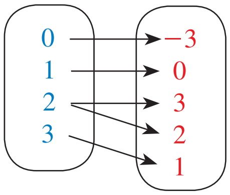
3.1 Practice WITH CalcChat® AND CalcView®

In Exercises 1–6, determine whether the relation is a function. Explain. *(See Example 1.)*

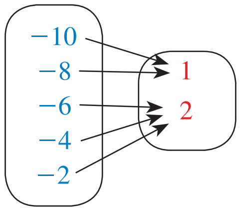
1. (1, −2), (2, 1), (3, 6), (4, 13), (5, 22)

2. (7, 4), (5, −1), (3, −8), (1, −5), (3, 6)

3. **Input, x** **Output, y**



4. **Input, x** **Output, y**



5.

Input, x	16	1	0	1	16
Output, y	−2	−1	0	1	2

6.

Input, x	−3	0	3	6	9
Output, y	$\frac{2}{3}$	$\frac{1}{3}$	0	$-\frac{1}{3}$	$-\frac{2}{3}$

In Exercises 7–10, determine whether the graph represents a function. Explain. *(See Example 2.)*

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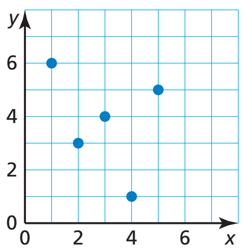

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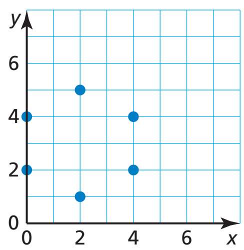

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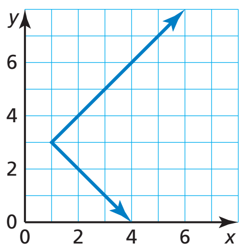

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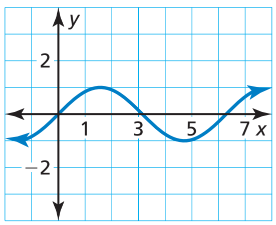

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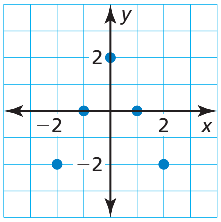
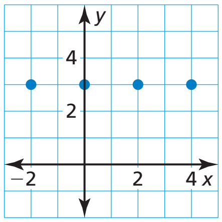
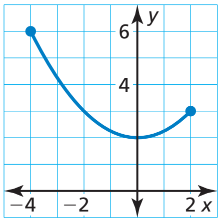
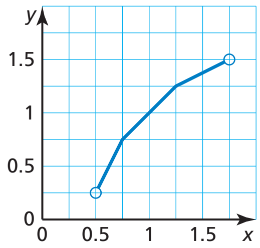
In Exercises 11–14, find the domain and range of the function represented by the graph. *(See Example 3.)*

11. 

12. 

13. 

14. 

ANALYZE A PROBLEM **In Exercises 15 and 16, identify the independent and dependent variables.**

15. the number of quarters you put into a parking meter and the amount of time on the meter

16. the amount of gasoline in a car's fuel tank and the amount of time spent driving

17. **MODELING REAL LIFE** A cell phone plan costs $30 for each line. *(See Example 4.)*

a. Does the situation represent a function? If so, identify the independent and dependent variables.

b. You can have a maximum of four lines on a plan. Find the domain and range.

18. **MODELING REAL LIFE** A taxi company charges an initial fee of $2.80 plus $3.50 per mile traveled.

a. Does the situation represent a function? If so, identify the independent and dependent variables.

b. You have enough money to travel at most 20 miles in the taxi. Find the domain and range.

ERROR ANALYSIS In Exercises 19 and 20, describe and correct the error in the statement about the relation shown in the table.

Input, x	1	2	3	4	5
Output, y	$6\frac{1}{2}$	$7\frac{1}{2}$	$8\frac{1}{2}$	$6\frac{1}{2}$	$9\frac{1}{2}$

19. ✗ The relation is *not* a function. One output is paired with two inputs.

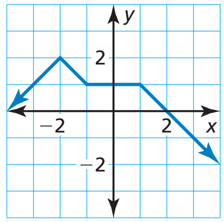
20. ✗ The relation is a function. The range is 1, 2, 3, 4, and 5.

21. **MULTIPLE REPRESENTATIONS** The table shows the balance of a savings account over time. Represent the situation in words and in a coordinate plane. Does the situation represent a function? Explain.

Month, x	0	1	2	3	4
Balance (dollars), y	100	125	150	175	200

22. **MULTIPLE REPRESENTATIONS** The equation $1.5x + 0.5y = 12$ represents the number x of hardcover books and the number y of softcover books you can buy at a used-book sale. Represent the situation in a table and in a coordinate plane. Does the situation represent a function? Explain.

23. **REASONING** The graph represents a function. Find the input value corresponding to an output of 2.

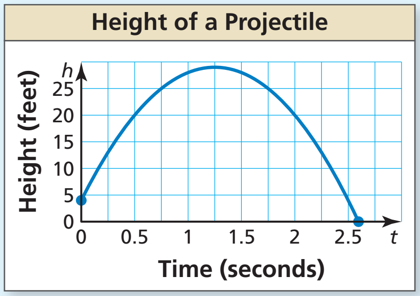


24. **OPEN-ENDED** Complete the table so that when t is the independent variable, the relation is a function, and when t is the dependent variable, the relation is not a function.

t				
v				

25. **MAKING AN ARGUMENT** Your friend says that a line always represents a function. Is your friend correct? Explain.

26. **HOW DO YOU SEE IT?**
The graph represents the height h of a projectile after t seconds.



a. Is h a function of t? Explain.

b. Approximate the height of the projectile after 0.5 second and after 1.25 seconds.

c. Approximate the domain and range.

d. Is t a function of h? Explain.

3 MTR MAINTAIN ACCURACY **In Exercises 27–30, determine whether the statement uses the word *function* in a way that is mathematically correct. Explain your reasoning.**

27. The selling price of an item is a function of the cost of making the item.

28. The sales tax on a purchased item in a given state is a function of the selling price.

29. A function pairs each student in your school with a homeroom teacher.

30. A function pairs each chaperone on a school trip with 10 students.

4 MTR DISCUSS MATHEMATICAL THINKING **In Exercises 31–34, tell whether the statement is *true* or *false*. If it is false, explain why.**

31. Every function is a relation.

32. Every relation is a function.

33. When you switch the inputs and outputs of any function, the resulting relation is a function.

34. When the domain of a function has an infinite number of values, the range always has an infinite number of values.

35. B.E.S.T. TEST PREP Which of the following values of x and y make the relation a function? Select all that apply.

$$(-3, 7), (-2, 3), (0, 8), (1, -1), (x, y)$$

Ⓐ $x = -4, y = 0$
Ⓑ $x = 1, y = -2$
Ⓒ $x = 5, y = -1$
Ⓓ $x = 2, y = 8$

36. THOUGHT PROVOKING
Describe a function in which the inputs and/or the outputs are not numbers. Identify the independent and dependent variables. Then find the domain and range of the function.

DIG DEEPER **In Exercises 37–40, find the domain and range of the function.**

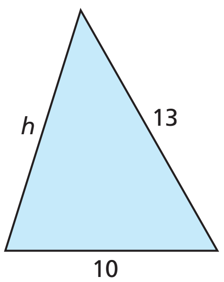
37. $y = |x|$

38. $y = -|x|$

39. $y = |x| - 6$

40. $y = 4 - |x|$

41. **5 MTR CONNECTING CONCEPTS** Find the domain and range of a function that represents the perimeter of the triangle shown.



REVIEW & REFRESH

42. Tell whether x and y are proportional.

x	4	6	12	16
y	6	8	16	20

43. What number is 60% of 35?

44. 27 is what percent of 75?

In Exercises 45–48, solve the equation.

45. $n - 5.3 = -7.4$

46. $|2x + 5| = 3x$

47. $7c + 10 - 12c = -11 - 2c$

48. $-\frac{1}{2}(3h + 8) + 2 = 13$

49. Determine whether the relation is a function. Explain.

$(0, -6), (1, -3), (3, 2), (5, 1), (2, -3)$

50. Write the sentence as an inequality.

Seven is at most the quotient of a number d and -5.

7 MTR **51.** **MODELING REAL LIFE** There is a 6% sales tax on your clothing purchase. You pay \$1.80 in tax. What is the total amount you pay?

In Exercises 52–57, solve the inequality. Graph the solution.

52. $z + 9 < 5$

53. $-5y + 9.2 \geq -18.3$

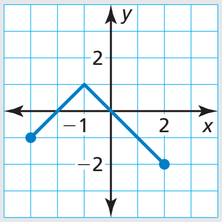
54. $\frac{x}{-7} \leq 2$

55. $7w + 6 > 3(-2 + w)$

56. $2|t - 5| + 3 \geq 9$

57. $-12 \leq 2m + 8 < 4$

58. Find the domain and range of the function represented by the graph.



59. **REASONING** Find the value of a for which the equation

$$12x - 15 = a(5 - 4x)$$

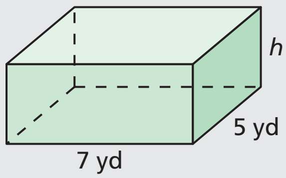
is an identity.

60. Write an inequality that represents the graph.

7 MTR **61.** **MODELING REAL LIFE** You want to score an average of at least 10 points per game in the last six basketball games of the season. In five of the six remaining games, you score 8, 14, 6, 12, and 8 points. How many points do you need to score in the sixth game to achieve your goal?

62. Solve the literal equation $3x - 6y = 18$ for y.

63. The volume of the rectangular prism is 105 cubic yards. What is the surface area of the prism in square feet?



3.2 Characteristics of Functions

Learning Target: Describe characteristics of functions.

Success Criteria:
- I can estimate intercepts of a graph of a function.
- I can approximate when a function is positive, negative, increasing, or decreasing.
- I can sketch a graph of a function from a verbal description.

Algebraic Reasoning
MA.912.AR.2.5 Solve and graph mathematical and real-world problems that are modeled with linear functions. Interpret key features and determine constraints in terms of the context.
MA.912.AR.3.8 Solve and graph mathematical and real-world problems that are modeled with quadratic functions. Interpret key features and determine constraints in terms of the context.
Also MA.912.AR.2.4, MA.912.AR.3.7, MA.912.F.1.6

EXPLORE IT! Describing Characteristics of Functions

Work with a partner. Consider the function $y = x^3 - 3x$.

a. What do you think it means for a function to be *positive*? *negative*? *increasing*? *decreasing*?

b. Write a pair of numbers greater than 50 and a pair of numbers less than -50 to use as inputs for the function.

NUMBERS GREATER THAN 50	NUMBERS LESS THAN −50
First Number: ________	First Number: ________
Second Number: ________	Second Number: ________

Find the outputs for each pair of inputs. Do you think the function is increasing? decreasing? Explain your reasoning using your input-output pairs.

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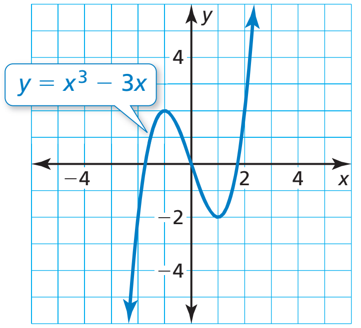
c. The graph of the function is shown below. Approximate when the function is positive, negative, increasing, or decreasing over its entire domain.



d. Explain whether it is possible for a graph to be decreasing over its entire domain but never negative. Justify your answer using a sketch.

4 MTR **COMPARE METHODS**

Compare using *input-output pairs* with using a graph when describing characteristics of a function. Which method is generally more accurate?

Intercepts of Graphs of Functions

Vocabulary (AZ VOCAB)

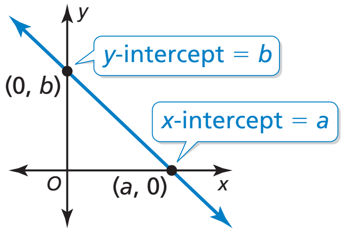
x-intercept, *p. 189*
y-intercept, *p. 189*
increasing, *p. 190*
decreasing, *p. 190*
end behavior, *p. 190*

KEY IDEA

Intercepts

An ***x*-intercept** of a graph is the *x*-coordinate of a point where the graph intersects the *x*-axis. It occurs when $y = 0$.

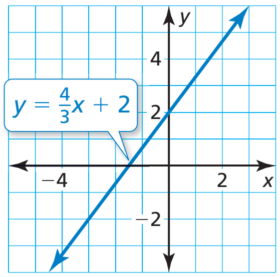
A ***y*-intercept** of a graph is the *y*-coordinate of a point where the graph intersects the *y*-axis. It occurs when $x = 0$.



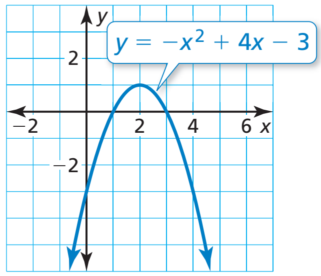
EXAMPLE 1 **Estimating Intercepts** (WATCH)

Estimate the intercepts of the graph of each function.

a.



b.



STUDY TIP

You can use a graph to estimate intercepts, but your estimates may not be exact. Substitute your estimates into the equation to check whether they are exact.

SOLUTION

a. The graph appears to intersect the *x*-axis at $(-1.5, 0)$. It appears to intersect the *y*-axis at $(0, 2)$.

▶ So, the *x*-intercept is about -1.5, and the *y*-intercept is about 2.

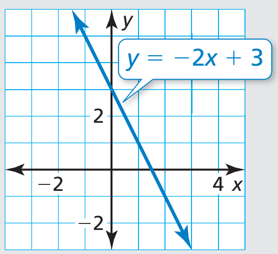
b. The graph appears to intersect the *x*-axis at $(1, 0)$ and $(3, 0)$. It appears to intersect the *y*-axis at $(0, -3)$.

▶ So, the *x*-intercepts are about 1 and 3, and the *y*-intercept is about -3.

SELF-ASSESSMENT 1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Estimate the intercepts of the graph of the function.

1.



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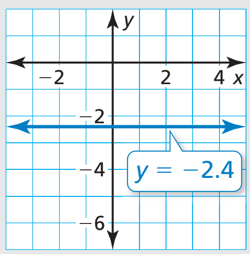
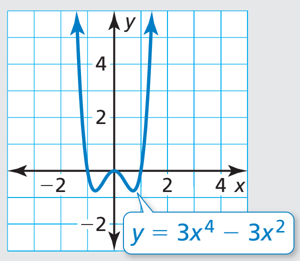


3.



4. **REASONING** Can the graph of a function have more than one *y*-intercept? Can the graph of a function have an infinite number of *x*-intercepts? Explain your reasoning.

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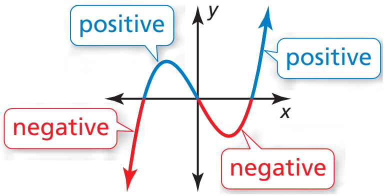
Other Characteristics of Functions

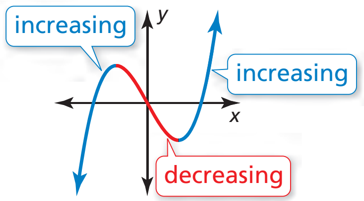
KEY IDEAS

Positive, Negative, Increasing, Decreasing, and End Behavior

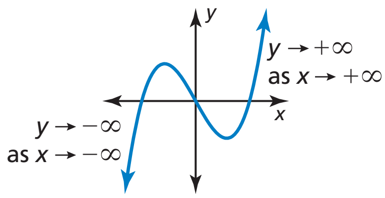
A function is *positive* when its graph lies above the x-axis. A function is *negative* when its graph lies below the x-axis.



A function is **increasing** when its graph moves up as x moves to the right. A function is **decreasing** when its graph moves down as x moves to the right.

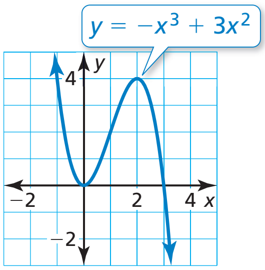


The **end behavior** of a function is the behavior of the graph as x approaches positive infinity ($+\infty$) or negative infinity ($-\infty$).



EXAMPLE 2 **Describing Characteristics**

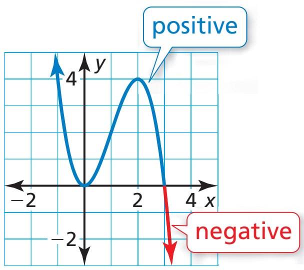
WATCH



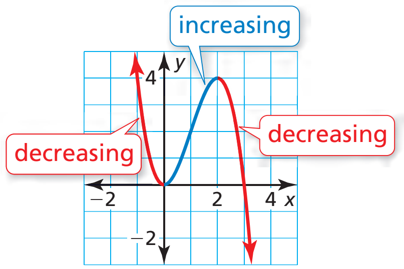
Approximate when the function $y = -x^3 + 3x^2$ is positive, negative, increasing, or decreasing. Then describe the end behavior of the function.

SOLUTION

Positive and Negative:
The function appears to be positive when $x < 0$, positive when $0 < x < 3$, and negative when $x > 3$.



Increasing and Decreasing:
The function appears to be decreasing when $x < 0$, increasing when $0 < x < 2$, and decreasing when $x > 2$.



End behavior: The graph shows that the function values increase as x approaches negative infinity and the function values decrease as x approaches positive infinity. So, $y \to +\infty$ as $x \to -\infty$ and $y \to -\infty$ as $x \to +\infty$.

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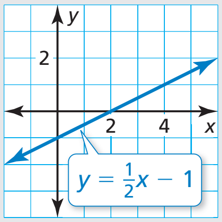
SELF-ASSESSMENT

1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

Approximate when the function is positive, negative, increasing, or decreasing. Then describe the end behavior of the function.

5.



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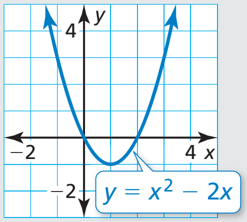

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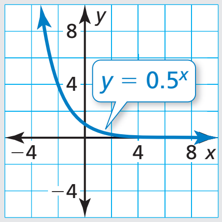
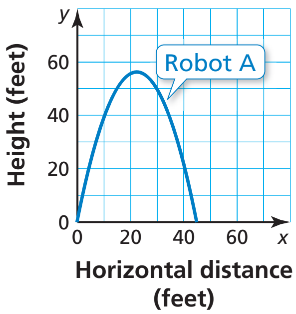


A Japanese mission used hopping robots to conduct experiments on an asteroid. Because of the asteroid's low gravity, a single hop could last 15 minutes or more.

Solving Real-Life Problems

7 MTR **EXAMPLE 3** **Modeling Real Life** WATCH

Researchers send two robots to explore an asteroid. The robots move by "hopping" between locations. The graph shows the path of Robot A's first hop. Robot B lands 60 feet from where it starts its first hop, reaching a maximum height of 48 feet after traveling a horizontal distance of 30 feet. Compare the two hops.

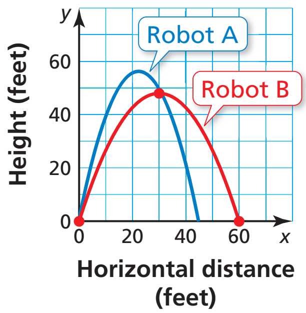


SOLUTION

Use the verbal description to sketch a graph that represents Robot B's hop. First identify several points on the graph.

- the start of the hop: (0, 0)
- the maximum height of the hop: (30, 48)
- the end of the hop: (60, 0)

So, the graph increases from (0, 0) to (30, 48) and then decreases to (60, 0). Sketch the graph using a curve similar to the graph that represents Robot A's hop.



▶ **Intercepts:** Robot A's hop has x-intercepts of about 0 and 45. So, the robot lands about 45 feet from where it starts. This is about $60 - 45 = 15$ feet shorter than Robot B's hop.

Increasing and decreasing: Each hop reaches its maximum height when the height changes from increasing to decreasing. The graph shows that the maximum height of Robot A's hop is between 50 and 60, or about 55. So, Robot A's hop is about $55 - 48 = 7$ feet higher than Robot B's hop at its maximum height.

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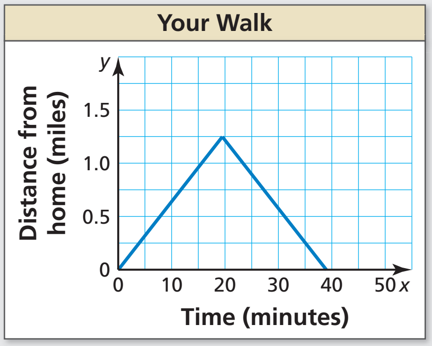
1 MTR ANALYZE A PROBLEM

In Example 3, what are the domain and range of each function in this context? Write your answers in both inequality and set-builder notations.

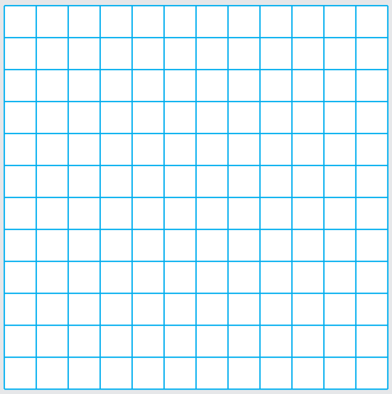
SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

8. The graph shows your distance from home while out on a walk. The next day, you jog the same route twice at a constant speed. The entire jog takes 1 hour. Compare your walk to your jog.



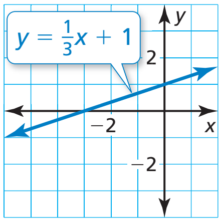
2 MTR **9.** **MODEL A PROBLEM** You throw a ball straight up into the air and notice that the speed of the ball decreases as it approaches its maximum height, then increases again on the way down. Sketch a graph that represents the relationship between time and height in this situation. Explain your reasoning.



3.2 Practice WITH CalcChat® AND CalcView®

In Exercises 1–4, estimate the intercepts of the graph of the function. *(See Example 1.)*

1.



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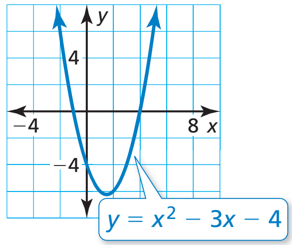
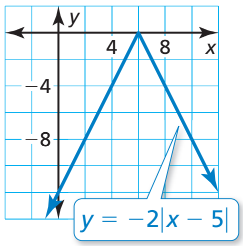


▶ 3.



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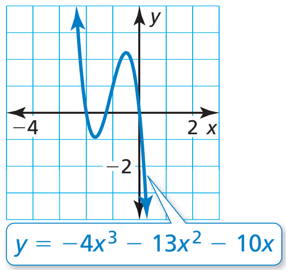
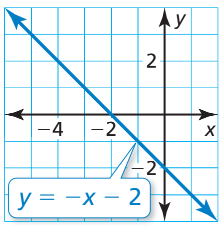


In Exercises 5–8, approximate when the function is positive, negative, increasing, or decreasing. Then describe the end behavior of the function. *(See Example 2.)*

5.



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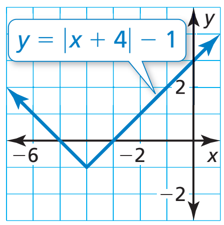
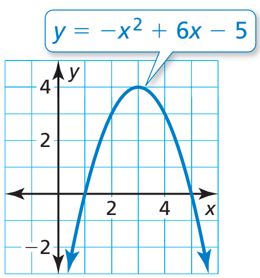


▶ 7.



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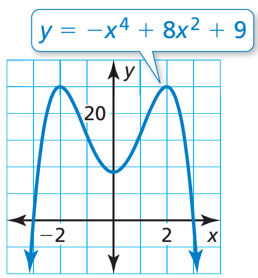

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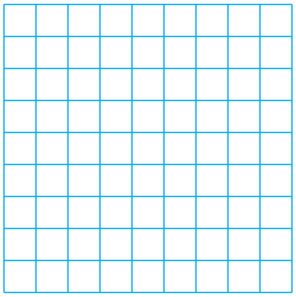
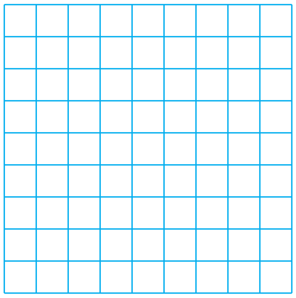
In Exercises 9 and 10, sketch a graph of a function with the given characteristics.

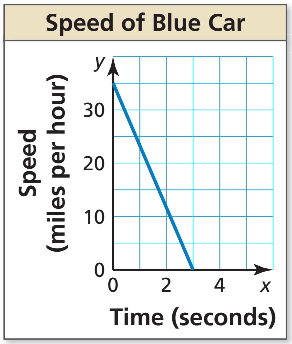
9.
- The function is increasing when $x < -6$ and decreasing when $x > -6$.
- The function is negative when $x < -8$, positive when $-8 < x < -4$, and negative when $x > -4$.



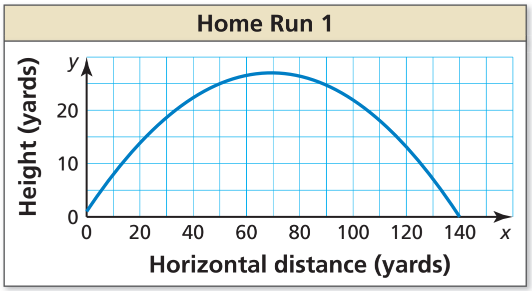
10.
- The x-intercepts are -0.5, 1, and 3.25.
- $y \to +\infty$ as $x \to -\infty$ and $y \to -\infty$ as $x \to +\infty$.

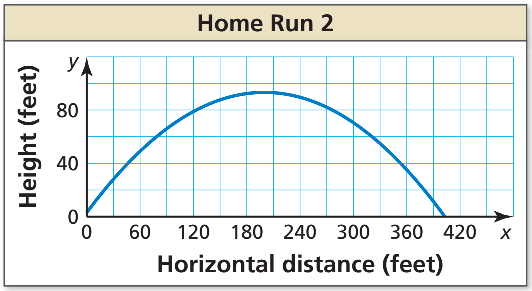


11. MODELING REAL LIFE The graph shows the speed of a blue car after the driver applies the brakes, represented by $y = -\frac{35}{3}x + 35$, where x is the time (in seconds) and y is the speed (in miles per hour). The driver of a red car applies the brakes while traveling 30 miles per hour. The speed of the red car decreases at a constant rate until the car comes to a complete stop 4 seconds later. Compare the initial speeds and stopping times. *(See Example 3.)*

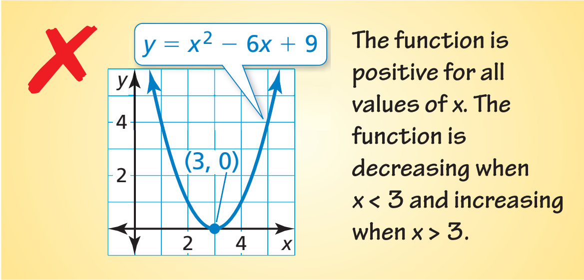


12. MODELING REAL LIFE The graphs show the path of two home runs. Compare the two home runs.

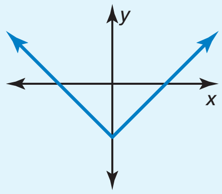




13. ERROR ANALYSIS Describe and correct the error in describing characteristics of the function.



14. HOW DO YOU SEE IT?
The graph of a function is shown.



a. How many x-intercepts does the graph have? Is the y-intercept *positive* or *negative*?

b. Is the function *increasing* or *decreasing* when $x < 0$? $x > 0$?

15. B.E.S.T. TEST PREP The graph of a function is a line that is decreasing for all values of x and has an x-intercept of 4. Which of the following are true? Select all that apply.

Ⓐ The y-intercept is negative.

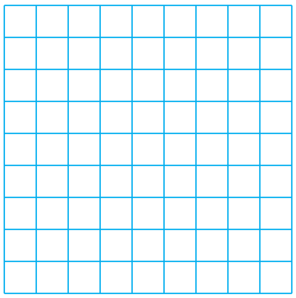
Ⓑ The graph has only one x-intercept.

Ⓒ The function is positive when $x < 4$ and negative when $x > 4$.

Ⓓ $y \to -\infty$ as $x \to -\infty$ and $y \to +\infty$ as $x \to +\infty$.

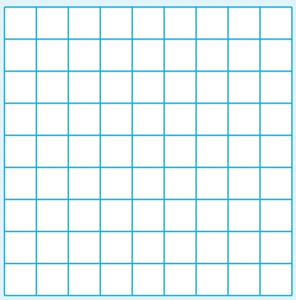
16. MAKING AN ARGUMENT Consider the graph of a function that is negative over its entire domain. Can the graph have an x-intercept? Explain.

17. DIG DEEPER You board a car at the bottom of a Ferris wheel. The Ferris wheel then makes several complete rotations before stopping again to let you off where you boarded. Sketch a graph that represents the relationship between time and your height above the ground, and describe the relationship.



18. THOUGHT PROVOKING
Sketch a graph of a function with the given characteristics.

- The function is decreasing for $x < 0$ and increasing for $x > 0$.
- As x approaches negative infinity, the function does *not* approach positive infinity or negative infinity.
- As x approaches positive infinity, the function does *not* approach positive infinity or negative infinity.
- The graph does *not* have a y-intercept.



REVIEW & REFRESH

In Exercises 19 and 20, write the sentence as an inequality. Graph the inequality.

19. A number n is greater than or equal to -5 and less than -1.

20. A number k is no more than $-\frac{1}{2}$ or at least $2\frac{1}{2}$.

In Exercises 21–24, solve the equation.

21. $7 + b = -21$

22. $-3.2x = 16$

23. $-3(t - 5) = 14$

24. $30 = 2m - 8m + 12$

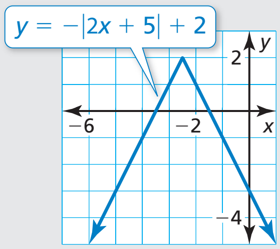
7 MTR **25.** **MODELING REAL LIFE** You have a \$25 gift card to a coffee shop. You have already used \$19.85. You want to purchase one drink and one bakery item. Which pairs of items can you purchase with the amount left on the gift card?

Drink	Price
Coffee	\$2.09
Cappuccino	\$3.79

Bakery item	Price
Muffin	\$2.59
Bagel	\$1.49

26. Find the greatest common factor of 30 and 42.

27. Estimate the intercepts of the graph of the function.



In Exercises 28–31, solve the inequality. Graph the solution.

28. $5a > 20$

29. $\frac{r}{-2} + 6 \leq 11$

30. $1.5x + 7 - 5x > 11 - 3x$

31. $\left|\frac{1}{3}x + 6\right| + 2 > 3$

32. **REASONING** Complete the relation so that it is (a) a function and (b) *not* a function. Explain your reasoning.

$(-6, -1), (-4, 0), (-2, 1), (2, 2),$ ______

3.3 Linear Functions

Learning Target: Identify and graph linear functions.

Success Criteria:
- I can identify linear functions using graphs, tables, and equations.
- I can determine whether a domain is discrete or continuous in a real-life situation.
- I can graph linear functions with discrete and continuous domains.

Algebraic Reasoning
MA.912.AR.2.4 Given a table, equation or written description of a linear function, graph that function, and determine and interpret its key features.
Functions
MA.912.F.1.1 Given an equation or graph that defines a function, determine the function type. Given an input-output table, determine a function type that could represent it.
Also MA.912.AR.2.5

EXPLORE IT! Finding a Pattern

Work with a partner. Use a piece of rope that is at least 100 centimeters long. Record your data in the table.

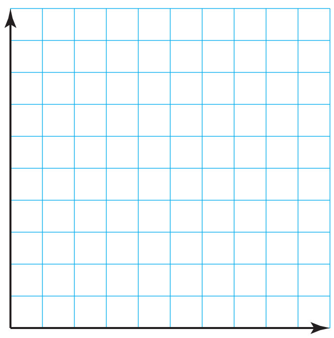
Number of knots	Length of rope
0	
1	
2	
3	
4	
5	
6	
7	
8	

a. Measure the length of the rope. Describe your measurement.

b. Make a knot in the rope, and then measure the length of the rope again. Continue to make identical knots in the rope, measuring the length of the rope after each knot is tied.

c. Write several observations about the data. What pattern(s) do you notice in the data? Explain.

d. Make a scatter plot of the data. What pattern(s) do you notice in the scatter plot? Explain.



e. How can you predict the length of the rope when it has 10 knots? Explain your reasoning.

f. Does it matter where you tie the knots on the rope? Is there a maximum number of knots you can tie? Does the thickness of the rope or the type of knot you tie affect your results? Explain your reasoning.

USE ANOTHER METHOD (2 MTR)
Is there more than one way to predict the length in part (e)? Explain.

GO DIGITAL

Identifying Linear Functions

Vocabulary

linear equation in two variables, *p. 198*
linear function, *p. 198*
nonlinear function, *p. 198*
solution of a linear equation in two variables, *p. 201*
discrete domain, *p. 201*
continuous domain, *p. 201*

A **linear equation in two variables**, x and y, is an equation that can be written in the form

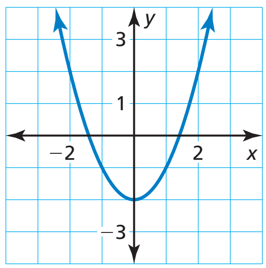
$$y = mx + b$$

where m and b are constants. The graph of a linear equation is a line. Likewise, a **linear function** is a function whose graph is a nonvertical line. A linear function has a constant rate of change and can be represented by a linear equation in two variables. A **nonlinear function** does not have a constant rate of change. So, its graph is *not* a line.

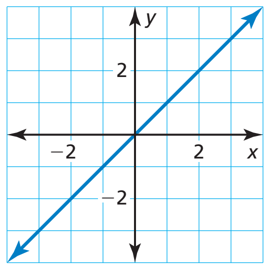
EXAMPLE 1 Identifying Linear Functions Using Graphs

Does the graph represent a *linear* or *nonlinear* function? Explain.

a.



b.



SOLUTION

a. The graph is *not* a line.

▶ So, the function is nonlinear.

b. The graph is a nonvertical line.

▶ So, the function is linear.

EXAMPLE 2 Identifying Linear Functions Using Tables

Does the table represent a *linear* or *nonlinear* function? Explain.

a.

x	3	6	9	12
y	36	30	24	18

b.

x	1	3.5	6	8.5
y	2	9	20	35

SOLUTION

a.

+3 +3 +3

x	3	6	9	12
y	36	30	24	18

−6 −6 −6

As x increases by 3, y decreases by 6. The rate of change is constant.

▶ So, the function is linear.

b.

+2.5 +2.5 +2.5

x	1	3.5	6	8.5
y	2	9	20	35

+7 +11 +15

As x increases by 2.5, y increases by different amounts. The rate of change is *not* constant.

▶ So, the function is nonlinear.

STUDY TIP

A constant rate of change describes a quantity that changes by equal amounts over equal intervals.

GO DIGITAL

EXAMPLE 3 B.E.S.T. Test Prep: Identifying Linear Functions Using Equations

WATCH

Which of the following equations represent linear functions? Explain.

Ⓐ $0 = -x - y$ Ⓒ $y = 2^x$ Ⓔ $y = -1 + x$

Ⓑ $y = \sqrt{x}$ Ⓓ $y = |x| - 1$ Ⓕ $x^2 - y = 0$

SOLUTION

You cannot rewrite the equations $y = \sqrt{x}$, $y = 2^x$, $y = |x| - 1$, and $x^2 - y = 0$ in the form $y = mx + b$. So, they cannot represent linear functions.

You can rewrite the equation $0 = -x - y$ as $y = -x$ and the equation $y = -1 + x$ as $y = x - 1$.

▶ So, the correct answers are Ⓐ and Ⓔ.

CONCEPT SUMMARY

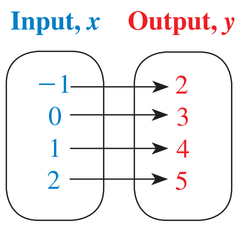
Representations of Functions

Words An output is 3 more than the input. **Equation** $y = x + 3$

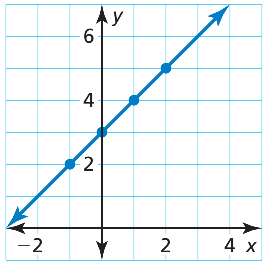
Input-Output Table

Input, x	Output, y
−1	2
0	3
1	4
2	5

Mapping Diagram



Graph

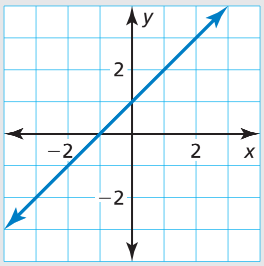


SELF-ASSESSMENT

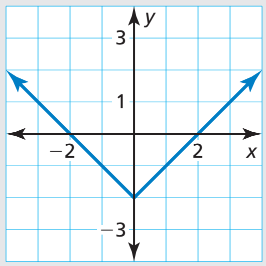
1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Does the graph or table represent a *linear* or *nonlinear* function? Explain.

1.



2.



3.

x	0	1	2	3
y	3	5	7	9

4.

x	1	2	3	4
y	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$

Does the equation represent a *linear* or *nonlinear* function? Explain.

5. $y = x + 9$

6. $5 = -x + y$

7. $y = 5 - 2x^2$

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Graphing Linear Functions

EXAMPLE 4 Using Tables to Graph Linear Functions

Graph the linear function represented by each table.

a.

x	y
−2	−4
0	−3
2	−2
4	−1

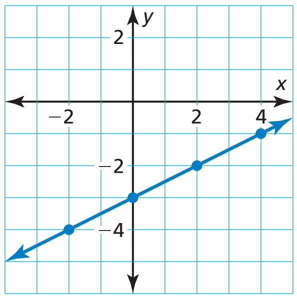
b.

x	y
−1	5
2	2.5
5	0
8	−2.5

SOLUTION

a. Write each ordered pair (x, y). Then plot each point in a coordinate plane and draw a line through the points.

x	y	(x, y)
−2	−4	(−2, −4)
0	−3	(0, −3)
2	−2	(2, −2)
4	−1	(4, −1)

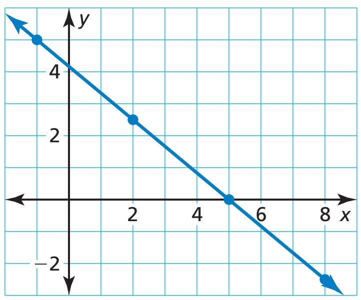


STUDY TIP

When the domain of a linear function is not specified or cannot be obtained from a real-life context, it is understood to be all real numbers.

b. Write each ordered pair (x, y). Then plot each point in a coordinate plane and draw a line through the points.

x	y	(x, y)
−1	5	(−1, 5)
2	2.5	(2, 2.5)
5	0	(5, 0)
8	−2.5	(8, −2.5)



4 MTR CONSTRUCT AN ARGUMENT

How many points do you need in order to graph a linear function? Explain your reasoning.

GO DIGITAL

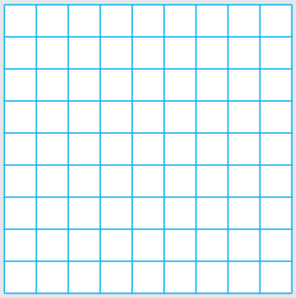
SELF-ASSESSMENT

1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

Graph the linear function represented by the table.

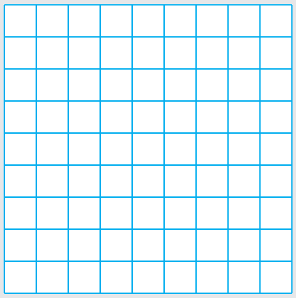
8.

x	0	1	2	3
y	6	4	2	0

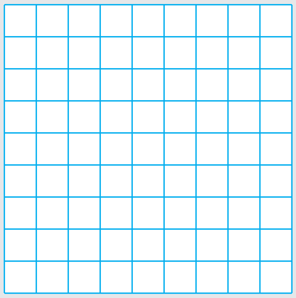


9.

x	−2	−1	0	1
y	−1	0.5	2	3.5



5 MTR **10.** **PATTERNS** The graph of a linear function passes through the point $(-3, 2.25)$. As x increases by 4, y decreases by 1.25. Graph the function.



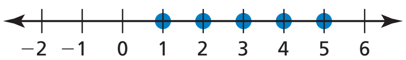
A **solution of a linear equation in two variables** is an ordered pair (x, y) that makes the equation true. The graph of a linear equation in two variables is the set of points (x, y) in a coordinate plane that represents all solutions of the equation. Sometimes the points are distinct, and other times the points are connected.

KEY IDEAS

Discrete and Continuous Domains

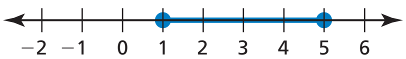
A **discrete domain** is a set of input values that consists of only certain numbers in an interval.

Example: Integers from 1 to 5



A **continuous domain** is a set of input values that consists of all numbers in an interval.

Example: All numbers from 1 to 5



EXAMPLE 5 Graphing Discrete Data

The linear function $y = 15.95x$ represents the cost y (in dollars) of x tickets for a museum. Each customer can buy a maximum of four tickets.

a. Find the domain of the function. Is the domain discrete or continuous? Explain.

b. Graph the function using its domain.

STUDY TIP

The domain of a function depends on the real-life context of the function, not just the equation that represents the function.

SOLUTION

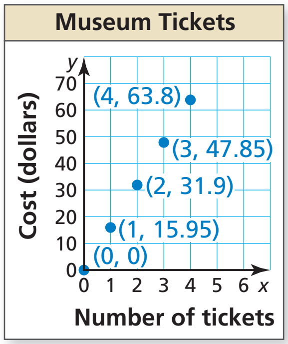
a. You cannot buy part of a ticket, only a certain number of tickets. Because x represents the number of tickets, it must be a whole number. The maximum number of tickets a customer can buy is four.

▶ So, the domain is 0, 1, 2, 3, and 4, and it is discrete.

b. Step 1 Make an input-output table to find the ordered pairs.

Input, x	$15.95x$	Output, y	(x, y)
0	15.95(0)	0	(0, 0)
1	15.95(1)	15.95	(1, 15.95)
2	15.95(2)	31.9	(2, 31.9)
3	15.95(3)	47.85	(3, 47.85)
4	15.95(4)	63.8	(4, 63.8)

Step 2 Plot the ordered pairs. The domain is discrete. So, the graph consists of individual points.



SELF-ASSESSMENT

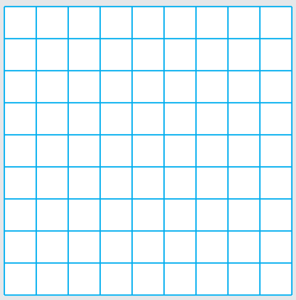
1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

11. The linear function $m = 50 - 9d$ represents the amount m (in dollars) of money you have left after buying d DVDs.

a. Interpret the terms and coefficient in the equation.

b. Find the domain of the function. Is the domain discrete or continuous? Explain.

c. Graph the function using its domain.



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EXAMPLE 6 Graphing Continuous Data

Each year, thousands of peregrine falcons fly over the Florida Keys as they migrate to South America. A peregrine falcon can fly 350 feet per second.

a. Explain why the distance traveled is a function of the number of seconds.

b. Find the domain of the function. Is the domain discrete or continuous? Explain.

c. Graph the function using its domain.

SOLUTION

a. As the number n of seconds increases by 1, the distance d traveled increases by 350 feet. The rate of change is constant.

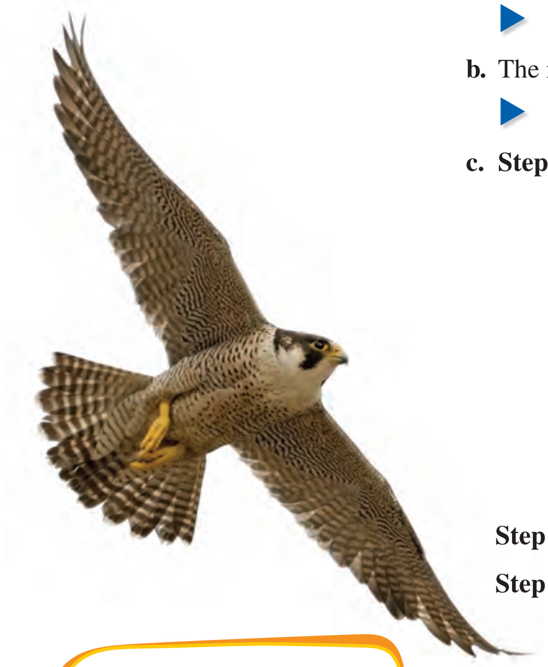
▶ So, this situation represents a linear function.

b. The number n of seconds can be any value greater than or equal to 0.

▶ So, the domain is $n \geq 0$, and it is continuous.

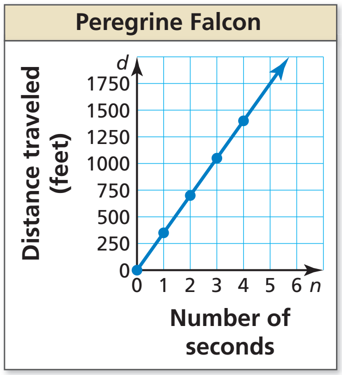
c. Step 1 Make an input-output table to find ordered pairs.

Input, n	Output, d	(n, d)
0	0	(0, 0)
1	350	(1, 350)
2	700	(2, 700)
3	1050	(3, 1050)
4	1400	(4, 1400)



Step 2 Plot the ordered pairs.

Step 3 Draw a line through the points. The line should start at (0, 0) and continue to the right. Use an arrow to indicate that the line continues without end, as shown. The domain is continuous. So, the graph is a line with a domain of $n \geq 0$.



Hawk Mania is an annual bird-watching event held by the Florida Keys Audubon Society. The event teaches about the falcons, eagles, and ospreys that migrate through the Keys each fall.

GO DIGITAL

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

12. A 20-gallon bathtub is draining at a rate of 2.5 gallons per minute.

a. Explain why the number of gallons remaining is a function of the number of minutes.

b. Find the domain of the function. Is the domain discrete or continuous? Explain.

c. Graph the function using its domain.

13. When feeding, a juvenile whale shark filters about 600 cubic meters of water through its mouth each hour. About 2.8 kilograms of food are filtered out from the water each hour.

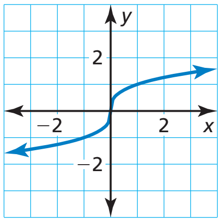
a. Graph the function that represents the amount *a* of water filtered by the whale shark as a function of the number *m* of minutes.

b. The whale shark feeds for 7.5 hours each day. Graph the function that represents the amount *f* of food (in pounds) filtered by the whale shark as a function of the number *d* of days.

3.3 Practice WITH CalcChat® AND CalcView®

In Exercises 1–6, determine whether the graph represents a *linear* or *nonlinear* function. Explain. *(See Example 1.)*

1.



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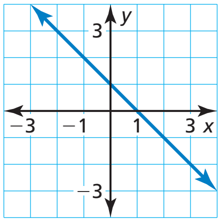

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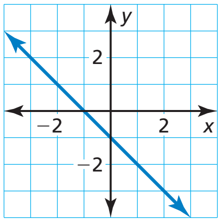

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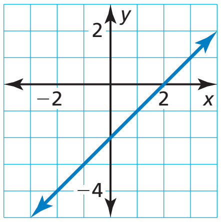

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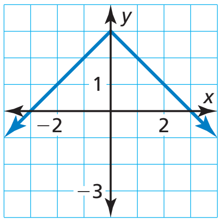

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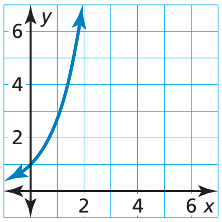


In Exercises 7–10, determine whether the table represents a *linear* or *nonlinear* function. Explain. *(See Example 2.)*

7.

x	1	2	3	4
y	5	10	15	20

8.

x	5	7	9	11
y	−9	−3	−1	3

9.

x	4	8	12	16
y	16.8	12.6	7.4	1.2

10.

x	−1	0	1	2
y	35	20	5	−10

11. REASONING Explain why a V-shaped graph does *not* represent a linear function.

12. REASONING How can you tell whether a graph shows a discrete domain or a continuous domain?

GO DIGITAL

4 MTR **ERROR ANALYSIS** **In Exercises 13 and 14, describe and correct the error in determining whether the table or graph represents a linear function.**

13.

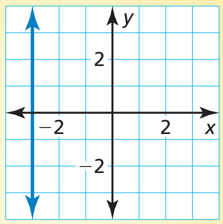
+2 +2 +2

x	2	4	6	8
y	4	16	64	256

×4 ×4 ×4

As *x* increases by 2, *y* increases by a constant factor of 4. So, the function is linear.

14.



The graph is a line. So, the graph represents a linear function.

In Exercises 15–22, determine whether the equation represents a *linear* or *nonlinear* function. Explain.
(See Example 3.)

15. $y = x^2 + 13$

16. $y = 7 - x$

17. $y = \sqrt[3]{8} - x$

18. $y = x \cdot x \cdot x$

19. $2 + y = x + 4$

20. $7 - y = 2^x$

21. $x - y = |-54|$

22. $|x| + 12 = y$

23. **B.E.S.T. TEST PREP** Which of the following equations do *not* represent linear functions? Select all that apply.

Ⓐ $12 = -x^2 + y$

Ⓑ $y - x + 3 = 0$

Ⓒ $x = 8$

Ⓓ $-x = y$

Ⓔ $y = \left(\frac{1}{2}\right)^x + 9$

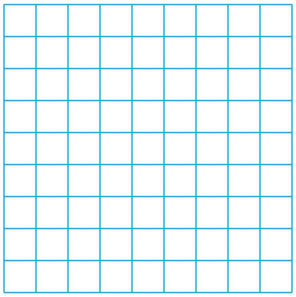
Ⓕ $y = \sqrt{x} + 3$

24. **WRITING** Compare discrete domains and continuous domains.

In Exercises 25–28, graph the linear function represented by the table. *(See Example 4.)*

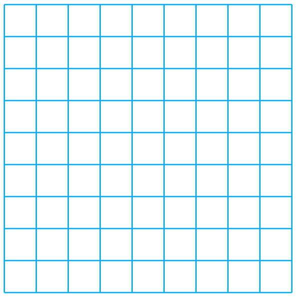
25.

x	−2	−1	0	1	2
y	7	4	1	−2	−5



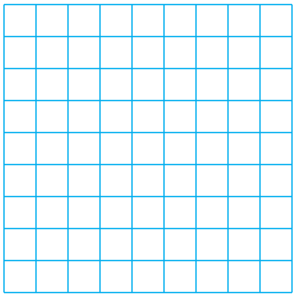
26.

x	−6	−3	0	3	6
y	−4	−3	−2	−1	0



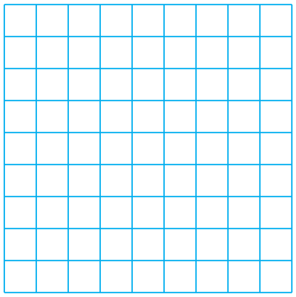
27.

x	−5	−3	3	5	8
y	−15	−11	1	5	11



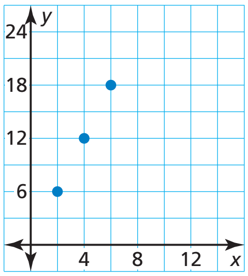
28.

x	−6.3	−3	2.1	3	7
y	9.3	6	0.9	0	−4

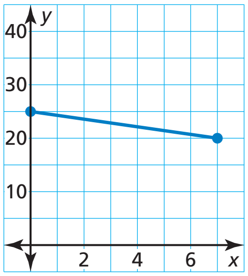


In Exercises 29 and 30, find the domain of the function represented by the graph. Determine whether the domain is *discrete* or *continuous*. Explain.

29.



30.



In Exercises 31 and 32, determine whether the domain is *discrete* or *continuous*. Explain.

31.

Input **Time (hours), *x***	3	6	9
Output **Distance (miles), *y***	150	300	450

32.

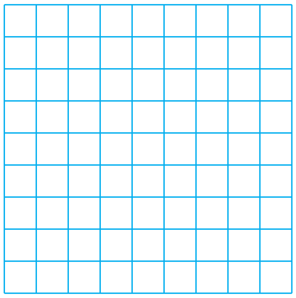
Input **Relay teams, *x***	0	1	2
Output **Athletes, *y***	0	4	8

33. MODELING REAL LIFE The linear function $m = 10 - 1.44p$ represents the amount m (in dollars) of money that you have after printing p photographs. *(See Example 5.)*

a. Interpret the terms and coefficient in the equation.

b. Find the domain of the function. Is the domain discrete or continuous? Explain.

c. Graph the function using its domain.

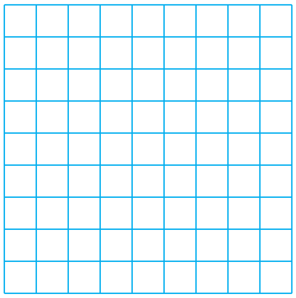


34. MODELING REAL LIFE The linear function $y = 145 + 30x$ represents the cost y (in dollars) of an airline ticket after adding x checked bags. At most 5 bags can be checked.

a. Interpret the terms and coefficient in the equation.

b. Find the domain of the function. Is the domain discrete or continuous? Explain.

c. Graph the function using its domain.

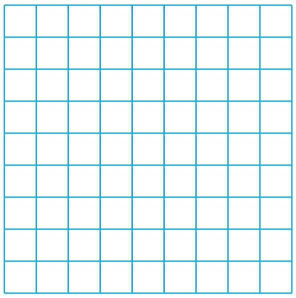


35. MODELING REAL LIFE You fill a swimming pool with water at a rate of 17 gallons per minute. *(See Example 6.)*

a. Is the amount of water in the pool a function of the number of minutes? Explain.

b. Find the domain of the function. Is the domain discrete or continuous? Explain.

c. Graph the function using its domain.

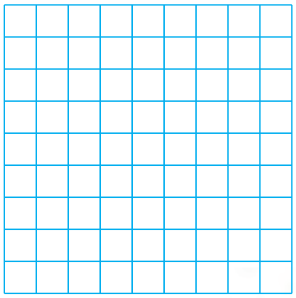


36. MODELING REAL LIFE The amount of air in a scuba diving tank with a capacity of 2400 liters is decreasing at a rate of 48 liters per minute.

a. Is the amount of air in the tank a function of the number of minutes? Explain.

b. Find the domain of the function. Is the domain discrete or continuous? Explain.

c. Graph the function using its domain.



37. STRUCTURE Complete the table so it represents a linear function.

x	5	10	15	20	25
y	−1				11

2 MTR **38. MULTIPLE REPRESENTATIONS** You are researching the speed of sound waves in dry air at 86°F. The linear function $d = 0.217t$ represents the distances d (in miles) sound waves travel in t seconds.

a. Represent the situation using a table and a graph.

b. Which of the three representations would you use to find how long it takes sound waves to travel 0.1 mile in dry air at 86°F? Explain.

39. OPEN-ENDED Identify a real-life situation where the domain is continuous. Justify your answer.

6 MTR **40. ASSESS REASONABLENESS** You and a friend observe the Mug Race along the St. Johns River. The table shows how many sailboats you see. Graph the linear function. Your friend claims you will see 110 sailboats by hour 6. Is your friend's claim reasonable? Explain.

Time (hours), *x*	0	1	2	3	4
Number of boats, *y*	0	15	30	45	60

41. REASONING Is the function represented by the ordered pairs linear or nonlinear? Explain your reasoning.

(0, 2), (3, 14), (5, 22), (9, 38), (11, 46)

42. WRITING Describe the end behavior of an increasing linear function and a decreasing linear function.

5 MTR **43. STRUCTURE** The table shows your earnings y (in dollars) for working x hours. You work no more than 18 hours each week.

Time (hours), *x*	Earnings (dollars), *y*
4	40.80
5	51.00
6	61.20
7	71.40

a. What is your hourly pay rate?

b. Find the domain and range of the function.

4 MTR **44. MAKING AN ARGUMENT** The linear function $d = 50t$ represents the distance d (in miles) Car A is from a car rental store after t hours. The table shows the distances Car B is from the rental store.

Time (hours), *t*	Distance (miles), *d*
1	60
3	180
5	310
7	450
9	540

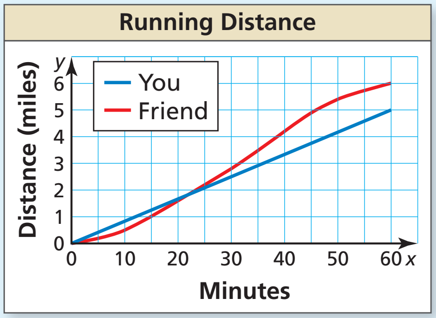
a. Does the table represent a linear or nonlinear function? Explain.

b. Which car is moving at a faster rate? Explain.

45. REASONING A water company fills two different-sized jugs. The first jug can hold x gallons of water. The second jug can hold y gallons of water. The company fills A jugs of the first size and B jugs of the second size. What does each expression represent? Does each expression represent a set of discrete or continuous values?

a. $x + y$ **b.** $A + B$ **c.** Ax **d.** $Ax + By$

46. HOW DO YOU SEE IT?
You and your friend go running. The graph shows the distances you and your friend run.



a. Describe your run and your friend's run. Who runs at a constant rate? How do you know? Why might a person not run at a constant rate?

b. Find the domain of each function. Describe the domains using the context of the problem.

7 MTR **47. PERFORMANCE TASK** You are ordering T-shirts for a school fundraiser and receive bids from three options. You are not sure exactly how many T-shirts you need, but you know it will be no more than 200 shirts. Create a proposal for the school principal. Include the number of T-shirts, which option you chose, and the cost per T-shirt.

Option 1

Charge includes an initial fee plus a cost per T-shirt. Sample pricing is shown in the table.

Number of shirts	Price (dollars)
40	200
80	340
120	480
160	620
200	760

Option 2

Cost for x T-shirts:
$c = 2x + 80$

Option 3

Cost is $5 per T-shirt.

48. THOUGHT PROVOKING
A movie complex is open from 10:00 A.M. to 2:00 A.M. daily. It contains 8 theaters, each of which has 225 seats. The number of viewers in the theaters is a function of the number of hours after 10:00 A.M. each day. Describe a reasonable domain and range of the function. Then determine whether the function must be linear. Explain.

49. REASONING Is a linear function always increasing or always decreasing? Explain.

50. DISCUSS MATHEMATICAL THINKING Explain the relationship between an equation and its graph. Why does the graph of a linear equation form a line?
4 MTR

2 MTR **51. MAKE A CONNECTION** What can you determine about the range of a linear function with a discrete domain? a continuous domain? Explain.

DIG DEEPER **In Exercises 52 and 53, describe a real-life situation for the constraints.**

52. The function has at least one negative number in the domain. The domain is continuous.

53. The function gives at least one negative number as an output. The domain is discrete.

REVIEW & REFRESH

In Exercises 54–57, solve the equation. Check your solution.

54. $h + 6 = -7$

55. $15 = 24 - 3y$

56. $9g - 12 = 6g$

57. $-18 = -2|w + 3|$

In Exercises 58–61, multiply or divide.

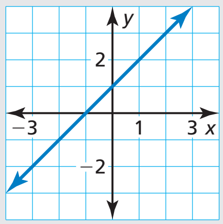
58. $\frac{3}{2} \cdot \frac{4}{7}$

59. $2\frac{5}{8} \cdot 3\frac{1}{3}$

60. $\frac{7}{8} \div \frac{1}{16}$

61. $\frac{8}{9} \div 4$

62. Tell whether x and y are proportional. Explain your reasoning.



63. Write the sentence as an inequality. Then graph the inequality.

The sum of a number and 12 is at least 35.

In Exercises 64 and 65, determine whether the relation is a function.

64. $(-5, 6), (0, 3), (2, 10), (4, -3), (-5, -2)$

65. $(-3, 4), (-1, -1), (1, 6), (2, 2), (4, -1)$

66. Is the domain discrete or continuous? Explain.

Input **Number of stories, *x***	1	2	3
Output **Height of building (feet), *y***	12	24	36

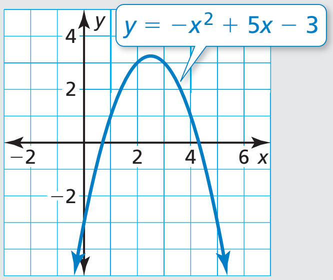
67. **OPEN-ENDED** Write an inequality that can be solved using the Division Property of Inequality where the inequality symbol needs to be reversed.

68. **MODELING REAL LIFE** An event center charges \$59.95 for each concert ticket.

a. Does this situation represent a function? If so, identify the independent and dependent variables.

b. There are 9350 seats in the arena. Find the domain and range. Is the domain discrete or continuous? Explain.

69. Approximate when the function is positive, negative, increasing, or decreasing. Then describe the end behavior of the function.



In Exercises 70–73, solve the inequality. Graph the solution.

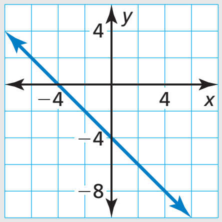
70. $-4 < -2x + 9 \le 32$

71. $-1 \le q - 6$ *or* $\frac{1}{8}q < -\frac{3}{4}$

72. $|8y + 3| > -1$

73. $4|3 - 2z| - 9 \le 3$

74. Does the graph represent a *linear* or *nonlinear* function? Explain.



In Exercises 75 and 76, write the number in scientific notation.

75. 107,000,000

76. 0.000002

3.4 Function Notation

Learning Target: Understand and use function notation.

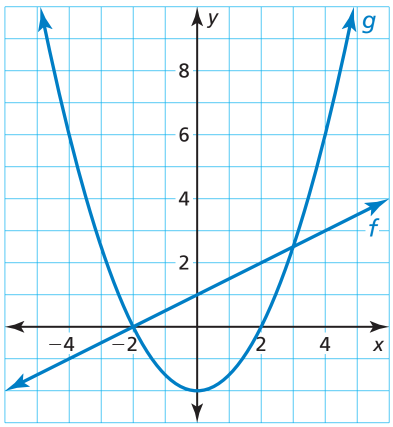
Success Criteria:
- I can evaluate functions using function notation.
- I can interpret statements that use function notation.
- I can graph functions represented using function notation.

Algebraic Reasoning
MA.912.AR.2.4 Given a table, equation, or written description of a linear function, graph that function, and determine and interpret its key features.
Functions
MA.912.F.1.5 Compare key features of linear functions each represented algebraically, graphically, in tables or written descriptions.
Also MA.912.AR.2.5, MA.912.F.1.2

EXPLORE IT! Using Function Notation

4 MTR COMMUNICATE CLEARLY
In part (b), for a function $y = f(x)$, explain the meaning of f, x, and $f(x)$.

Work with a partner. Consider the functions f and g represented by the graph.



a. Find a point on g and explain how you found the point.

b. Find a point on f and explain what $f(x) = y$ means.

c. Find the points on the appropriate graph(s) at which

- $f(0) = 1$.
- $g(-2) = 0$ and $g(2) = 0$.
- $f(3) = g(3)$.
- $g(x) < f(x)$.

d. Why is it helpful to use f and g to describe the functions instead of using y?

e. Use function notation and inequalities to complete each definition.

- A function h is positive when ____________________ and negative when ____________________.
- A function h is increasing on an interval when, for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies ____________________.
- A function h is decreasing on an interval when, for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies ____________________.

GO DIGITAL

Using Function Notation to Evaluate and Interpret

Vocabulary AZ VOCAB

function notation, *p. 214*

You learned that a linear function can be written in the form $y = mx + b$. By naming a linear function f, you can also write the function using **function notation**.

$f(x) = mx + b$ Function notation

The notation $f(x)$ is another name for y. If f is a function, and x is in its domain, then $f(x)$ represents the output of f corresponding to the input x. You can use letters other than f to name a function, such as g or h.

READING

The notation $f(x)$ is read as "the value of f at x" or "f of x." It does not mean "f times x."

EXAMPLE 1 Evaluating a Function WATCH

Evaluate $f(x) = -4x + 7$ when $x = 2$ and $x = -2$.

SOLUTION

$f(x) = -4x + 7$	Write the function.	$f(x) = -4x + 7$
$f(2) = -4(2) + 7$	Substitute for x.	$f(-2) = -4(-2) + 7$
$= -8 + 7$	Multiply.	$= 8 + 7$
$= -1$	Add.	$= 15$

▶ When $x = 2$, $f(x) = -1$, and when $x = -2$, $f(x) = 15$.

EXAMPLE 2 Interpreting Function Notation WATCH

Let $f(t)$ be the outside temperature (in degrees Fahrenheit) t hours after 6 A.M. Explain the meaning of each statement.

a. $f(0) = 58$ **b.** $f(6) = n$ **c.** $f(3.5) < f(9)$

SOLUTION

a. The initial value of the function is 58. So, the temperature at 6 A.M. is 58°F.

b. The output of f when $t = 6$ is n. So, the temperature at noon (6 hours after 6 A.M.) is n°F.

c. The output of f when $t = 3.5$ is less than the output of f when $t = 9$. So, the temperature at 9:30 A.M. (3 hours and 30 minutes after 6 A.M.) is less than the temperature at 3 P.M. (9 hours after 6 A.M.).

READING

The *initial value* of f is the value of the function when $x = 0$.

SELF-ASSESSMENT 1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Evaluate the function when $x = -4$, 0, and $\frac{1}{2}$.

1. $f(x) = 2x + 1$ **2.** $g(x) = -x - 1$ **3.** $n(x) = \frac{1}{2} - \frac{5}{6}x$

GO DIGITAL

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

4. You cook a turkey in an oven for 3 hours and 45 minutes. Let $f(t)$ be the temperature (in degrees Fahrenheit) of the turkey t hours after being placed in the oven. Explain the meaning of each statement.

a. $f(0) = 67$

b. $f(3.75) = 165$

c. $f(3) = f(4)$

d. $f(3.75) > f(4)$

Using Function Notation to Solve and Graph

EXAMPLE 3 Solving for the Independent Variable

WATCH

For $h(x) = \frac{2}{3}x - 5$, find the value of x for which $h(x) = -7$.

SOLUTION

$h(x) = \frac{2}{3}x - 5$ Write the function.

$-7 = \frac{2}{3}x - 5$ Substitute -7 for $h(x)$.

$-2 = \frac{2}{3}x$ Add 5 to each side.

$-3 = x$ Multiply each side by $\frac{3}{2}$.

▶ When $x = -3$, $h(x) = -7$.

GO DIGITAL

EXAMPLE 4 Graphing a Linear Function in Function Notation

Graph $f(x) = 2x + 5$.

WATCH

SOLUTION

Step 1 Make an input-output table to find ordered pairs.

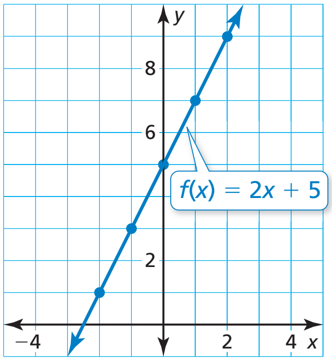
x	-2	-1	0	1	2
$f(x)$	1	3	5	7	9

STUDY TIP

The graph of $y = f(x)$ consists of the points $(x, f(x))$.

Step 2 Plot the ordered pairs.

Step 3 Draw a line through the points.



SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

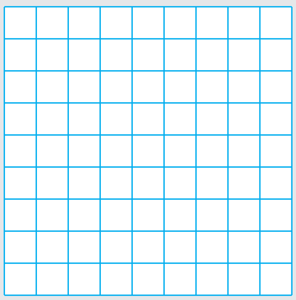
Find the value of x so that the function has the given value.

5. $f(x) = 6x + 9;\ f(x) = 21$

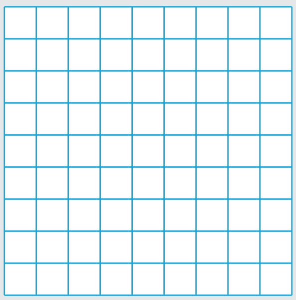
6. $g(x) = -\frac{1}{2}x + 3;\ g(x) = -1$

Graph the linear function.

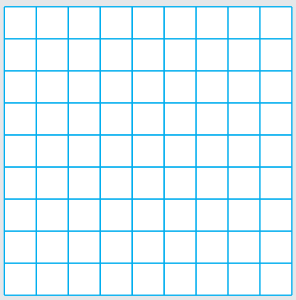
7. $f(x) = 3x - 2$



8. $g(x) = 4 - x$



9. $h(x) = -\frac{3}{4}x - 1$



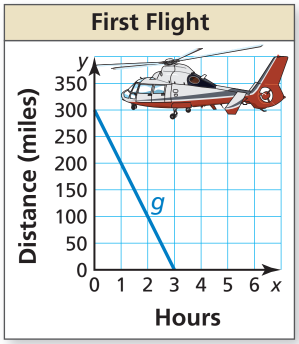
10. REASONING Let g be a function.

a. Given that $13 = g(-5)$, find a point that is on the graph of g.

b. Given that $-\frac{1}{2}$ is a solution of $g(x) = 4$, find a point that is on the graph of g.

Solving Real-Life Problems

7 MTR **EXAMPLE 5** **Modeling Real Life** WATCH



The graph of g shows how far a helicopter is from its destination after taking off. The function $f(x) = 350 - 125x$ represents a second flight, where $f(x)$ is the number of miles the helicopter is from its destination after x hours. Which flight takes less time?

SOLUTION

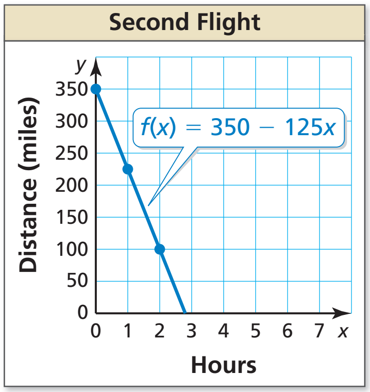
1. **Understand the Problem** You are given a graph of the first flight and an equation of the second flight. You are asked to determine which flight takes less time.

2. **Make a Plan** Graph the function that represents the second flight. Because both flights involve the same quantities with the same units, you can compare the graphs to determine which flight takes less time. The x-values that correspond to $f(x) = 0$ and $g(x) = 0$ represent the total flight times.

3. **Solve and Check** Graph $f(x) = 350 - 125x$.

Step 1 Make an input-output table to find the ordered pairs.

x	0	1	2	3
$f(x)$	350	225	100	−25



Step 2 Plot the ordered pairs. Draw a line through the points. Note that the function makes sense only when x and $f(x)$ are nonnegative. So, draw the line in the first quadrant only.

▶ From the graph of the first flight, you can see that when $g(x) = 0$, $x \approx 3$. From the graph of the second flight, you can see that when $f(x) = 0$, $x < 3$. So, the second flight takes less time.

Another Way For the second flight, use the equation to find the value of x for which $f(x) = 0$.

$f(x) = 350 - 125x$ — Write the function.

$0 = 350 - 125x$ — Substitute 0 for $f(x)$.

$-350 = -125x$ — Subtract 350 from each side.

$2.8 = x$ — Divide each side by −125.

Because $2.8 < 3$, the second flight takes less time.

2 MTR **MAKE A CONNECTION**

What happens to $f(x)$ as x increases from 2 to 3? What does this tell you about the time it takes the helicopter to reach its destination?

GO DIGITAL

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

11. WHAT IF? Let $f(x) = 250 - 75x$ represent the second flight in Example 5, where $f(x)$ is the number of miles the helicopter is from its destination after x hours. Which flight takes less time? Explain.

12. You stand at zero on a number line and flip a coin. When the coin is heads, you move one unit to the right. When the coin is tails, you move one unit to the left. After each flip, you record your position on the number line. Let g represent your position after the nth flip.

a. Explain why g is a function.

b. What does $g(5) = 3$ represent?

c. What is the probability that $g(3) = 0$? Explain your reasoning.

3.4 Practice WITH CalcChat® AND CalcView®

In Exercises 1–8, evaluate the function when $x = -2, 0,$ and 5. *(See Example 1.)*

1. $f(x) = x + 6$

2. $g(x) = 3x$

3. $h(x) = -2x + 9$

4. $r(x) = -x - 7$

5. $p(x) = -3 + \frac{1}{4}x$

6. $b(x) = 18 - 0.5x$

7. $v(x) = 12 - 2x - 5.8$

8. $n(x) = -1 - \frac{1}{3}x + 1\frac{2}{3}$

9. INTERPRETING FUNCTION NOTATION Let $c(t)$ be the number of customers in a restaurant t hours after 8 A.M. Explain the meaning of each statement. *(See Example 2.)*

a. $c(0) = 0$

b. $c(3) = c(8)$

c. $c(n) = 29$

d. $c(13.5) < c(12)$

10. INTERPRETING FUNCTION NOTATION Let $H(x)$ be the percent of U.S. households with Internet use x years after 1980. Explain the meaning of each statement.

a. $H(27) = 61.7$

b. $H(4) = k$

c. $H(37) > H(30)$

d. $H(17) + H(21) \approx H(29)$

In Exercises 11–16, find the value of x so that the function has the given value. *(See Example 3.)*

11. $h(x) = -7x;\ h(x) = 63$

12. $t(x) = 3x;\ t(x) = 24$

13. $m(x) = 4x + 15;\ m(x) = 7$

14. $k(x) = 6x - 12;\ k(x) = 15$

15. $q(x) = 0.5x - 3;\ q(x) = -4$

16. $j(x) = -\frac{4}{5}x + 7;\ j(x) = -5$

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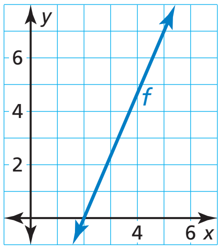
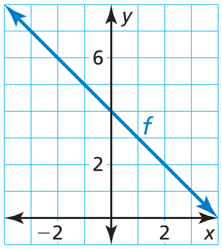
In Exercises 17 and 18, find the value of x so that $f(x) = 7$.

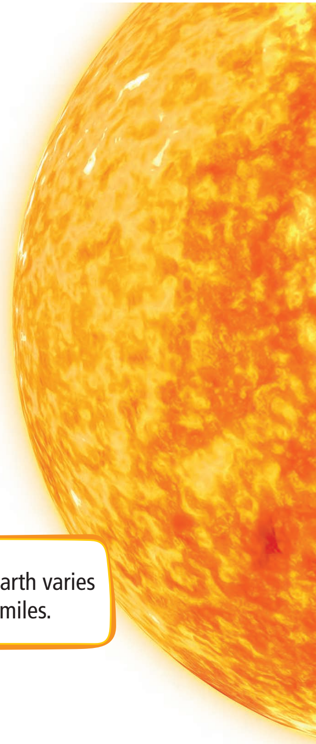
17.



18.



19. **MODELING REAL LIFE** The function $C(x) = 17.5x - 10$ represents the cost (in dollars) of buying x tickets to the orchestra with a \$10 coupon. How much does it cost to buy five tickets?



The distance from the Sun to Earth varies from 91.4 to 94.5 million miles.

20. **MODELING REAL LIFE** The function $d(t) = 300{,}000t$ represents the distance (in kilometers) that light travels in t seconds.

a. How far does light travel in 15 seconds?

b. How long does it take light to travel from the Sun to Earth?

In Exercises 21–28, graph the linear function. *(See Example 4.)*

21.

x	y
−2	2
0	1
2	0
4	−1

22.

x	y
−6	−1
−3	0
0	1
3	2

23. $p(x) = 4x$

24. $h(x) = -5x$

25. $d(x) = -\frac{1}{2}x - 3$

26. $w(x) = 0.6x + 2$

27. $g(x) = -4 + 7x$

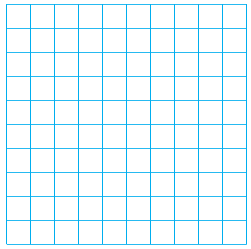
28. $f(x) = 3 - 6x$

29. PROBLEM SOLVING The function $C(x) = 25x + 50$ represents the labor cost (in dollars) for Contractor A to build a deck, where x is the number of hours of labor. The table shows sample labor costs from its main competitor, Contractor B. The deck is estimated to take 8 hours of labor. Which contractor would you hire? Explain. *(See Example 5.)*

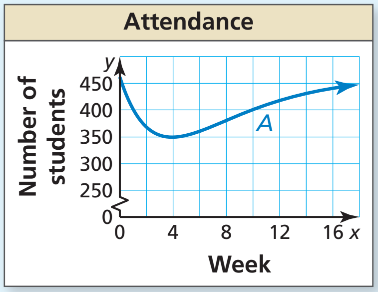
Hours	Cost
2	\$130
4	\$160
6	\$190

30. MODELING REAL LIFE The function $g(x) = 1.0 - 0.2x$ represents the percent (in decimal form) of battery power remaining on a laptop x hours after it is turned on. The function $f(x) = 0.75 - 0.125x$ represents the percent (in decimal form) of battery power remaining on a tablet x hours after it is turned on. Which device stays on longer? Explain.

31. MODELING REAL LIFE The cost of a wedding venue can be represented by the function $C(x) = 1000 + 45x$, where $C(x)$ represents the total cost of the space and x represents the number of people attending. A couple wants to spend no more than \$8900 for the venue. Graph the function in terms of the context. Then interpret the domain and range.



32. HOW DO YOU SEE IT? The function A represents the attendance at a high school since the beginning of a flu outbreak. The graph of the function is shown.



a. What happens to the school's attendance during the flu outbreak?

b. Estimate $A(13)$ and explain its meaning.

c. Use the graph to estimate the solution(s) of the equation $A(x) = 400$. Explain the meaning of the solution(s).

d. What was the least attendance? When did that occur?

e. How many students do you think are enrolled at this high school? Explain your reasoning.

33. REASONING A student's height can be represented by a function h, where the input is the student's age.

a. What does $h(14)$ represent? Can you determine the units for height? Explain.

b. What does $h(15) = 58$ represent? What can you conclude about the units for height?

34. THOUGHT PROVOKING Let $B(t)$ be your bank account balance after t days. Describe a situation in which $B(0) < B(4) < B(2)$.

35. REASONING Given a function f, tell whether the statement

$$f(a + b) = f(a) + f(b)$$

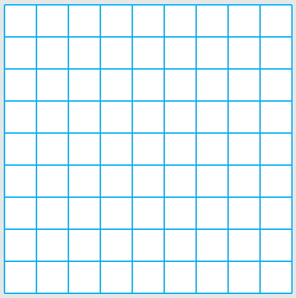
is true or false for all values of a and b. If it is false, explain why.

36. DIG DEEPER Let $f(x) = -5 - 10x$ and $g(x) = \frac{1}{5}x + 1$. Find $f(g(x))$. Simplify the expression.

REVIEW & REFRESH

37. Sketch a graph of a function with the given characteristics.

- The function is decreasing when $x < 1$ and increasing when $x > 1$.
- The function is positive when $x < -2.5$, negative when $-2.5 < x < 4.5$, and positive when $x > 4.5$.



In Exercises 38 and 39, solve the inequality. Graph the solution.

38. $5a < -35$ *or* $a - 14 > 1$

39. $-16 \le 6k + 2 < 0$

40. Determine whether the relation is a function. Explain.

Input, *x*	−1	0	1	2	3
Output, *y*	0	1	4	4	8

41. Write the sentence as an inequality.

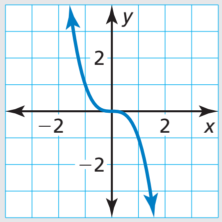
Four times a number *n* minus 10.5 is no less than −1.5.

42. For $g(x) = 10 - 8x$, find the value of x for which $g(x) = 42$.

43. Solve the literal equation $m = 5x + 6xy$ for x.

44. 7 MTR **MODELING REAL LIFE** You order two hamburgers and a drink. The drink costs \$1.50. You pay a total of \$5.83, including a 6% sales tax. How much does one hamburger cost?

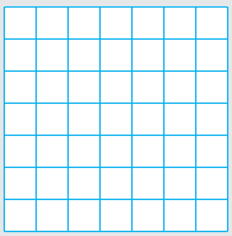
45. Determine whether the graph represents a *linear* or *nonlinear* function. Explain.



46. **NUMBER SENSE** Three times the greater of two consecutive odd integers is 15 less than two-thirds of the lesser integer. What are the integers?

47. Write $1\frac{3}{11}$ as a decimal.

48. Graph $f(x) = \frac{3}{2}x - 1$.



3.5 Graphing Linear Equations in Standard Form

Learning Target: Graph and interpret linear equations written in standard form.

Success Criteria:
- I can graph equations of horizontal and vertical lines.
- I can graph linear equations written in standard form using intercepts.
- I can solve real-life problems using linear equations in standard form.

Algebraic Reasoning
MA.912.AR.2.4 Given a table, equation or written description of a linear function, graph that function, and determine and interpret its key features.
MA.912.AR.2.5 Solve and graph mathematical and real-world problems that are modeled with linear functions. Interpret key features and determine constraints in terms of the context.

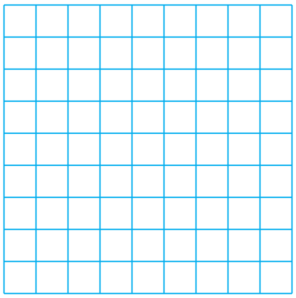
EXPLORE IT! Analyzing and Graphing a Linear Equation

Work with a partner. You sold a total of \$80 in tickets to a fundraiser. You lost track of how many of each type of ticket you sold. Adult tickets are \$4 each. Child tickets are \$2 each. The equation $4x + 2y = 80$ represents this situation, where x is the number of adult tickets and y is the number of child tickets.

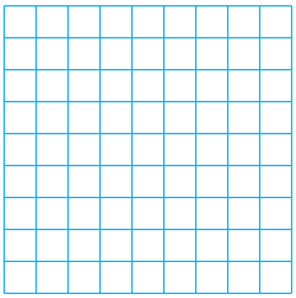
a. If you sold a large quantity of adult tickets, does that mean you also sold a large quantity of child tickets? Explain.

b. Construct a table of values to show different combinations of tickets you might have sold. Then plot the points and describe any patterns you notice.

x					
y					



c. Graph the equation. Find the intercepts. Explain the meanings of the intercepts in the context of the problem.



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d. Use technology to check your results in parts (b) and (c). Describe the characteristics of the graph.

e. If you know how many adult tickets you sold, can you determine how many child tickets you sold? Explain your reasoning.

f. Determine whether each statement is correct. Explain your reasoning.

i. As the value of x increases, the value of $2y$ decreases.

ii. As the value of y decreases, the value of $4x$ decreases.

iii. For $x < 10$, $y > 20$.

iv. $x = 20$ makes the equation true.

2 MTR **MODEL A PROBLEM**

How many different ways did you model the problem? Describe the benefits of each representation.

Horizontal and Vertical Lines

Vocabulary

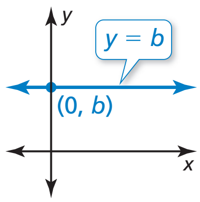
standard form, *p. 225*

The **standard form** of a linear equation is $Ax + By = C$, where A, B, and C are real numbers and A and B are not both zero.

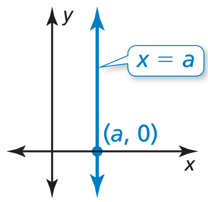
Consider what happens when $A = 0$ or when $B = 0$. When $A = 0$, the equation becomes $By = C$, or $y = \frac{C}{B}$. Because $\frac{C}{B}$ is a constant, you can write $y = b$. Similarly, when $B = 0$, the equation becomes $Ax = C$, or $x = \frac{C}{A}$, and you can write $x = a$.

KEY IDEAS

Horizontal and Vertical Lines



The graph of $y = b$ is a horizontal line. The line passes through the point $(0, b)$.



The graph of $x = a$ is a vertical line. The line passes through the point $(a, 0)$.

STUDY TIP

For a horizontal line, notice that for every value of *x*, the value of *y* is *b*.

For a vertical line, notice that for every value of *y*, the value of *x* is *a*.

EXAMPLE 1 Graphing Horizontal and Vertical Lines

WATCH

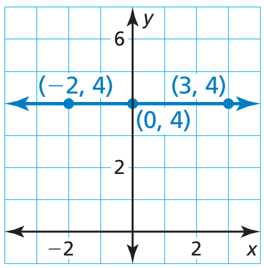
Graph each linear equation.

a. $y = 4$

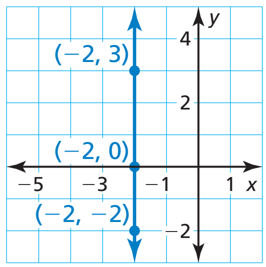
b. $x = -2$

SOLUTION

a. For every value of x, the value of y is 4. The graph of the equation $y = 4$ is a horizontal line 4 units above the x-axis.



b. For every value of y, the value of x is -2. The graph of the equation $x = -2$ is a vertical line 2 units to the left of the y-axis.



STUDY TIP

For every value of *x*, the ordered pair (*x*, 4) is a solution of *y* = 4.

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SELF-ASSESSMENT

1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

Graph the linear equation.

1. $y = -2.5$

2. $x = 5$

3. $x = -\frac{4}{3}$

4. $y = 18$

5. WRITING Describe the x- and y-intercepts of the horizontal line that passes through the origin and the vertical line that passes through the origin.

6. REASONING Graph $x = -1$ and $y = -1$. Does each graph represent a function? If so, find the domain and range.

Using Intercepts to Graph Linear Equations

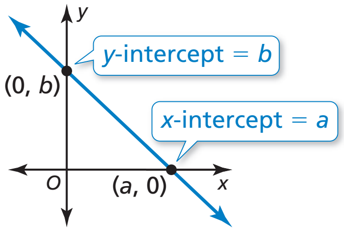
You can use the fact that two points determine a line to graph a linear equation. Two convenient points are the x- and y-intercepts.

KEY IDEA

Using Intercepts to Graph Equations

To graph the linear equation $Ax + By = C$ using intercepts, find the intercepts and draw the line that passes through them.

- To find the x-intercept, let $y = 0$ and solve for x.
- To find the y-intercept, let $x = 0$ and solve for y.



EXAMPLE 2 Using Intercepts to Graph a Linear Equation

Use intercepts to graph the equation $3x + 4y = 12$.

WATCH

SOLUTION

Step 1 Find the intercepts.

To find the x-intercept, substitute 0 for y and solve for x.

$3x + 4y = 12$	Write the original equation.
$3x + 4(0) = 12$	Substitute 0 for y.
$x = 4$	Solve for x.

To find the y-intercept, substitute 0 for x and solve for y.

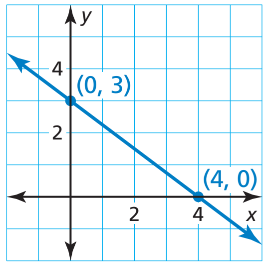
$3x + 4y = 12$	Write the original equation.
$3(0) + 4y = 12$	Substitute 0 for x.
$y = 3$	Solve for y.

Step 2 Plot the points and draw the line.

The x-intercept is 4, so plot the point (4, 0).

The y-intercept is 3, so plot the point (0, 3).

Draw a line through the points.



STUDY TIP

You can check your answer by finding other solutions of the equation and verifying that the corresponding points are on the graph.

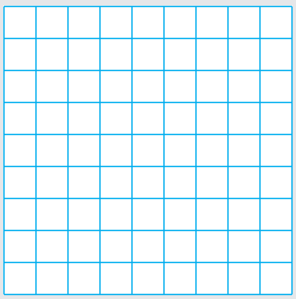
SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

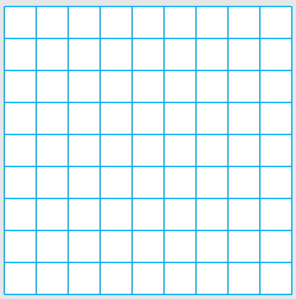
4 MTR **7. DISCUSS MATHEMATICAL THINKING** What are some advantages of using the standard form of a linear equation?

Use intercepts to graph the linear equation. Label the points corresponding to the intercepts.

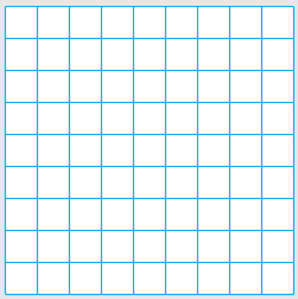
8. $2x - y = 4$



9. $x + 3y = -9$



10. $\frac{3}{4}x + 2y = 6$



2 MTR **11. MAKE A CONNECTION** Describe the graph of a linear equation written in the form $Ax + By = C$ when $C = 0$.

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Solving Real-Life Problems

7 MTR **EXAMPLE 3** **Modeling Real Life** WATCH

DECOMPOSE A PROBLEM 5 MTR
What do the terms $6x$ and $10y$ represent in this situation?

You are planning an awards banquet and need to rent tables to seat 180 people. There are two table sizes available. Small tables seat 6 people, and large tables seat 10 people. The equation $6x + 10y = 180$ models this situation, where x is the number of small tables and y is the number of large tables.

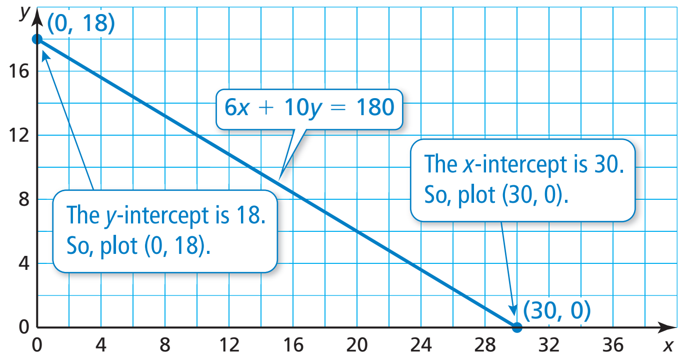
a. Graph the equation. Interpret the intercepts.

b. Find three possible solutions in the context of the problem.

SOLUTION

a. Use intercepts to graph the equation. Neither x nor y can be negative, so graph the equation only in the first quadrant.

STUDY TIP
Although x and y represent discrete data, it is convenient to draw a line segment that includes points whose coordinates are not whole numbers.



Use the graph to interpret the intercepts.

▶ The x-intercept shows that you can rent 30 small tables when you do not rent any large tables. The y-intercept shows that you can rent 18 large tables when you do not rent any small tables.

b. Only whole-number values of x and y make sense in the context of the problem. Besides the intercepts, it appears that the line passes through the point (10, 12). To verify that this point is a solution, check it in the equation.

$$6x + 10y = 180$$

$$6(10) + 10(12) \stackrel{?}{=} 180$$

$$180 = 180 \checkmark$$

▶ So, three possible combinations of tables that will seat 180 people are 0 small and 18 large, 10 small and 12 large, and 30 small and 0 large.

SELF-ASSESSMENT

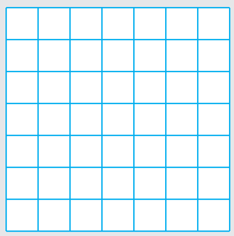
1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

12. WHAT IF? You decide to rent tables from a different company. The situation can be modeled by the equation $4x + 6y = 180$, where x is the number of small tables and y is the number of large tables.

a. Interpret the terms and coefficients in the equation.

b. Graph the equation. Interpret the intercepts.

c. Find three possible solutions in the context of the problem.



13. The number of people attending the banquet in Example 3 increases by 25%. Your friend claims that not all of the tables will be completely filled. Is your friend correct? Explain.

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3.5 Practice WITH CalcChat® AND CalcView®

In Exercises 1–4, graph the linear equation. *(See Example 1.)*

1. $x = 4$

2. $y = -3$

▶ **3.** $y = \frac{1}{2}$

4. $x = -1.5$

In Exercises 5–8, find the *x*- and *y*-intercepts of the graph of the linear equation.

5. $2x + 3y = 12$

6. $-6x + 9y = -18$

7. $3x = 6y + 2$

8. $\frac{3}{4} + x = \frac{1}{2}y$

In Exercises 9–18, use intercepts to graph the linear equation. Label the points corresponding to the intercepts. *(See Example 2.)*

9. $5x + 3y = 30$

10. $4x + 6y = 12$

▶ **11.** $-12x + 3y = 24$

12. $-2x + 6y = 18$

13. $-4x + 3y = -30$

14. $-2x + 7y = -21$

15. $2y - x = 7$

16. $3x + 5 = y$

17. $\frac{4}{3} + \frac{2}{3}x = \frac{1}{6}y$

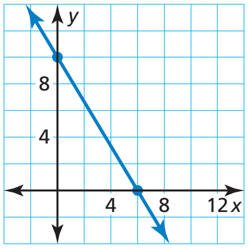
18. $y = \frac{1}{4} - \frac{5}{2}x$

GO DIGITAL

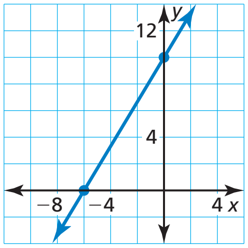
MULTIPLE REPRESENTATIONS In Exercises 19–22, match the equation with its graph.

19. $5x + 3y = 30$ **20.** $5x + 3y = -30$ **21.** $5x - 3y = 30$ **22.** $5x - 3y = -30$

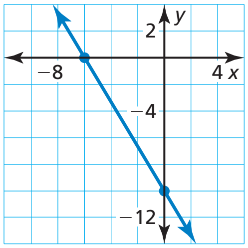
A.



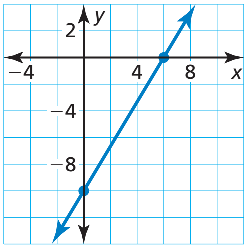
B.



C.



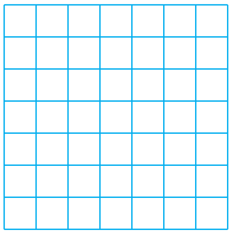
D.



23. **MODELING REAL LIFE** You have a budget of \$300 to order shirts for a math club. The equation $10x + 12y = 300$ models the total cost, where x is the number of short-sleeved shirts and y is the number of long-sleeved shirts. *(See Example 3.)*

a. Interpret the terms and coefficients in the equation.

b. Graph the equation. Interpret the intercepts.

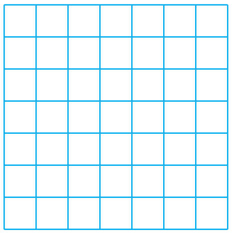


c. Find three possible solutions in the context of the problem.

24. **MODELING REAL LIFE** Your goal is to bike and jog a total of 150 miles this month. You want to bike no more than 120 miles this month. The equation $12.5x + 6y = 150$ models this situation, where x is the number of hours you bike and y is the number of hours you jog.

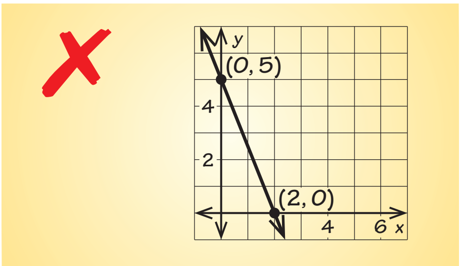
a. Interpret the terms and coefficients in the equation.

b. Graph the equation. Interpret the intercept(s).



c. You bike for 9 hours this month. How many hours must you jog to reach your goal? How many miles do you bike? jog?

25. **ERROR ANALYSIS** Describe and correct the error in using intercepts to graph the linear equation $4x + 10y = 20$.



26. **MAKING AN ARGUMENT** To find the x-intercept of the graph of a linear equation, can you substitute 0 for x and solve the equation? Explain.

CONNECTING CONCEPTS **In Exercises 27–30, write a set of linear equations that intersect to form the enclosed shape.**

27. rectangle

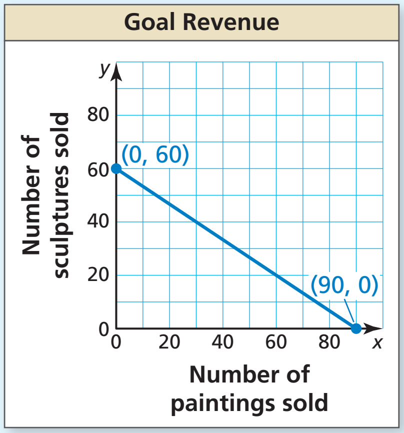
28. square

29. right triangle

30. trapezoid

31. **REASONING** Are the equations of horizontal and vertical lines written in standard form? Explain.

32. **HOW DO YOU SEE IT?**
An artist wants to earn a revenue of \$2700 by selling paintings for \$30 each and sculptures for \$45 each.



a. Interpret the intercepts of the graph.

b. Describe the domain and range in the context of the problem.

33. **B.E.S.T. TEST PREP** Which of the following is *not* true about the graph of $-\frac{2}{5}x + \frac{1}{10}y = -\frac{4}{5}$?

Ⓐ The x-intercept is 2 and the y-intercept is -8.

Ⓑ The function is decreasing when $x < 2$ and increasing when $x > 2$.

Ⓒ The graph passes through $(1, -4)$ and $(5, 12)$.

Ⓓ $y \to -\infty$ as $x \to -\infty$ and $y \to +\infty$ as $x \to +\infty$.

34. **THOUGHT PROVOKING**
The x- and y-intercepts of the graph of $ax + by = k$ are integers. Describe the possible values of k. Explain your reasoning.

35. **DIG DEEPER** You have \$99 to buy stamps and envelopes. A sheet of 20 stamps costs \$11. A box of 50 envelopes costs \$7.50.

a. Write an equation in standard form that models this situation. Do the intercepts of the graph make sense in this context? Explain.

b. Can you use all of the money to buy the same numbers of stamps and envelopes? Explain.

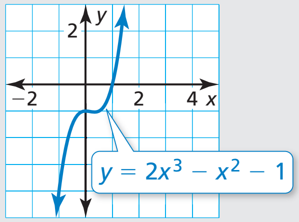
REVIEW & REFRESH

36. **MODELING REAL LIFE** The function $D(t) = 75 - 0.3t$ represents the number of gigabytes left after downloading a video game for t minutes.

a. How many gigabytes are left to download after 90 minutes?

b. How long will it take to download the entire video game?

37. Estimate the intercepts of the graph of the function.

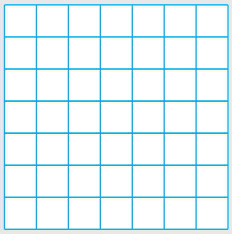


38. **WRITING** Explain how you can determine whether a graph represents a *linear* or a *nonlinear* function.

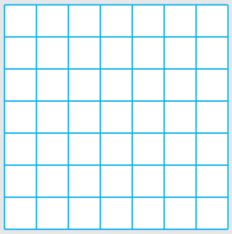
39. Determine whether the equation $y = x(2 - x)$ represents a *linear* or *nonlinear* function. Explain.

In Exercises 40 and 41, solve the inequality. Graph the solution.

40. $b + 5 \leq -12$



41. $-\frac{c}{3} > -15$



42. **REASONING** Complete the equation ____ x + ____ $y = 30$ so that the x-intercept of the graph is -10 and the y-intercept of the graph is 5.

In Exercises 43–46, solve the equation. Check your solutions.

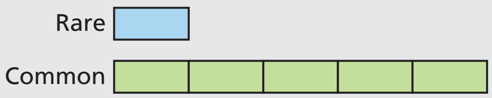
43. $6.8 + g = 14.1$

44. $-11 = 7 - 3(h + 2)$

45. $3(4 - 8k) = -4(6k - 3)$

46. $5|6n - 9| + 4 = 29$

47. The tape diagram represents the ratio of rare cards to common cards in a collection. There are 9 rare cards. How many common cards are in the collection?



48. For $f(x) = -\frac{2}{3}x + 1$, find the value of x for which $f(x) = 9$.

49. Find the x- and y-intercepts of the graph of $-4x + 8y = -16$.

3.6 Graphing Linear Equations in Slope-Intercept Form

Learning Target: Find the slope of a line and use slope-intercept form.

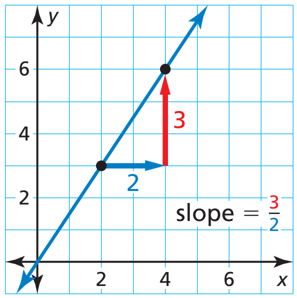
Success Criteria:
- I can find the slope of a line.
- I can use the slope-intercept form of a linear equation.
- I can solve real-life problems using slopes and y-intercepts.

Algebraic Reasoning
MA.912.AR.2.4 Given a table, equation or written description of a linear function, graph that function, and determine and interpret its key features.
MA.912.AR.2.5 Solve and graph mathematical and real-world problems that are modeled with linear functions. Interpret key features and determine constraints in terms of the context.

Slope is the rate of change between any two points on a line. It is the measure of the *steepness* of the line.

To find the slope of a line, find the value of the ratioof the change in y (vertical change) to the change in x (horizontal change).

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$



EXPLORE IT! Analyzing Linear Equations

USE STRUCTURE (5 MTR)
If you complete a similar table for a line with a negative or fractional slope or y-intercept, do your results change?

Work with a partner.

a. Complete the table for $y = 2x$. What do you notice about the values in Columns 2 and 4?

x	Change in x	y	Change in y
1	---	2	---
2	1	4	2
3			
4			
5			

b. Complete a similar table for each equation. Interpret your results.

i. $y = 2x + 1$

x	Change in x	y	Change in y
1	---		---
2			
3			
4			
5			

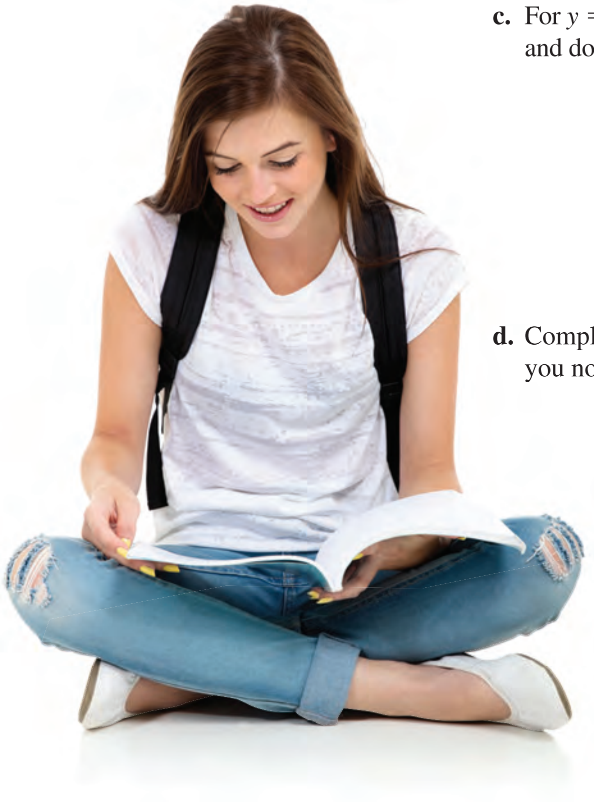
ii. $y = 4x - 3$

x	Change in x	y	Change in y
1	---		---
2			
3			
4			
5			

iii. $y = px + q$

x	Change in x	y	Change in y
1	---		---
2			
3			
4			
5			

GO DIGITAL



c. For $y = px + q$, when x increases by 1, explain why the change in y is constant and does not depend on the value of x. What does this constant represent?

d. Complete the table for each of the equations in part (a) and part (b). What do you notice?

x	Change in x	y	Change in y
1	---		---
3			
5			
7			

x	Change in x	y	Change in y
1	---		---
3			
5			
7			

x	Change in x	y	Change in y
1	---		---
3			
5			
7			

x	Change in x	y	Change in y
1	---		---
3			
5			
7			

e. For $y = px + q$, when x increases by a constant c, explain why the change in y is constant and does not depend on the valueof x. What does this constant represent?

f. What is the relationship between the graph of $y = px + q$ and the values of p and q?

The Slope of a Line

Vocabulary

slope, *p. 235*
rise, *p. 235*
run, *p. 235*
slope-intercept form, *p. 237*
constant function, *p. 237*

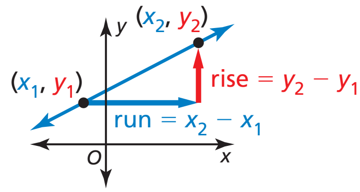
READING

In the slope formula, x_1 is read as "*x* sub one" and y_2 is read as "*y* sub two." The numbers 1 and 2 in x_1 and y_2 are called *subscripts*.

KEY IDEA

Slope

The **slope** m of a nonvertical line passing through two points (x_1, y_1) and (x_2, y_2) is the value of the ratio of the **rise** (change in y) to the **run** (change in x).



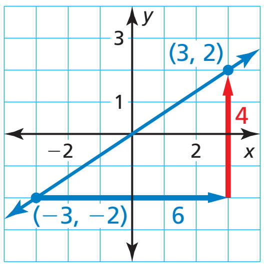
$$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

When the line rises from left to right, the slope is positive. When the line falls from left to right, the slope is negative.

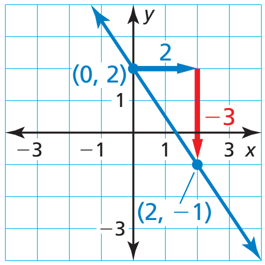
EXAMPLE 1 Finding Slopes of Lines

Describe the slope of each line. Then find the slope.

a.



b.



SOLUTION

a. The line rises from left to right. So, the slope is positive. Let $(x_1, y_1) = (-3, -2)$ and $(x_2, y_2) = (3, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-2)}{3 - (-3)} = \frac{4}{6} = \frac{2}{3}$$

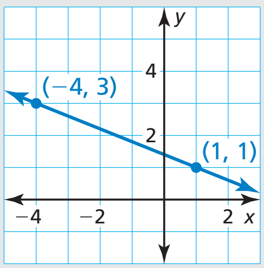
b. The line falls from left to right. So, the slope is negative. Let $(x_1, y_1) = (0, 2)$ and $(x_2, y_2) = (2, -1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 2}{2 - 0} = \frac{-3}{2} = -\frac{3}{2}$$

SELF-ASSESSMENT 1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Describe the slope of the line. Then find the slope.

1.



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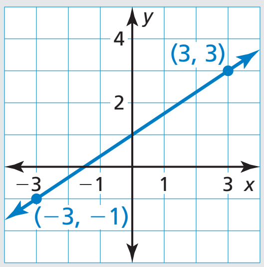

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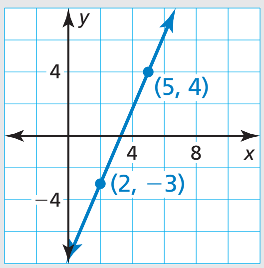


GO DIGITAL

SELF-ASSESSMENT

1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

5 MTR **4.** **STRUCTURE** When finding slope, can you label either point as (x_1, y_1) and (x_2, y_2)? Explain.

5. **WRITING** When the graph of a line is not horizontal or vertical, how can you tell whether the graph has a positive or a negative slope?

6. **REASONING** Line p has a slope of -4. Line q has a slope of $\frac{7}{4}$. Which line is steeper? Explain your reasoning.

EXAMPLE 2 Finding Slopes from Tables

WATCH

The points represented by each table lie on a line. How can you find the slope of each line from the table? What is the slope of each line?

a.

x	y
4	20
7	14
10	8
13	2

b.

x	y
-1	2
1	2
3	2
5	2

c.

x	y
-3	-3
-3	0
-3	6
-3	9

SOLUTION

Choose any two points from the table and use the slope formula.

a. Let $(x_1, y_1) = (4, 20)$ and $(x_2, y_2) = (7, 14)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14 - 20}{7 - 4} = \frac{-6}{3}, \text{ or } -2$$

▶ The slope is -2.

b. Let $(x_1, y_1) = (-1, 2)$ and $(x_2, y_2) = (5, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{5 - (-1)} = \frac{0}{6}, \text{ or } 0$$

The change in y is 0.

▶ The slope is 0.

c. Let $(x_1, y_1) = (-3, 0)$ and $(x_2, y_2) = (-3, 6)$.

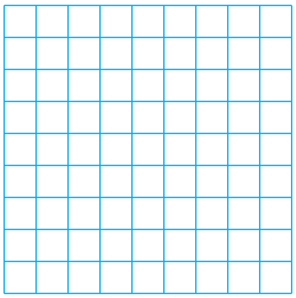
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 0}{-3 - (-3)} = \frac{6}{0} \text{ ✗}$$

The change in x is 0.

▶ Because division by zero is undefined, the slope of the line is undefined.

ASSESS REASONABLENESS 6 MTR

Plot the points in the table to show that your answer in part (a) is reasonable.

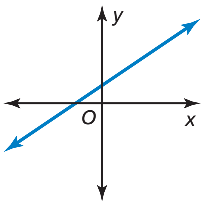


GO DIGITAL

CONCEPT SUMMARY

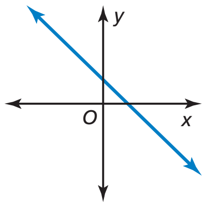
Slope

Positive slope



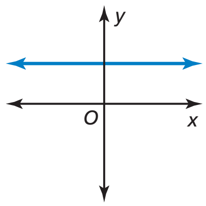
The line rises from left to right.

Negative slope



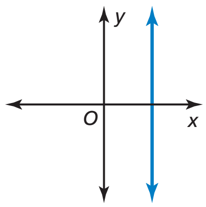
The line falls from left to right.

Slope of 0



The line is horizontal.

Undefined slope



The line is vertical.

SELF-ASSESSMENT

1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

The points represented by each table lie on a line. How can you find the slope of each line from the table? What is the slope of each line?

7.

x	2	4	6	8
y	10	15	20	25

8.

x	5	5	5	5
y	−12	−9	−6	−3

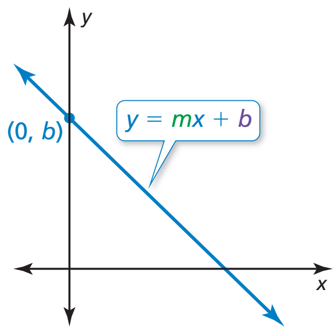
Using the Slope-Intercept Form of a Linear Equation

KEY IDEA

Slope-Intercept Form

Words A linear equation written in the form $y = mx + b$ is in **slope-intercept form**. The slope of the line is m, and the y-intercept of the line is b.

Algebra $y = mx + b$ (slope: m; y-intercept: b)



A linear equation written in the form $y = 0x + b$, or $y = b$, is a **constant function**. The graph of a constant function is a horizontal line.

EXAMPLE 3 Identifying Slopes and y-Intercepts

WATCH

Find the slope and the y-intercept of the graph of each linear equation.

a. $y = 3x - 4$ **b.** $y = 6.5$ **c.** $-5x - y = -2$

SOLUTION

a. $y = mx + b$ Write the slope-intercept form.

slope (m), y-intercept (b)

$y = 3x + (-4)$ Rewrite the original equation in slope-intercept form.

▶ The slope is 3, and the y-intercept is -4.

b. The equation represents a constant function. The equation can also be written as $y = 0x + 6.5$.

▶ The slope is 0, and the y-intercept is 6.5.

c. Rewrite the equation in slope-intercept form by solving for y.

$-5x - y = -2$ Write the original equation.

$-y = 5x - 2$ Add $5x$ to each side.

$y = -5x + 2$ Divide each side by -1.

▶ The slope is -5, and the y-intercept is 2.

STUDY TIP

For a constant function, every input has the same output. For instance, in Example 3(b), every input has an output of 6.5.

STUDY TIP

When you rewrite a linear equation in slope-intercept form, you are expressing y as a function of x.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Find the slope and the y-intercept of the graph of the linear equation.

9. $y = -6x + 1$ **10.** $y = -\frac{1}{2}$ **11.** $x + 4y = -10$

4 MTR **12. WHICH ONE DOESN'T BELONG?** Which equation does *not* belong with the other three? Explain your reasoning.

$y = -5x - 1$	$2x - y = 8$	$y = x + 4$	$y = -3x + 13$

GO DIGITAL

EXAMPLE 4 Using Slope-Intercept Form to Graph an Equation

WATCH

STUDY TIP

You can use the slope to find points on a line in either direction. In Example 4, note that the slope can be written as $\frac{2}{-1}$. So, you can move 1 unit left and 2 units up from (0, 4) to find the point (−1, 6).

Graph $2x + y = 4$. Identify the x-intercept.

SOLUTION

Step 1 Rewrite the equation in slope-intercept form.

$$y = -2x + 4$$

Step 2 Find the slope and the y-intercept.

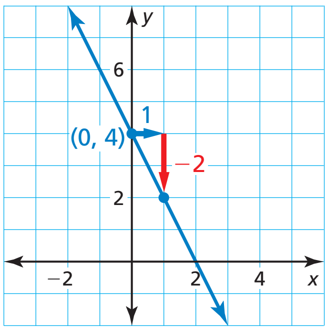
$$m = -2 \text{ and } b = 4$$

Step 3 The y-intercept is 4. So, plot (0, 4).

Step 4 Use the slope to find another point on the line.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-2}{1}$$

Plot the point that is 1 unit right and 2 units down from (0, 4). Draw a line through the two points.



▶ The line appears to intersect the x-axis at (2, 0). So, the x-intercept is 2.

2 MTR USE ANOTHER METHOD

Show how you can find the x-intercept by substituting 0 for y in the equation.

GO DIGITAL

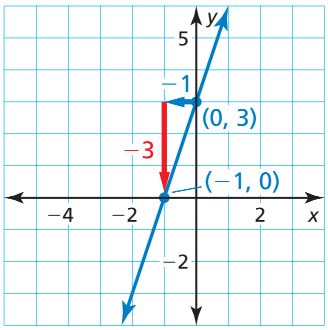
EXAMPLE 5 Graphing from a Verbal Description

WATCH

A linear function g models a relationship in which the dependent variable increases 3 units for every 1 unit the independent variable increases. Graph g when $g(0) = 3$. Identify the slope and the intercepts of the graph.

SOLUTION

Because the function g is linear, it has a constant rate of change. Let x represent the independent variable and y represent the dependent variable.



Step 1 Find the slope. When the dependent variable increases by 3, the change in y is $+3$. When the independent variable increases by 1, the change in x is $+1$.

So, the slope is $\frac{3}{1}$, or 3.

Step 2 Find the y-intercept. The statement $g(0) = 3$ indicates that when $x = 0$, $y = 3$. So, the y-intercept is 3. Plot $(0, 3)$.

Step 3 Use the slope to find another point on the line. A slope of 3 can be written as $\frac{-3}{-1}$. Plot the point that is 1 unit left and 3 units down from $(0, 3)$. Draw a line through the two points. The line crosses the x-axis at $(-1, 0)$. So, the x-intercept is -1.

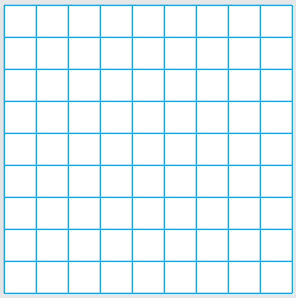
▶ The slope is 3, the y-intercept is 3, and the x-intercept is -1.

SELF-ASSESSMENT

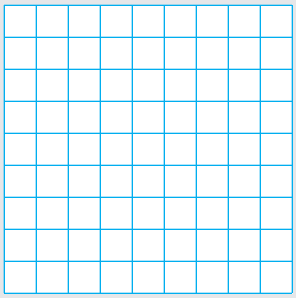
1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Graph the linear equation. Identify the x-intercept.

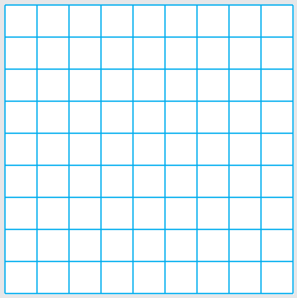
13. $y = 4x - 4$



14. $3x + y = -3$



15. $x + 2y = 6$



16. A linear function h models a relationship in which the dependent variable decreases 2 units for every 5 units the independent variable increases. Graph h when $h(0) = 4$. Identify the slope and the intercepts of the graph.

Solving Real-Life Problems

In most real-life problems, slope indicates a rate, such as miles per hour, dollars per hour, or people per year.

7 MTR

EXAMPLE 6 Modeling Real Life

WATCH

The function $h(t) = -75t - 4000$ represents the elevation $h(t)$ (in feet) of a submersible t minutes after it begins to descend. The ocean floor is at a depth of 13,000 feet.

a. Find the domain and range in this context. Then graph the function.

b. Interpret the slope and the intercepts of the graph.

SOLUTION

1. Understand the Problem You know the function that models the elevation. You are asked to find the domain and range in this context and to graph the function. Then you are asked to interpret the slope and intercepts of the graph.

2. Make a Plan Use the equation to find the domain and range in this context. Then graph the function. Examine the graph to interpret the slope and the intercepts.

3. Solve and Check

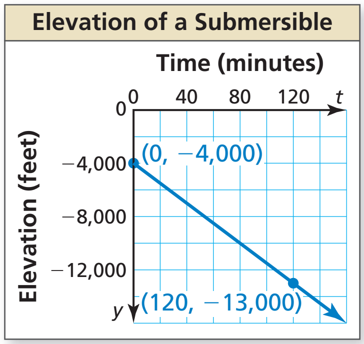
a. Because the ocean floor is at a depth of 13,000 feet, $h(t) \geq -13{,}000$. Find the value of t when $h(t) = -13{,}000$.

$$h(t) = -75t - 4000 \quad \text{Write the equation.}$$

$$-13{,}000 = -75t - 4000 \quad \text{Substitute } -13{,}000 \text{ for } h(t).$$

$$120 = t \quad \text{Solve for } t.$$

The time t must be greater than or equal to 0, and the y-intercept is -4000. So, the domain is $\{t \mid 0 \leq t \leq 120\}$, and the range is $\{h(t) \mid -13{,}000 \leq h(t) \leq -4{,}000\}$. Use the slope of -75 and the y-intercept of -4000 to graph the function in this context.



b. The slope is -75. So, the elevation changes at a rate of -75 feet per minute. The term -4000 is the y-intercept. So, the elevation of the submersible when the descent begins is -4000 feet. There is no t-intercept in this context.

In 1975, scientists used submersibles to discover the Oculina coral reefs off Florida's east coast. The discovery led to the world's first Marine Protected Area for deepwater coral.

Look Back Show that the slope between the points $(0, -4000)$ and $(120, -13{,}000)$ is -75.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-13{,}000 - (-4000)}{120 - 0} = -75 \checkmark$$

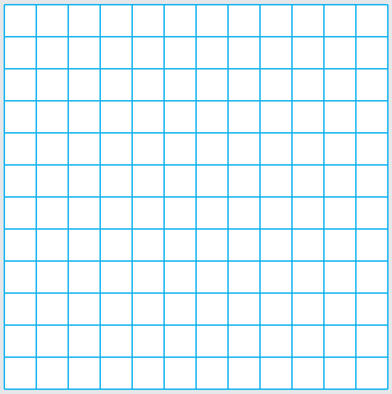
GO DIGITAL

SELF-ASSESSMENT 1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

17. The table shows the elevation $p(t)$ (in meters) of a submersible t minutes after it begins to descend. The ocean floor is at a depth of 3810 meters.

Time (minutes), t	Elevation (meters), $p(t)$
0	-1800
5	-1900
10	-2000

a. Does this submersible descend faster than the submersible in Example 6? Use a graph to justify your answer.

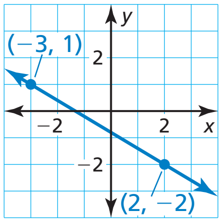


b. Both submersibles begin to descend at the same time. Which one reaches the ocean floor first? Explain.

3.6 Practice WITH CalcChat® AND CalcView®

In Exercises 1–4, describe the slope of the line. Then find the slope. *(See Example 1.)*

1.



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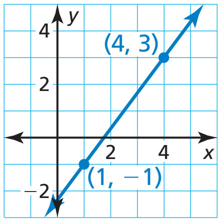
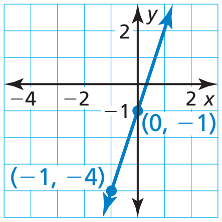


3.



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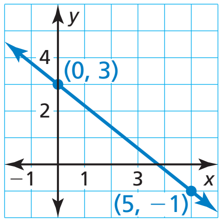


In Exercises 5 and 6, find the slope of the line that passes through the given points.

5. (1, 4), (3, −6)

6. (2, −2), (−7, −5)

In Exercises 7–10, the points represented by the table lie on a line. Find the slope of the line. *(See Example 2.)*

7.

x	−9	−5	−1	3
y	−2	0	2	4

8.

x	−1	2	5	8
y	−6	−6	−6	−6

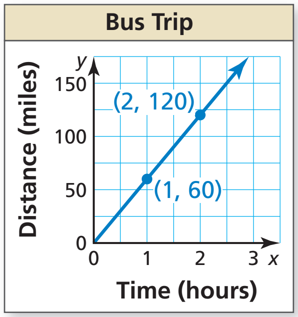
9.

x	0	0	0	0
y	−4	0	4	8

10.

x	−4	−3	−2	−1
y	2	−5	−12	−19

11. ANALYZING A GRAPH The graph shows the distance y (in miles) that a bus travels in x hours. Find and interpret the slope of the line.



12. ANALYZING A TABLE The table shows the amount x (in hours) of time you spend at a theme park and the admission fee y (in dollars) to the park. The points represented by the table lie on a line. Find and interpret the slope of the line.

Time (hours), x	Admission (dollars), y
6	54.99
7	54.99
8	54.99

In Exercises 13–20, find the slope and the y-intercept of the graph of the linear equation. *(See Example 3.)*

13. $y = -3x + 2$

14. $y = 4x - 7$

15. $y = 6x$

16. $y = -1$

17. $-0.75x + y = 4$

18. $x + y = -6\frac{1}{2}$

19. $\frac{1}{6}x = \frac{1}{3} - y$

20. $0 = 4.5 - 2y + 4.8x$

GO DIGITAL

ERROR ANALYSIS In Exercises 21 and 22, describe and correct the error in finding the slope and the y-intercept of the graph of the equation.

21. ✗ $x = -4y$

The slope is -4, and the y-intercept is 0.

22. ✗ $y = 3x - 6$

The slope is 3, and the y-intercept is 6.

In Exercises 23–30, graph the linear equation. Identify the x-intercept. *(See Example 4.)*

23. $y = -x + 7$

24. $y = \frac{1}{2}x + 3$

25. $y = 2x$

26. $y = -x$

27. $3x + y = -1$

28. $x + 4y = 8$

29. $-y + \frac{3}{5}x = 0$

30. $2.5x - y - 7.5 = 0$

In Exercises 31 and 32, graph the function with the given description. Identify the slope and the intercepts of the graph. *(See Example 5.)*

31. A linear function f models a relationship in which the dependent variable decreases 4 units for every 2 units the independent variable increases, and $f(0) = -2$.

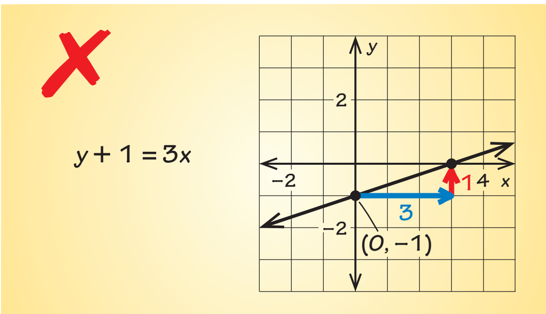
32. A linear function h models a relationship in which the dependent variable increases 1 unit for every 5 units the independent variable decreases, and $h(0) = 3$.

33. MODELING REAL LIFE A linear function r models the growth of your right index fingernail. The length of the fingernail increases 0.7 millimeter every week. Graph r when $r(0) = 12$. Identify the slope and interpret the y-intercept of the graph.

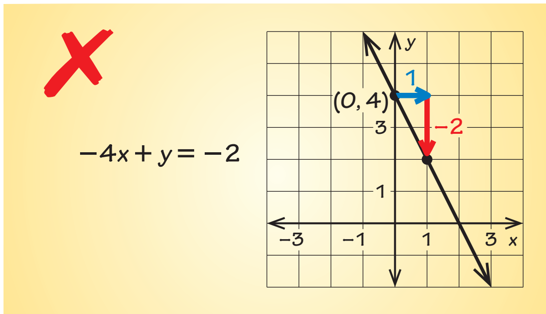
34. MODELING REAL LIFE A linear function m models the amount of milk sold by a farm per month. The amount decreases 500 gallons for every \$1 increase in price. Graph m when $m(0) = 3000$. Identify the slope and interpret the intercepts of the graph.

ERROR ANALYSIS **In Exercises 35 and 36, describe and correct the error in graphing the function.**

35.



36.



37. MODELING REAL LIFE The function $d(t) = \frac{1}{2}t + 2$ represents the amount (in inches) of water in a rain barrel for six rainy days, where t is the time (in days) since the rainfall began. *(See Example 6.)*

a. Find the domain and range in this context. Then graph the function.

b. Interpret the slope and intercepts of the graph.

38. **MODELING REAL LIFE** The function $f(x) = -200x + 1000$ represents the altitude (in feet) of a paraglider x minutes from the time the paraglider begins a descent to a landing site located 100 feet above sea level.

7 MTR

a. Find the domain and range in this context. Then graph the function.

b. Interpret the slope and intercepts of the graph.

c. The function $g(x) = -150x + 900$ represents the altitude (in feet) of a second paraglider x minutes from the time the paraglider begins a descent to the same landing site. Both paragliders start their descent at the same time. Who reaches an altitude of 100 feet first? Explain.

39. **CHOOSE A METHOD** Describe two ways to graph the equation $4x - 6y = 18$. Which method do you prefer? Explain.

3 MTR

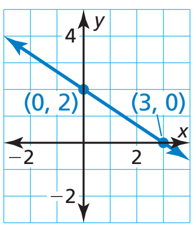
40. **PROBLEM SOLVING** A linear function represents the cost of catering an event from a Puerto Rican food truck. The table shows the cost y (in dollars) for x dishes. The function $c(x) = 9x + 600$ represents the cost (in dollars) of catering an event from a Dominican food truck, where x is the number of dishes ordered. Graph each function in terms of the context. Which truck charges a greater initial fee? Which truck can cater the greatest number of dishes for $695?

Dishes, x	Cost (dollars), y
0	500
5	550
10	600

The national dish of Puerto Rico is arroz con gandules, which features rice and pigeon peas seasoned with sofrito.

41. **B.E.S.T. TEST PREP** Which of the following linear functions has a slope of $-\frac{2}{3}$ and a y-intercept of 2? Select all that apply.

Ⓐ



Ⓑ

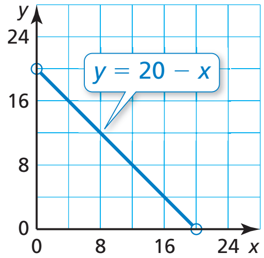
x	y
−2	6
0	3
2	0
4	−3

Ⓒ $f(x)$ decreases by 3 units for every 2 units x increases, and $f(0) = 2$.

Ⓓ $-y + 2 = \frac{2}{3}x$

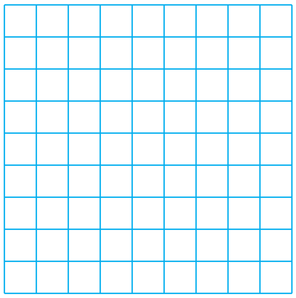
42. **WRITING** Describe the end behavior of the function $y = mx + b$ when (a) $m > 0$ and (b) $m < 0$.

5 MTR **43. CONNECTING CONCEPTS** The graph shows the relationship between the width y (in inches) and the length x (in inches) of a rectangle. The perimeter of a second rectangle is 10 inches less than the perimeter of the first rectangle.

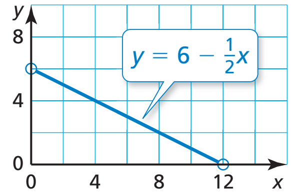


a. Graph the relationship between the width and length of the second rectangle.

b. How does your graph in part (a) compare to the graph shown?

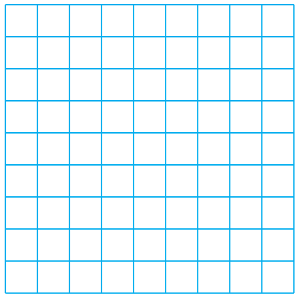


5 MTR **44. CONNECTING CONCEPTS** The graph shows the relationship between the base length x (in meters) and the lengths y (in meters) of the two equal sides of an isosceles triangle. The perimeter of a second isosceles triangle is 8 meters more than the perimeter of the first triangle.



a. Graph the relationship between the base length and the side lengths of the second triangle.

b. How does your graph in part (a) compare to the graph shown?



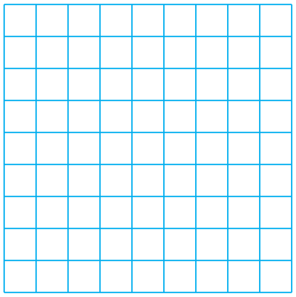
5 MTR **45. CONNECTING CONCEPTS** Graph the equations in the same coordinate plane. What is the area of the enclosed figure?

$3y = -9$

$2y - 14 = 4x$

$-4x + 5 - y = 0$

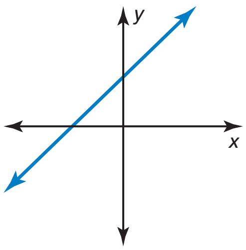
$y - 1 = 0$



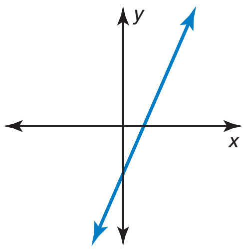
4 MTR **46. MAKING AN ARGUMENT** Your friend says that you can write the equation of any line in slope-intercept form. Is your friend correct? Explain your reasoning.

47. ANALYZING EQUATIONS Which equations could be represented by each graph? (The graphs are not drawn to scale.)

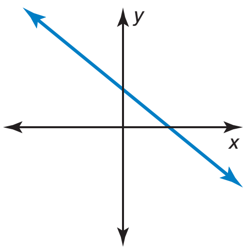
a.



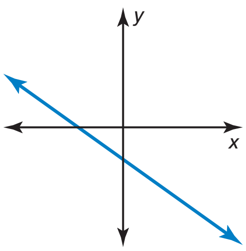
b.



c.

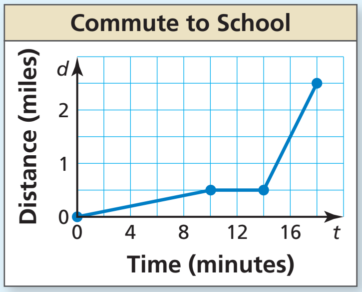


d.



$y = -3x + 8$	$y = -x - \frac{4}{3}$
$y = -7x$	$y = 2x - 4$
$y = \frac{7}{4}x - \frac{1}{4}$	$y = \frac{1}{3}x + 5$
$y = -4x - 9$	$y = 6$

48. HOW DO YOU SEE IT? You commute to school by walking and by riding a bus. The graph represents your commute.



a. Describe your commute in words.

b. Calculate and interpret the slopes of the different parts of the graph.

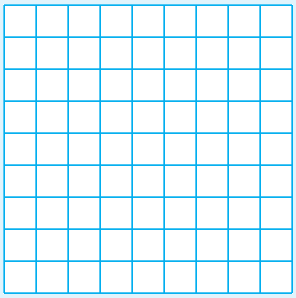
PROBLEM SOLVING **In Exercises 49 and 50, find the value of *k* so that the graph of the equation has the given slope or *y*-intercept.**

49. $16kx - 4y = 20;\ m = \frac{1}{2}$

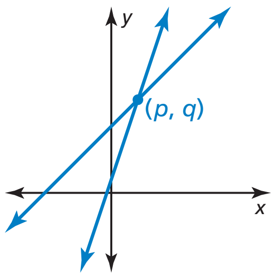
50. $\frac{2}{3}x + 2y - \frac{5}{3}k = 0;\ b = -10$

51. DIG DEEPER To show that the slope of a line is constant, let (x_1, y_1) and (x_2, y_2) be any two points on the line $y = mx + b$. Use the equation of the line to express y_1 in terms of x_1 and y_2 in terms of x_2. Then show that the slope between the points is m.

52. THOUGHT PROVOKING
Your family goes on vacation to a beach 300 miles from your house. You stop along the way and reach your destination 6 hours after departing. Draw a graph that describes your trip. Explain what each part of your graph represents.

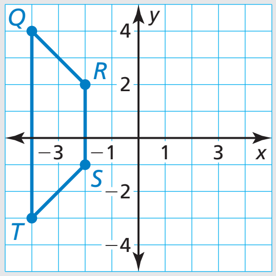


53. DIG DEEPER The graphs of the functions $g(x) = 6x + a$ and $h(x) = 2x + b$, where a and b are constants, are shown. They intersect at the point (p, q). Which is greater, $g(p + 2)$ or $h(p + 2)$? How much greater? Explain your reasoning.



REVIEW & REFRESH

In Exercises 54–56, find the coordinates of thefigure after the transformation.



54. Translate the trapezoid 3 units right.

55. Dilate the trapezoid with respect to the origin using a scale factor of $\frac{1}{2}$.

56. Reflect the trapezoid in the y-axis.

In Exercises 57–60, solve the equation.

57. $-\frac{x}{7} = 2.5$

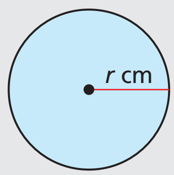
58. $\frac{1}{3}n - 4 = -4 + \frac{1}{3}n$

59. $-4(7 - a) + 10 = -12$

60. $5(2q - 1) = -3(q + 6)$

61. Find the slope and y-intercept of the graph of $7x - 4y = 10$.

62. The circumference of the circle is at least 14π centimeters. Find the possible values of r.

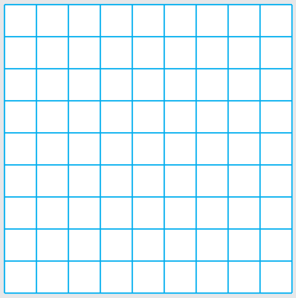


63. **MAKE A CONNECTION** Let f be a function. Use each statement to write a point on the graph of f.

a. $f(3)$ is equal to -8.

b. A solution of the equation $f(x) = \frac{3}{4}$ is -1.

64. Graph $y = -\frac{3}{2}x - 6$. Identify the x-intercept.



In Exercises 65 and 66, determine whether the table or equation represents a *linear* or *nonlinear* function. Explain.

65.

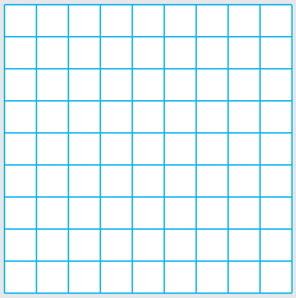
x	−2	−1	0	1
y	−4	−1	3	8

66. $\frac{x}{4} + \frac{y}{12} = 1$

67. **MODELING REAL LIFE** The table shows the prices of four video games. You have a coupon for 20% off one game, and you want to spend a total of \$30 with an absolute deviation of at most \$5. Which video games can you buy? Use an absolute value inequality to justify your answer.

Video game	Price
A	\$44.99
B	\$41.99
C	\$49.99
D	\$39.99

68. Use intercepts to graph the equation $-3x + 2y = 9$. Label the points corresponding to the intercepts.



In Exercises 69 and 70, graph the inequality.

69. $h < -4$

70. $\frac{9}{2} \leq t$

3.7 Transformations of Linear Functions

Learning Target: Graph transformations of linear functions.

Success Criteria:
- I can identify a transformation of a linear graph.
- I can graph transformations of linear functions.
- I can explain how translations, reflections, stretches, and shrinks affect graphs of functions.

Algebraic Reasoning

MA.912.F.2.1 Identify the effect on the graph or table of a given function after replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$ and $f(x + k)$ for specific values of k.

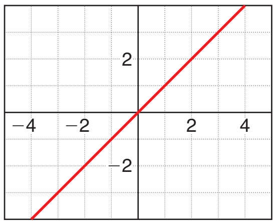
H **MA.912.F.2.3** Given the graph or table of $f(x)$ and the graph or table of $f(x) + k$, $kf(x)$, $f(kx)$ and $f(x + k)$, state the type of transformation and find the value of the real number k.

Also MA.912.AR.2.4, MA.912.AR.2.5

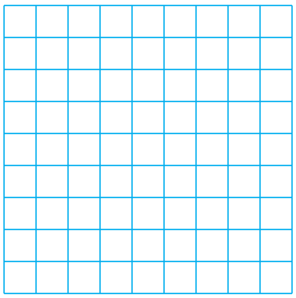
EXPLORE IT! Comparing Graphs of Functions

Work with a partner.

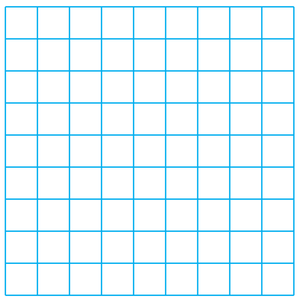
a. The graph of $f(x) = x$ is shown. Graph f and g on the same set of coordinate axes. Compare the graphs of f and g.



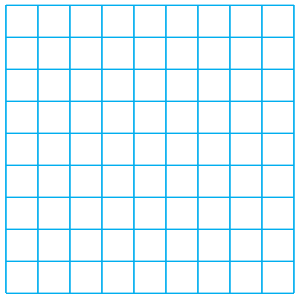
i. $g(x) = x + 4$



ii. $g(x) = 2x$



iii. $g(x) = -x$



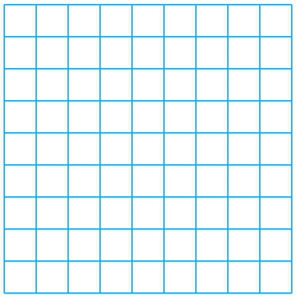
GO DIGITAL

USE STRUCTURE 5 MTR

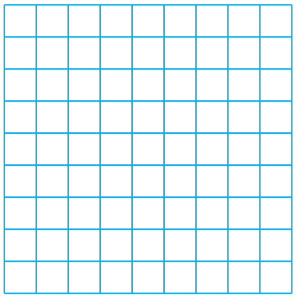
How can you use the right side of each equation in part (b) to compare the values of $n(x)$ and $m(x)$? What does this tell you about the graph of n?

b. Write any linear function m in terms of x. Compare the graphs of m and n. Explain your reasoning.

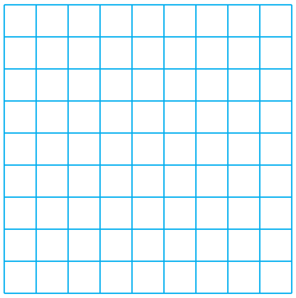
i. $n(x) = m(x) + 3$



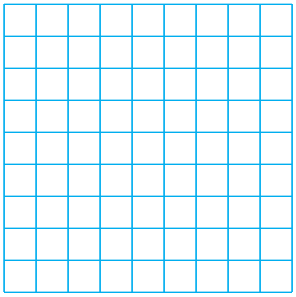
ii. $n(x) = m(x) - 3$



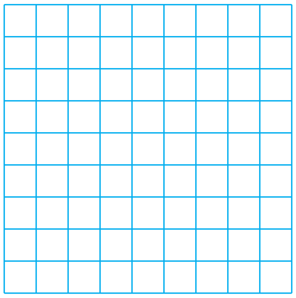
iii. $n(x) = \frac{1}{3} \cdot m(x)$



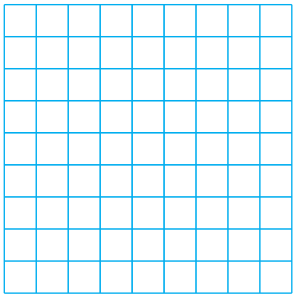
iv. $n(x) = 3 \cdot m(x)$



v. $n(x) = -m(x)$



vi. $n(x) = m(-x)$



c. Discuss your results in part (b) with other students. What do you notice?

d. How does the graph of a function p compare to the graph of each of the following functions? Explain your reasoning.

i. $q(x) = p(x) + k$

ii. $q(x) = k \cdot p(x)$, where $k > 0$

iii. $q(x) = -p(x)$

iv. $q(x) = p(-x)$

Translations and Reflections

A **family of functions** is a group of functions with similar characteristics. The most basic function in a family of functions is the **parent function**. For nonconstant linear functions, the parent function is $f(x) = x$. The graphs of all other nonconstant linear functions are *transformations* of the graph of the parent function. A **transformation** changes the size, shape, position, or orientation of a graph.

WORDS AND MATH

Used as an adjective, *parent* can refer to "being the original source." A *parent function* is the original source of the family of functions.

Vocabulary

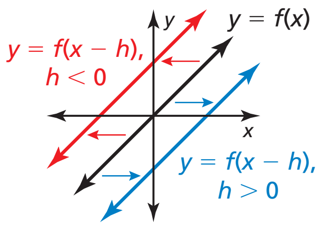
- family of functions, *p. 253*
- parent function, *p. 253*
- transformation, *p. 253*
- translation, *p. 253*
- reflection, *p. 254*
- horizontal shrink, *p. 255*
- horizontal stretch, *p. 255*
- vertical stretch, *p. 255*
- vertical shrink, *p. 255*

KEY IDEAS

A **translation** is a transformation that shifts a graph horizontally or vertically.

Horizontal Translations

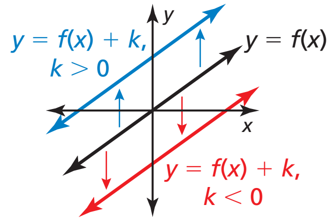
The graph of $y = f(x - h)$ is a horizontal translation of the graph of $y = f(x)$, where $h \neq 0$.



Subtracting h from the *inputs* before evaluating the function shifts the graph left when $h < 0$ and right when $h > 0$.

Vertical Translations

The graph of $y = f(x) + k$ is a vertical translation of the graph of $y = f(x)$, where $k \neq 0$.



Adding k to the *outputs* shifts the graph down when $k < 0$ and up when $k > 0$.

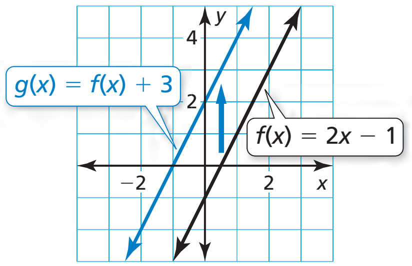
EXAMPLE 1 Describing Horizontal and Vertical Translations

Let $f(x) = 2x - 1$. Graph (a) $g(x) = f(x) + 3$ and (b) $t(x) = f(x + 3)$. Describe the transformations from the graph of f to the graphs of g and t.

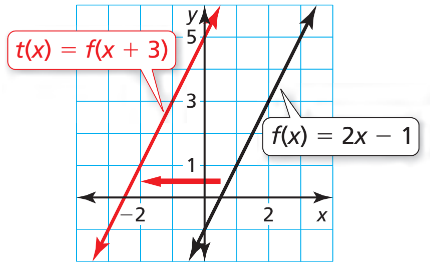
WATCH

SOLUTION

a. The function g is of the form $y = f(x) + k$, where $k = 3$. So, the graph of g is a vertical translation 3 units up of the graph of f.



b. The function t is of the form $y = f(x - h)$, where $h = -3$. So, the graph of t is a horizontal translation 3 units left of the graph of f.



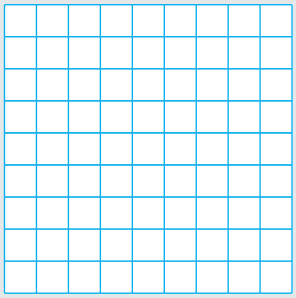
GO DIGITAL

SELF-ASSESSMENT

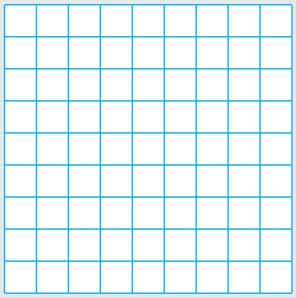
1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Using f, graph (a) g and (b) h. Describe the transformations from the graph of f to the graphs of g and h.

1. $f(x) = 3x + 1$; $g(x) = f(x) - 2$; $h(x) = f(x - 2)$



2. $f(x) = -2x$; $g(x) = f(x) + 1$; $h(x) = f(x + 1)$



KEY IDEAS

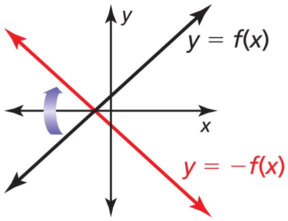
A **reflection** is a transformation that flips a graph over a line called the *line of reflection*.

STUDY TIP

A reflected point is the same distance from the line of reflection as the original point but on the opposite side of the line.

Reflections in the x-Axis

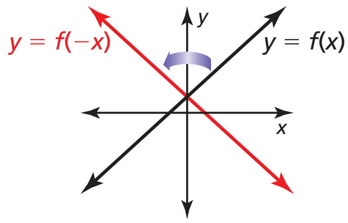
The graph of $y = -f(x)$ is a reflection in the x-axis of the graph of $y = f(x)$.



Multiplying the outputs by -1 changes their signs.

Reflections in the y-Axis

The graph of $y = f(-x)$ is a reflection in the y-axis of the graph of $y = f(x)$.



Multiplying the inputs by -1 changes their signs.

EXAMPLE 2 Describing Reflections in the x-Axis and the y-Axis

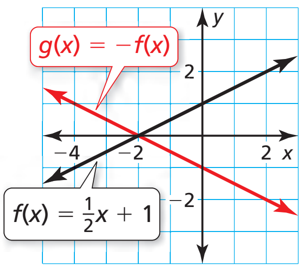
WATCH

Let $f(x) = \frac{1}{2}x + 1$. Graph (a) $g(x) = -f(x)$ and (b) $t(x) = f(-x)$. Describe the transformations from the graph of f to the graphs of g and t.

SOLUTION

a. To find the outputs of g, multiply the outputs of f by -1. The graph of g consists of the points $(x, -f(x))$.

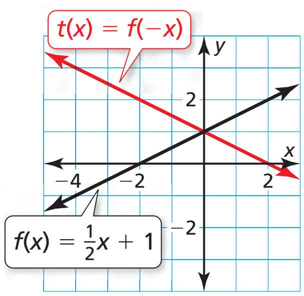
x	-4	-2	0
$f(x)$	-1	0	1
$-f(x)$	1	0	-1



▶ The graph of g is a reflection in the x-axis of the graph of f.

b. To find the outputs of t, multiply the inputs by -1 and then evaluate f. The graph of t consists of the points $(x, f(-x))$.

x	-2	0	2
$-x$	2	0	-2
$f(-x)$	2	1	0



▶ The graph of t is a reflection in the y-axis of the graph of f.

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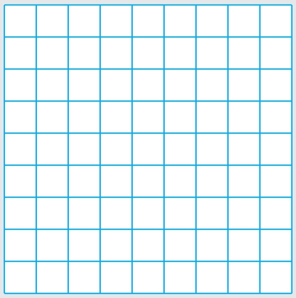
SELF-ASSESSMENT

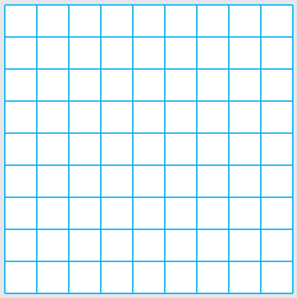
1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

Using f, graph g. Describe the transformation from the graph of f to the graph of g.

3. $f(x) = \frac{3}{2}x + 2;\ g(x) = -f(x)$



4. $f(x) = -4x - 2;\ g(x) = f(-x)$



5. OPEN-ENDED Write a linear function for which a reflection in the x-axis has the same graph as a reflection in the y-axis.

Stretches and Shrinks

You can transform a function by multiplying all the inputs (x-coordinates) by the same factor a. When $a > 1$, the transformation is a **horizontal shrink** because the graph shrinks toward the y-axis. When $0 < a < 1$, the transformation is a **horizontal stretch** because the graph stretches away from the y-axis. In each case, the y-intercept stays the same.

You can also transform a function by multiplying all the outputs (y-coordinates) by the same factor a. When $a > 1$, the transformation is a **vertical stretch** because the graph stretches away from the x-axis. When $0 < a < 1$, the transformation is a **vertical shrink** because the graph shrinks toward the x-axis. In each case, the x-intercept stays the same.

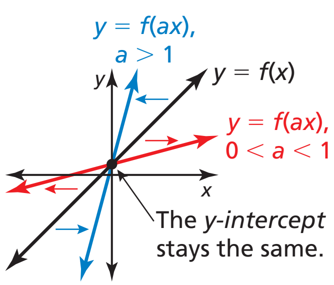
STUDY TIP

When $a < 0$ and $a \neq -1$, the graphs of $y = f(ax)$ and $y = a \cdot f(x)$ represent a stretch or shrink *and* a reflection in the x- or y-axis of the graph of $y = f(x)$.

KEY IDEAS

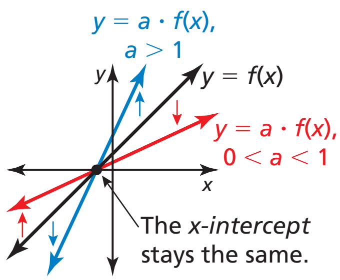
Horizontal Stretches and Shrinks

The graph of $y = f(ax)$ is a horizontal stretch or shrink by a factor of $\frac{1}{a}$ of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.



Vertical Stretches and Shrinks

The graph of $y = a \cdot f(x)$ is a vertical stretch or shrink by a factor of a of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.



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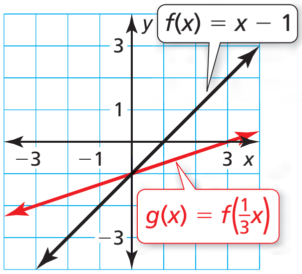
EXAMPLE 3 Describing Horizontal and Vertical Stretches

Let $f(x) = x - 1$. Graph (a) $g(x) = f\left(\frac{1}{3}x\right)$ and (b) $h(x) = 3f(x)$. Describe the transformations from the graph of f to the graphs of g and h.

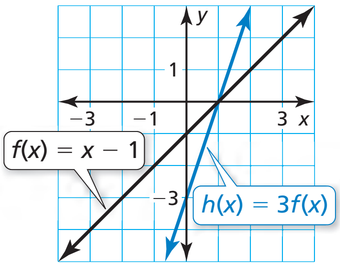
SOLUTION



a. To find the outputs of g, multiply the inputs by $\frac{1}{3}$. Then evaluate f. The graph of g consists of the points $\left(x, f\left(\frac{1}{3}x\right)\right)$.

x	-3	0	3
$\frac{1}{3}x$	-1	0	1
$f\left(\frac{1}{3}x\right)$	-2	-1	0

▶ The graph of g is a horizontal stretch of the graph of f by a factor of $1 \div \frac{1}{3} = 3$.



b. To find the outputs of h, multiply the outputs of f by 3. The graph of h consists of the points $(x, 3f(x))$.

x	0	1	2
$f(x)$	-1	0	1
$3f(x)$	-3	0	3

▶ The graph of h is a vertical stretch of the graph of f by a factor of 3.

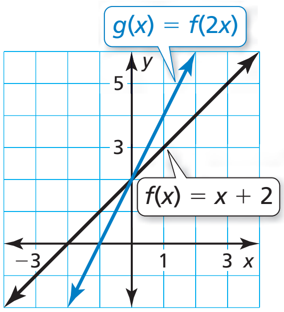
EXAMPLE 4 Describing Horizontal and Vertical Shrinks

Let $f(x) = x + 2$. Graph (a) $g(x) = f(2x)$ and (b) $h(x) = \frac{1}{4}f(x)$. Describe the transformations from the graph of f to the graphs of g and h.

SOLUTION

a. To find the outputs of g, multiply the inputs by 2. Then evaluate f. The graph of g consists of the points $(x, f(2x))$.

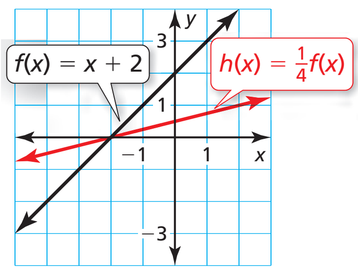
x	-1	0	1
$2x$	-2	0	2
$f(2x)$	0	2	4



▶ The graph of g is a horizontal shrink of the graph of f by a factor of $\frac{1}{2}$.

b. To find the outputs of h, multiply the outputs of f by $\frac{1}{4}$. The graph of h consists of the points $\left(x, \frac{1}{4}f(x)\right)$.

x	-2	0	2
$f(x)$	0	2	4
$\frac{1}{4}f(x)$	0	$\frac{1}{2}$	1



▶ The graph of h is a vertical shrink of the graph of f by a factor of $\frac{1}{4}$.

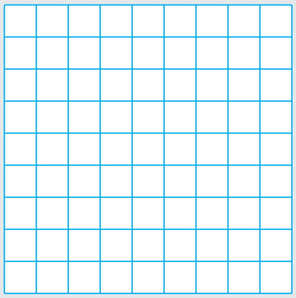
GO DIGITAL

SELF-ASSESSMENT

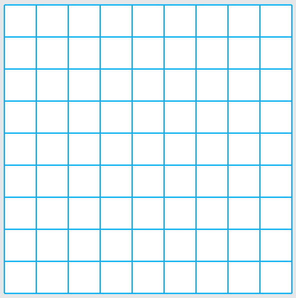
1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

Using f, graph (a) g and (b) h. Describe the transformations from the graph of f to the graphs of g and h.

6. $f(x) = 4x - 2$; $g(x) = f\left(\frac{1}{2}x\right)$; $h(x) = 2f(x)$

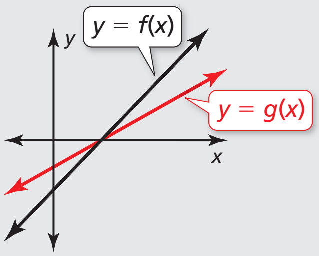


7. $f(x) = -3x + 4$; $g(x) = f(3x)$; $h(x) = \frac{1}{2}f(x)$



8. WRITING How does the value of a in the equation $y = h(ax)$ affect the graph of $y = h(x)$? How does the value of a in the equation $y = a \cdot h(x)$ affect the graph of $y = h(x)$?

9. REASONING The functions f and g are linear functions. The graph of g is a vertical shrink of the graph of f. What can you say about the intercepts of the graphs of f and g? Is this always true? Explain.



Describing Transformations

EXAMPLE 5 Describing a Transformation

WATCH H

The table represents two linear functions f and g. Describe the transformation from the graph of f to the graph of g.

x	−1	0	1	2	3
$f(x)$	−4	−1	2	5	8
$g(x)$	−2	1	4	7	10

SOLUTION

To determine the transformation, compare values of $f(x)$ and $g(x)$.

x	−1	0	1	2	3
$f(x)$	−4	−1	2	5	8
	+ 2	+ 2	+ 2	+ 2	+ 2
$g(x)$	−2	1	4	7	10

For each input, $g(x)$ is 2 more than $f(x)$. So, $g(x) = f(x) + 2$.

▶ The graph of g is a translation 2 units up of the graph of f.

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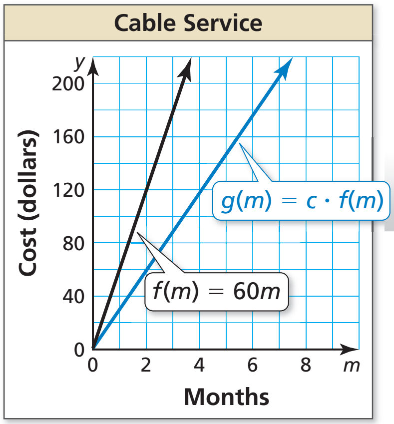
EXAMPLE 6 Modeling Real Life

The cost (in dollars) of cable service for m months is represented by the function f. To attract new customers, the cable company multiplies the monthly fee by a factor of c. Use the graph to find and interpret the value of c.



SOLUTION

To find c, compare f and g. The function $g(m) = c \cdot f(m)$ indicates that the graph of g is a vertical stretch or shrink of the graph of f. The graphs of f and g show that for any number of months, the new cost is one-half of the original cost. For example, the cost for two months ($m = 2$) decreases from \$120 to \$60. So, $c = \frac{1}{2}$.

▶ The factor $c = \frac{1}{2}$ indicates that the monthly price is halved.

SELF-ASSESSMENT

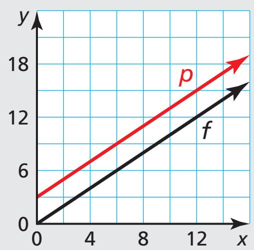
1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

10. The table represents two linear functions f and g. Describe the transformation from the graph of f to the graph of g.

x	0	2	4	6	8
$f(x)$	-8	-4	0	4	8
$g(x)$	-2	-1	0	1	2

11. A company pays x dollars per unit for a product. The selling price is represented by the function p.

a. What does $f(x) = x$ represent in this situation? Describe a transformation of the graph of f that results in the graph of p.

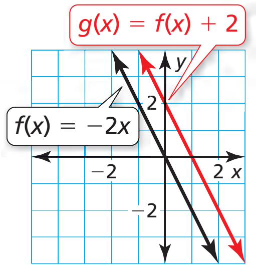


b. How does the company determine the selling price of a product?

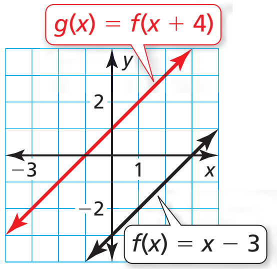
3.7 Practice WITH CalcChat® AND CalcView®

In Exercises 1–6, use the graphs of *f* and *g* to describe the transformation from the graph of *f* to the graph of *g*. *(See Example 1.)*

1.



2.



3. $f(x) = \frac{1}{3}x + 3$; $g(x) = f(x) - 3$

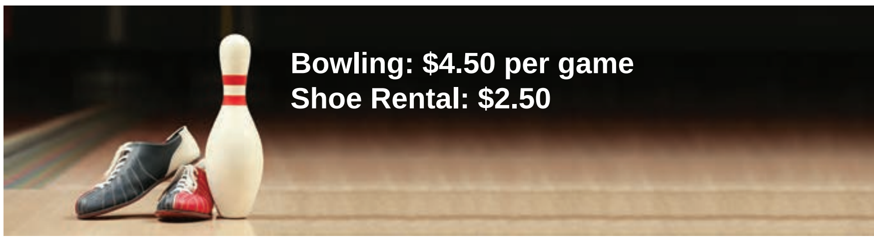
4. $f(x) = -3x + 4$; $g(x) = f(x) + 1$

5. $f(x) = -x - 2$; $g(x) = f(x + 5)$

6. $f(x) = \frac{1}{2}x - 5$; $g(x) = f(x - 3)$

7 MTR **7. MODELING REAL LIFE** You and your friend start biking from the same location. Your distance (in miles) after t minutes is represented by $d(t) = \frac{1}{5}t$. Your friend starts biking 5 minutes after you. Her distance is represented by $f(t) = d(t - 5)$. Describe the transformation from the graph of d to the graph of f.

7 MTR **8. MODELING REAL LIFE** The total cost (in dollars) to bowl n games is represented by $C(n) = 4.5n + 2.5$. The shoe rental price increases \$0.50. The new total cost is represented by $T(n) = C(n) + 0.5$. Describe the transformation from the graph of C to the graph of T.



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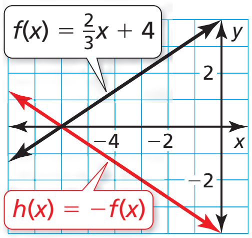
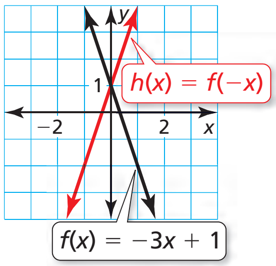
In Exercises 9–12, use the graphs of f and h to describe the transformation from the graph of f to the graph of h. *(See Example 2.)*

9.



10.

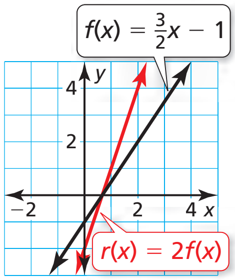


11. $f(x) = -5 - x;\ h(x) = f(-x)$

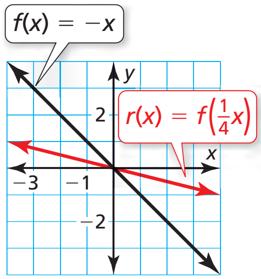
12. $f(x) = \frac{1}{4}x - 2;\ h(x) = -f(x)$

In Exercises 13–18, use the graphs of f and r to describe the transformation from the graph of f to the graph of r. *(See Example 3.)*

13.



14.



15. $f(x) = -2x - 4;\ r(x) = f\left(\frac{1}{2}x\right)$

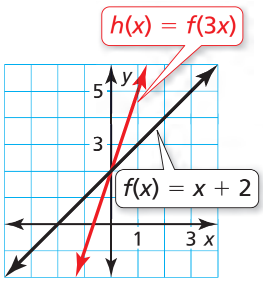
16. $f(x) = 3x + 5;\ r(x) = f\left(\frac{1}{3}x\right)$

17. $f(x) = \frac{2}{3}x + 1;\ r(x) = 3f(x)$

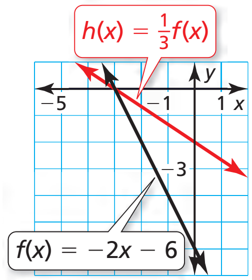
18. $f(x) = -\frac{1}{4}x - 2;\ r(x) = 4f(x)$

In Exercises 19–24, use the graphs of f and h to describe the transformation from the graph of f to the graph of h. *(See Example 4.)*

19.



20.



21. $f(x) = 3x - 12;\ h(x) = \frac{1}{6}f(x)$

22. $f(x) = -x + 1;\ h(x) = f(2x)$

23. $f(x) = -2x - 2;\ h(x) = f(5x)$

24. $f(x) = 4x + 8;\ h(x) = \frac{3}{4}f(x)$

25. **MODELING REAL LIFE** The temperature (in degrees Fahrenheit) x hours after 5 P.M. is represented by $t(x) = -4x + 72$. The temperature x hours after 10 A.M. is represented by $d(x) = 4x + 72$. Describe the transformation from the graph of t to the graph of d.

26. **MODELING REAL LIFE** The cost (in dollars) of a basic music streaming service for m months is represented by $B(m) = 5m$. The cost of the premium service is represented by $P(m) = 10m$. Describe the transformation from the graph of B to the graph of P.

In Exercises 27–32, use the graphs of f and g to describe the transformation from the graph of f to the graph of g.

27. $f(x) = x - 2;\ g(x) = f(x + 4)$

28. $f(x) = -4x + 8;\ g(x) = -f(x)$

29. $f(x) = -2x - 7;\ g(x) = f(x - 2)$

30. $f(x) = 3x + 8;\ g(x) = f\left(\frac{2}{3}x\right)$

31. $f(x) = x - 6;\ g(x) = 6f(x)$

32. $f(x) = -x;\ g(x) = f(x) - 3$

In Exercises 33–36, write a function g in terms of f so that the statement is true.

33. The graph of g is a horizontal translation 2 units right of the graph of f.

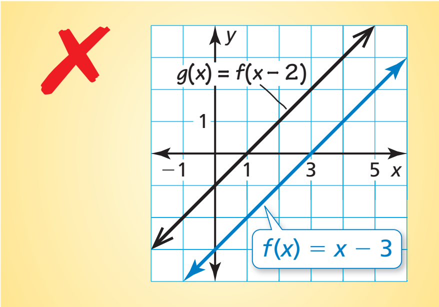
34. The graph of g is a reflection in the y-axis of the graph of f.

35. The graph of g is a vertical translation 4 units up of the graph of f.

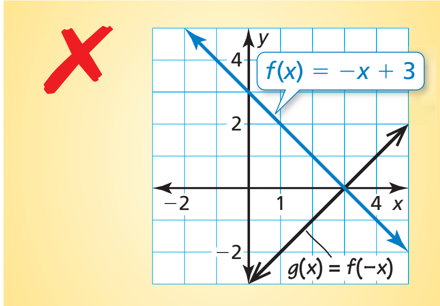
36. The graph of g is a horizontal shrink by a factor of $\frac{1}{5}$ of the graph of f.

4 MTR **ERROR ANALYSIS** **In Exercises 37 and 38, describe and correct the error in graphing *g*.**

37.



38.



In Exercises 39–46, describe the transformation from the graph of *f* to the graph of *g*. *(See Example 5.)*

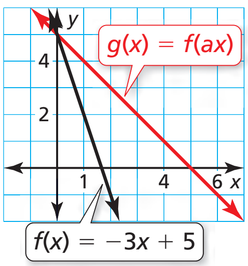
39.

x	−2	−1	0	1	2
***f*(*x*)**	14	11	8	5	2
***g*(*x*) = *f*(*x* + *k*)**	11	8	5	2	−1

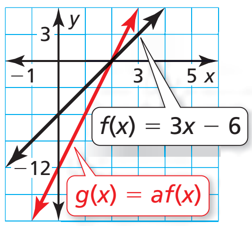
40.

x	−3	−2	−1	0	1
***f*(*x*)**	−10	−6	−2	2	6
g*(*x*) = *f*(*x*) + *k	−5	−1	3	7	11

41.



42.



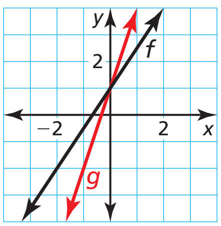
43.

x	−1	0	1	2	3
***f*(*x*)**	6	7	8	9	10
***g*(*x*)**	−6	−7	−8	−9	−10

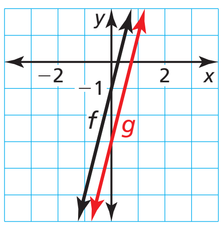
44.

x	3	5	7	9	11
***f*(*x*)**	2	0	−2	−4	−6
***g*(*x*)**	−2	−4	−6	−8	−10

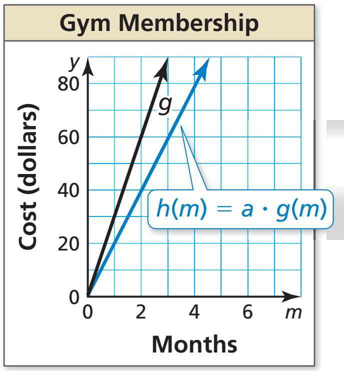
45.



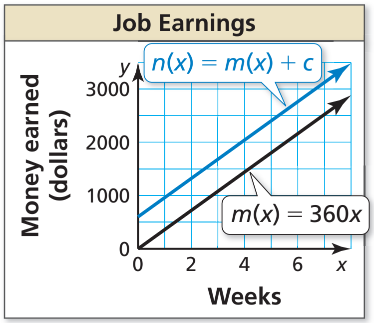
46.



7 MTR **47.** **MODELING REAL LIFE** The cost (in dollars) of a gym membership for m months is represented by the function g. In January, the gym multiplies the monthly fee by a factor of a. Use the graph to find and interpret the value of a. *(See Example 6.)*

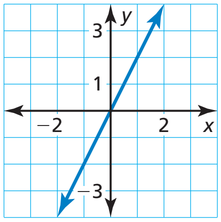


7 MTR **48.** **MODELING REAL LIFE** The function $m(x) = 360x$ represents the amount of money (in dollars) you make at your job after x weeks. You earn a bonus of c dollars. Use the graph to find and interpret the value of c.



49. **B.E.S.T. TEST PREP** Which of the graphs are related by only a translation? Explain.

Ⓐ



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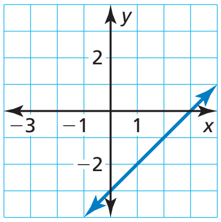

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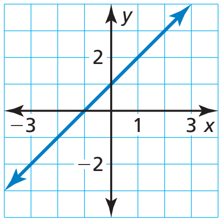

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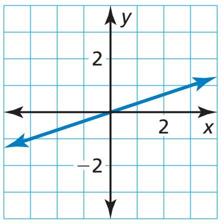

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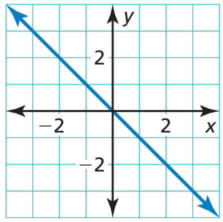

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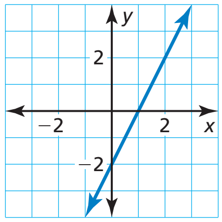


50. **WRITING** How does the value of p in the equation $y = g(x) + p$ affect the graph of $y = g(x)$? How does the value of p in the equation $y = g(x + p)$ affect the graph of $y = g(x)$?

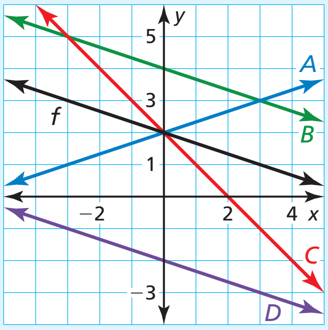
5 MTR **51.** **STRUCTURE** The graph of $g(x) = a \cdot f(x - b) + c$ is a transformation of the graph of the linear function f. Complete each statement.

a. The graph of g is a vertical ____________ of the graph of f when $a = 4$, $b = 0$, and $c = 0$.

b. The graph of g is a vertical translation 1 unit up of the graph of f when $a = 1$, $b = 0$, and $c =$ __________.

c. The graph of g is a reflection in the ____________ of the graph of f when $a = -1$, $b = 0$, and $c = 0$.

52. HOW DO YOU SEE IT? Match each function with its graph. Explain.



a. $a(x) = f(-x)$

b. $g(x) = f(x) - 4$

c. $h(x) = f(x) + 2$

d. $k(x) = f(3x)$

OPEN-ENDED In Exercises 53 and 54, write a function whose graph passes through the given point and is a transformation of the graph of $f(x) = x$.

53. $(4, 2)$

54. $\left(\frac{3}{2}, \frac{7}{2}\right)$

In Exercises 55–58, graph f and g. Write g in terms of f. Describe the transformation from the graph of f to the graph of g.

55. $f(x) = 2x - 5$; $g(x) = 2x - 8$

56. $f(x) = 3x + 9$; $g(x) = 3x + 15$

57. $f(x) = -x - 4$; $g(x) = x - 4$

58. $f(x) = x - 1$; $g(x) = 3x - 3$

59. REASONING The graph of $f(x) = x + 5$ is a vertical translation 5 units up of the graph of $f(x) = x$. How can you obtain the graph of $f(x) = x + 5$ from the graph of $f(x) = x$ using a horizontal translation?

60. REASONING A swimming pool is filled with water by a hose at a rate of 1020 gallons per hour. The amount (in gallons) of water in the pool after t hours is represented by the function $v(t) = 1020t$. How does the graph of v change in each situation?

a. A larger hose is found. Then the pool is filled at a rate of 1360 gallons per hour.

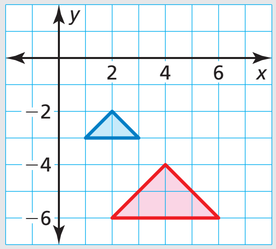
b. Before filling up the pool with a hose, a water truck adds 2000 gallons of water to the pool.

61. MAKING AN ARGUMENT 4 MTR Is it true that for all linear functions, a horizontal stretch by a factor of c produces the same result as a vertical shrink by a factor of $\frac{1}{c}$? Explain.

62. THOUGHT PROVOKING When is the graph of $y = f(x) + w$ the same as the graph of $y = f(x + w)$ for linear functions? Explain your reasoning.

REVIEW & REFRESH

63. The red figure is similar to the blue figure. Describe a similarity transformation between the figures.



In Exercises 64 and 65, solve the inequality. Graph the solution, if possible.

64. $5|x + 7| < 25$

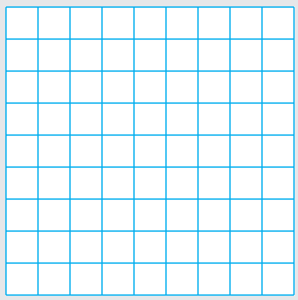
65. $-2|x + 1| \geq 18$

66. **REASONING** Complete the inequality $-\frac{1}{6}n$ ▭ $\frac{2}{3}$ with <, ≤, >, or ≥ so that the solution is $n \leq -4$.

67. Evaluate $g(x) = \frac{1}{4}x - 5$ when $x = 12$ and when $x = -2$.

68. **MODELING REAL LIFE** An elevator on the top floor of a building begins to descend to the ground floor. The function $h(t) = -8t + 250$ models the situation, where $h(t)$ is the height (in meters) of the elevator t seconds after it begins to descend.

a. Graph the function, and find its domain and range.



b. Interpret the terms and coefficient in the equation, and the x-intercept of its graph.

In Exercises 69–72, solve the equation. Check your solution.

69. $2.5b = 10$

70. $\frac{c}{5} + 1 = -2$

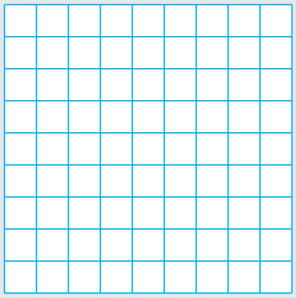
71. $14 - 3q = 4q - 14 + q$

72. $|-4r + 6| - 5 = 13$

73. **MODELING REAL LIFE** The linear function $m = 55 - 8.5b$ represents the amount m (in dollars) of money that you have after buying b books.

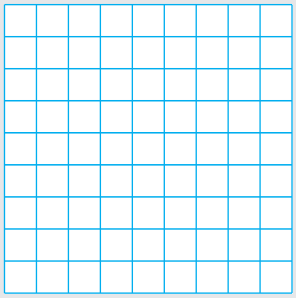
a. Find the domain of the function. Is the domain discrete or continuous? Explain.

b. Graph the function using its domain.

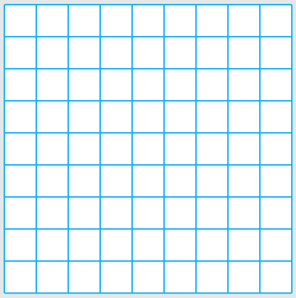


In Exercises 74 and 75, graph f and h. Describe the transformation from the graph of f to the graph of h.

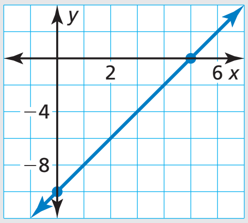
74. $f(x) = x$; $h(x) = \frac{1}{3}x$



75. $f(x) = x$; $h(x) = x - 4$



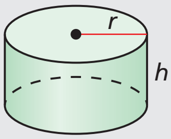
76. **STRUCTURE** The graph of the equation $Ax + By = 15$ is shown. Find the values of A and B.



77. Determine whether the relation is a function. Explain.

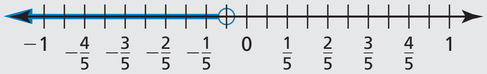
$(-10, 2), (-8, 3), (-6, 5), (-8, 8), (-10, 6)$

78. Solve the formula for h.



$V = \pi r^2 h$

79. Write an inequality that represents the graph.



3.8 Graphing Absolute Value Functions

Learning Target: Graph absolute value functions.

Success Criteria:
- I can identify characteristics of absolute value functions.
- I can graph absolute value functions.
- I can describe transformations of graphs of absolute value functions.

Algebraic Reasoning

MA.912.AR.4.3 Given a table, equation or written description of an absolute value function, graph that function and determine its key features.

Functions

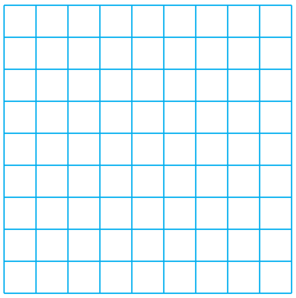
MA.912.F.2.1 Identify the effect on the graph or table of a given function after replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$ and $f(x + k)$ for specific values of k.

H **Also MA.912.F.2.3**

EXPLORE IT! Understanding Graphs of Absolute Value Functions

Work with a partner.

a. Think about the absolute value of a number. What characteristics might you observe in the graph of an *absolute value function*?



b. Graph $y = |x|$. Make several observations about the graph.

c. Let f be the parent absolute value function $f(x) = |x|$. Which of the following functions have a graph with one x-intercept? two x-intercepts? no x-intercept? Explain your reasoning.

$g(x) = f(x) - 1$	$h(x) = f(x - 1)$	$p(x) = -f(x)$	$q(x) = f(x) + 1$

RELATE CONCEPTS (5 MTR)

How can you use what you know about transformations of linear functions to make conclusions about transformations of absolute value functions?

d. Match each absolute value function with its graph. Explain your reasoning. Then use technology to check your answers.

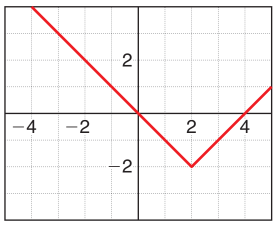
i. $g(x) = -|x - 2|$

ii. $g(x) = |x - 2| + 2$

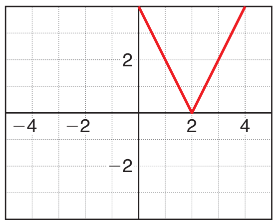
iii. $g(x) = |x - 2| - 2$

iv. $g(x) = 2|x - 2|$

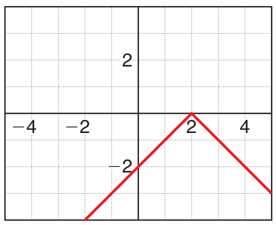
A.



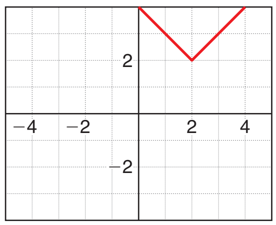
B.



C.



D.



GO DIGITAL

Characteristics of Absolute Value Functions

Vocabulary

absolute value function, *p. 268*
vertex, *p. 268*
vertex form, *p. 273*

WORDS AND MATH

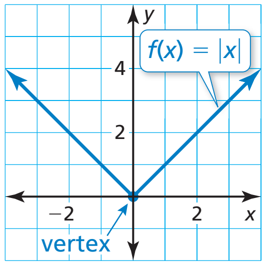
In geometry, a *vertex* is the point where the sides meet for an angle, polygon, or solid.

KEY IDEA

Absolute Value Function

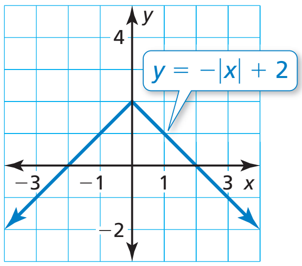
An **absolute value function** is a function that contains an absolute value expression. The parent absolute value function is $f(x) = |x|$. The graph of $f(x) = |x|$ is V-shaped and symmetric about the y-axis. The **vertex** is the point where the graph changes direction. The vertex of the graph of $f(x) = |x|$ is (0, 0).

The domain of $f(x) = |x|$ is all real numbers. The range is $y \geq 0$.



EXAMPLE 1 Describing Characteristics

Determine when the function $y = -|x| + 2$ is positive, negative, increasing, or decreasing. Then describe the end behavior of the function.



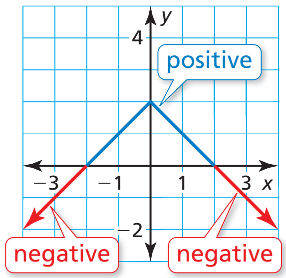
SOLUTION

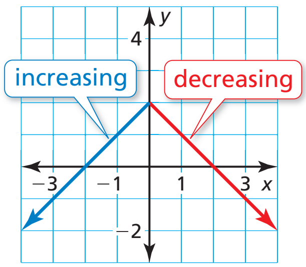
Positive and Negative:

The x-intercepts are ± 2. The function is negative when $x < -2$, positive when $-2 < x < 2$, and negative when $x > 2$.

Increasing and Decreasing:

The vertex is (0, 2). The function is increasing when $x < 0$ and decreasing when $x > 0$.





End behavior: The graph shows that the function values decrease as x approaches both positive and negative infinity. So, $y \to -\infty$ as $x \to -\infty$ and $y \to -\infty$ as $x \to +\infty$.

GO DIGITAL

Transforming Graphs of Absolute Value Functions

The graphs of all other absolute value functions are transformations of the graph of the parent function $f(x) = |x|$. The transformations presented in the previous section also apply to absolute value functions.

EXAMPLE 2 Graphing Absolute Value Functions

WATCH

Graph each function. Compare each graph to the graph of $f(x) = |x|$. Find the domain and range.

a. $q(x) = 2|x|$

b. $m(x) = |x - 2|$

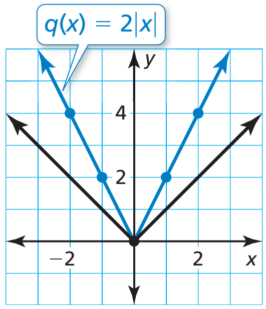
SOLUTION

a. Step 1 Make a table of values.

x	−2	−1	0	1	2
$q(x)$	4	2	0	2	4

Step 2 Plot the ordered pairs.

Step 3 Draw the graph.



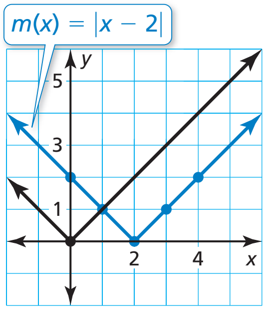
▶ The function q is of the form $y = a \cdot f(x)$, where $a = 2$. So, the graph of q is a vertical stretch of the graph of f by a factor of 2. The domain is all real numbers. The range is $y \geq 0$.

b. Step 1 Make a table of values.

x	0	1	2	3	4
$m(x)$	2	1	0	1	2

Step 2 Plot the ordered pairs.

Step 3 Draw the graph.



▶ The function m is of the form $y = f(x - h)$, where $h = 2$. So, the graph of m is a horizontal translation 2 units right of the graph of f. The domain is all real numbers. The range is $y \geq 0$.

STUDY TIP

A vertical stretch of the graph of $f(x) = |x|$ is *narrower* than the graph of $f(x) = |x|$.

A vertical shrink of the graph of $f(x) = |x|$ is *wider* than the graph of $f(x) = |x|$.

HELP A CLASSMATE (1 MTR)

Explain to a classmate why you can also describe the transformation in part (a) as a horizontal shrink.

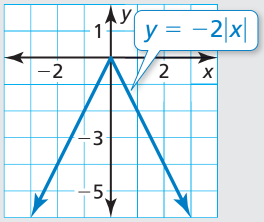
GO DIGITAL

SELF-ASSESSMENT

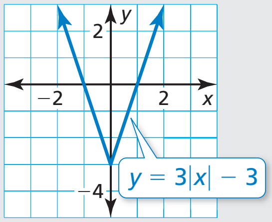
1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Determine when the function is positive, negative, increasing, or decreasing. Then describe the end behavior of the function.

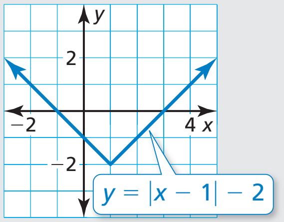
1.



2.



3.



1 MTR **4. HELP A CLASSMATE** Explain to a classmate how to use the vertex and x-intercepts to determine when an absolute value function is positive, negative, increasing, or decreasing.

Graph the function. Compare the graph to the graph of $f(x) = |x|$. Find the domain and range.

5. $h(x) = |x| - 1$

6. $n(x) = |x + 4|$

7. $t(x) = -3|x|$

8. $v(x) = \frac{1}{4}|x|$

9. REASONING Without graphing, explain how the graph of $g(x) = |x| + 5$ compares to the graph of $f(x) = |x|$.

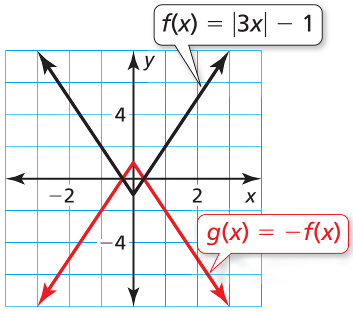
5 MTR **10. USE STRUCTURE** How do you know whether the graph of $g(x) = a|x|$ is a vertical stretch or a vertical shrink of the graph of $f(x) = b|x|$?

Describing Transformations

EXAMPLE 3 Describing a Transformation

WATCH

Let $f(x) = |3x| - 1$. Graph $g(x) = -f(x)$. Describe the transformation from the graph of f to the graph of g.



SOLUTION

To find the outputs of g, multiply the outputs of f by -1. The graph of g consists of the points $(x, -f(x))$.

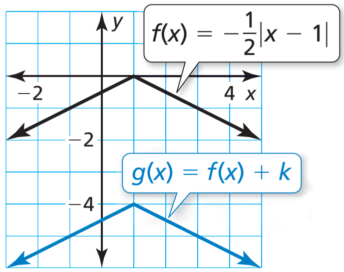
x	-2	-1	0	1	2
$f(x)$	5	2	-1	2	5
$-f(x)$	-5	-2	1	-2	-5

▶ The graph of g is a reflection in the x-axis of the graph of f.

EXAMPLE 4 Describing a Transformation

WATCH H

Describe the transformation from the graph of f to the graph of g.



SOLUTION

Compare f and g to find the value of k. The function $g(x) = f(x) + k$ indicates that the graph of g is a vertical translation of the graph of f. The graphs of f and g show that for any input, the output of g is 4 less than the output of f. For example, $g(1) = -4$ and $f(1) = 0$. So, $k = -4$.

▶ The graph of g is a translation 4 units down of the graph of f.

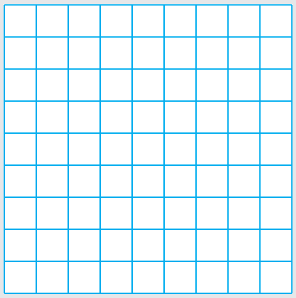
GO DIGITAL

SELF-ASSESSMENT

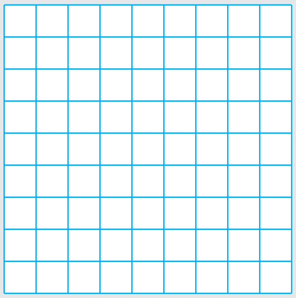
1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

Using f, graph (a) g and (b) h. Describe the transformations from the graph of f to the graphs of g and h.

11. $f(x) = |x + 3|$; $g(x) = f(x) - 4$; $h(x) = f(x - 4)$

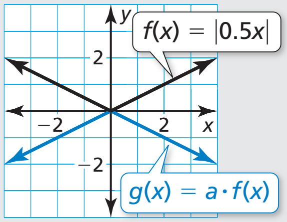


12. $f(x) = -|x|$; $g(x) = 2f(x)$; $h(x) = f(2x)$



Describe the transformation from the graph of f to the graph of g.

13.



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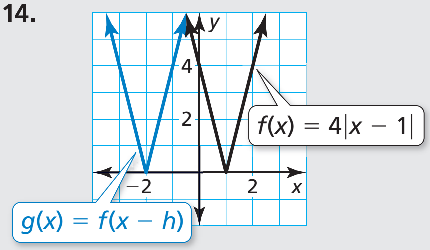

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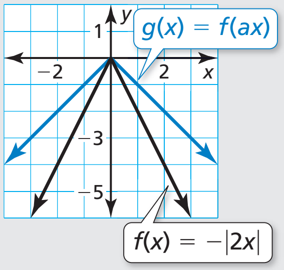
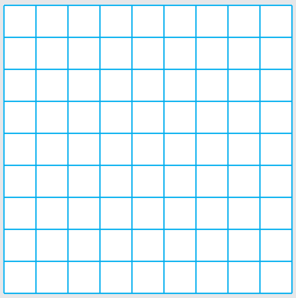


16. Graph the absolute value functions f and g represented by the table. Describe the transformation from the graph of f to the graph of g.

x	−2	−1	1	2
$f(x)$	2	4	4	2
$g(x)$	1	2	2	1



17. OPEN-ENDED Write an absolute value function that is a transformation of the parent absolute value function. Describe the end behavior of the function.

Using Vertex Form

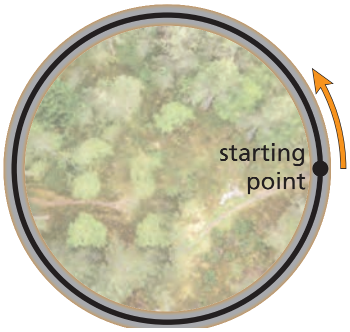
KEY IDEA

Vertex Form of an Absolute Value Function

An absolute value function written in the form $g(x) = a|x - h| + k$, where $a \neq 0$, is in **vertex form**. The vertex of the graph of g is (h, k).

Any absolute value function can be written in vertex form, and its graph is symmetric about the line $x = h$.

7 MTR **EXAMPLE 5** Modeling Real Life WATCH



You ride your bicycle around a circular trail one time. The function $f(x) = -\frac{1}{3}|x - 4.5| + 1.5$ represents the shortest distance (in miles) along the trail between you and your starting point after x minutes. (a) Graph the function. Find the domain and range in this context. (b) Interpret the intercepts and the vertex.

SOLUTION

a. The function is in vertex form, $g(x) = a|x - h| + k$, where $h = 4.5$ and $k = 1.5$. So, the vertex is $(h, k) = (4.5, 1.5)$. Substitute 0 for $f(x)$ to find any x-intercepts.

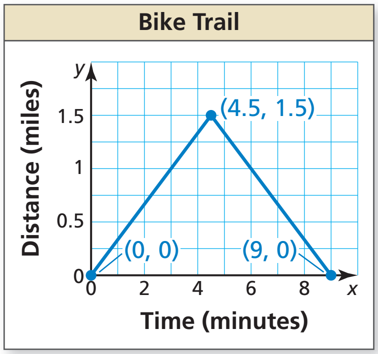
$$0 = -\frac{1}{3}|x - 4.5| + 1.5$$

$$4.5 = |x - 4.5|$$

$$x - 4.5 = 4.5 \quad \text{or} \quad x - 4.5 = -4.5$$

$$x = 9 \qquad\qquad x = 0$$

Graph the function using the points (0, 0), (4.5, 1.5), and (9, 0). The time x and distance $f(x)$ must be greater than or equal to 0 in this context. So, the domain is $\{x \mid 0 \leq x \leq 9\}$ and the range is $\{f(x) \mid 0 \leq f(x) \leq 1.5\}$.



b. The intercepts 0 and 9 indicate that it takes you $9 - 0 = 9$ minutes to ride around the trail. The vertex (4.5, 1.5) indicates that you are 1.5 miles from your starting point after 4.5 minutes. You can also determine that you are halfway around the trail at that time.

1 MTR **ANALYZE A PROBLEM**

Explain why it makes sense that the graph begins at (0, 0). How can you use symmetry to find the other intercept?

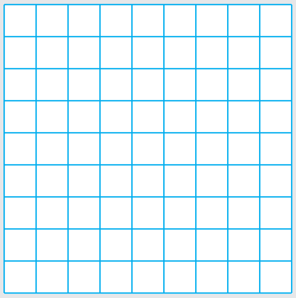
GO DIGITAL

SELF-ASSESSMENT

1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

18. The function $d(x) = \frac{1}{3}|x - 15| - 5$ represents the elevation (in meters) of a scuba diver x seconds after descending from the surface.

a. Graph the function. Find the domain and range in this context.



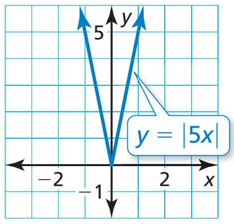
b. Interpret the intercepts and the vertex.

19. **WHAT IF?** The function $f(x) = -0.25|x - 8| + 2$ models the situation in Example 5. What is the radius of the trail? At what speed did you ride your bicycle? Explain your reasoning.

3.8 Practice WITH CalcChat® AND CalcView®

In Exercises 1–6, determine when the function is positive, negative, increasing, or decreasing. Then describe the end behavior of the function. *(See Example 1.)*

1.



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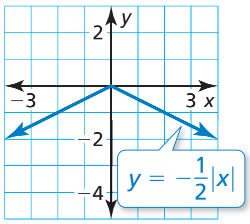

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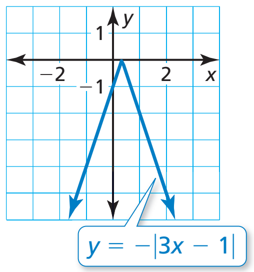

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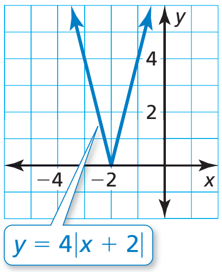

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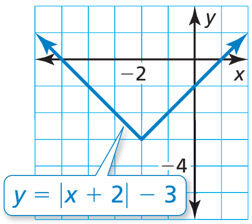

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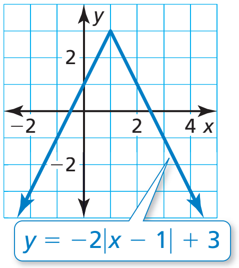
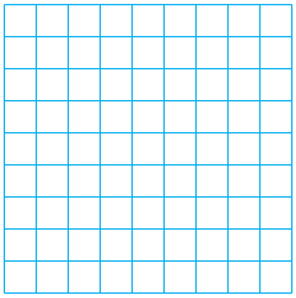
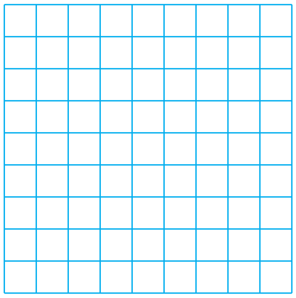


In Exercises 7–10, use the graphs of f and h to describe the transformation from the graph of f to the graph of h.

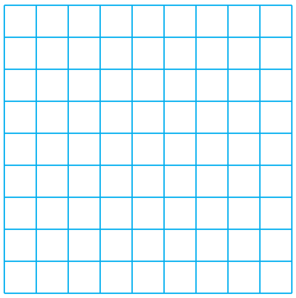
7. $f(x) = |x - 6|;\ h(x) = f(x) - 2$



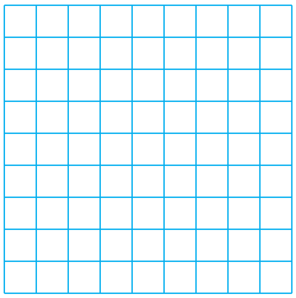
8. $f(x) = -2|x + 4|;\ h(x) = f(-x)$



9. $f(x) = -8x;\ h(x) = f\left(\frac{3}{4}x\right)$



10. $f(x) = \frac{1}{7}|x| + 1;\ h(x) = f(6x)$



GO DIGITAL

In Exercises 11–20, graph the function. Compare the graph to the graph of $f(x) = |x|$. Find the domain and range. *(See Example 2.)*

▶ **11.** $d(x) = |x| - 4$

12. $r(x) = |x| + 5$

13. $m(x) = |x + 1|$

14. $v(x) = |x - 3|$

15. $m(x) = -|x|$

16. $v(x) = |-x|$

17. $p(x) = \frac{1}{3}|x|$

18. $j(x) = 3|x|$

▶ **19.** $a(x) = |5x|$

20. $q(x) = \left|\frac{3}{2}x\right|$

4 MTR **ERROR ANALYSIS** **In Exercises 21 and 22, describe and correct the error in graphing the function.**

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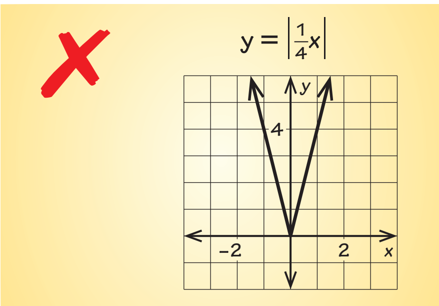

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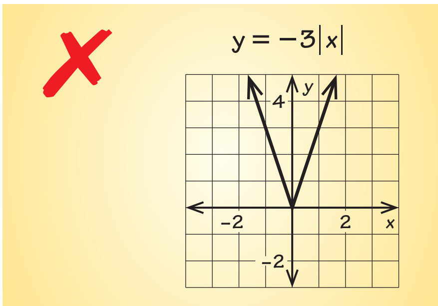


In Exercises 23–28, write an equation that represents the given transformation(s) of the graph of $g(x) = |x|$.

23. vertical translation 7 units down

24. horizontal translation 10 units left

25. reflection in the x-axis

26. reflection in the y-axis

27. vertical shrink by a factor of $\frac{1}{4}$

28. horizontal stretch by a factor of 3

In Exercises 29–32, using f, graph (a) g and (b) h. Describe the transformations from the graph of f to the graphs of g and h. *(See Example 3.)*

29. $f(x) = -|x + 7|$; $g(x) = f(x) + 4$; $h(x) = f(x + 4)$

30. $f(x) = 2|x| + 11$; $g(x) = -f(x)$; $h(x) = f(-x)$

31. $f(x) = \frac{1}{6}|x + 5|$; $g(x) = 3f(x)$; $h(x) = f(3x)$

32. $f(x) = 3|x| - 4$; $g(x) = \frac{1}{5}f(x)$; $h(x) = f\left(\frac{1}{5}x\right)$

In Exercises 33–42, describe the transformation from the graph of *f* to the graph of *g*. *(See Example 4.)*

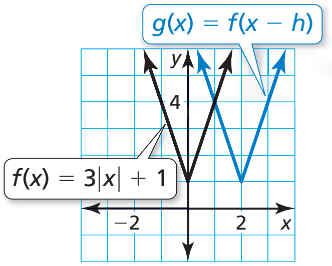
33.

x	-4	-3	-2	-1	0
$f(x)$	-1	0	-1	-2	-3
$g(x) = f(x) + k$	-6	-5	-6	-7	-8

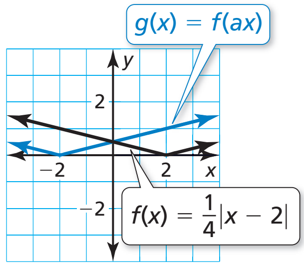
34.

x	-5	-3	-1	1	3
$f(x)$	2	0	-2	0	2
$g(x) = a \cdot f(x)$	1	0	-1	0	1

35.



36.



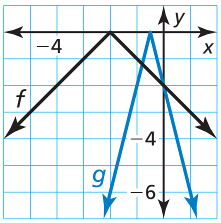
37.

x	1	2	3	4	5
$f(x)$	3	2	1	0	1
$g(x)$	9	6	3	0	3

38.

x	-4	-3	-2	-1	0
$f(x)$	-5	-6	-7	-6	-5
$g(x)$	5	6	7	6	5

39.



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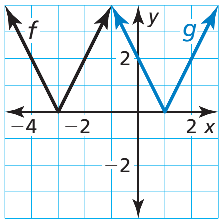

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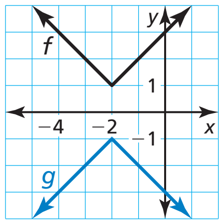

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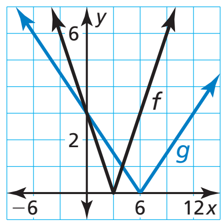
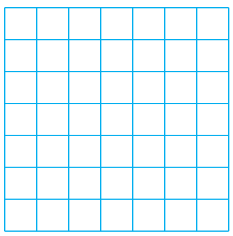


43. MODELING REAL LIFE You are running a ten-mile race. The function $d(t) = \frac{1}{8}|t - 40|$ represents the distance (in miles) you are from a water stop after t minutes. *(See Example 5.)*

a. Graph the function. Find the domain and range in this context.

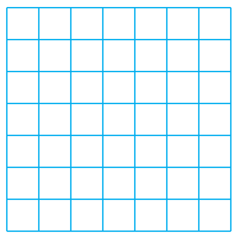
b. Interpret the intercepts and the vertex. When is the function decreasing? increasing? Explain what each represents in this context.



44. MODELING REAL LIFE A traveler is driving from Georgia to Florida. The function $d(t) = 60|t - 2.5|$ represents the distance (in miles) the car is from the state line after t hours.

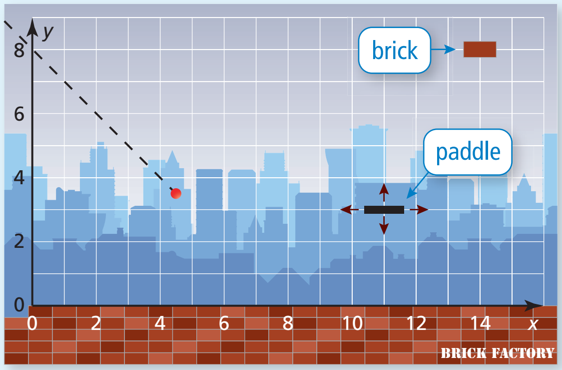
a. Graph the function. Find the domain and range in this context.

b. Interpret the intercepts and the vertex. When is the function decreasing? increasing? Explain what each represents in this context.



45. REASONING Is it possible for an absolute value function to always be increasing? decreasing? Explain your reasoning.

46. HOW DO YOU SEE IT?
The object of a computer game is to break bricks by deflecting a ball toward them using a paddle. The graph shows the current path of the ball and the location of the last brick.



a. You can move the paddle up, down, left, and right. At what coordinates should you place the paddle to break the last brick? Assume the ball deflects at a right angle.

b. You move the paddle to the coordinates in part (a), and the ball is deflected. How can you write an absolute value function that describes the path of the ball?

47. B.E.S.T. TEST PREP Which of the following functions have the same end behavior?

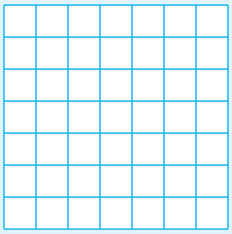
Ⓐ $a(x) = 4|x - 2| + 3$

Ⓑ $b(x) = -\frac{3}{2}|x + 5|$

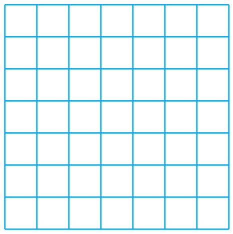
Ⓒ $c(x) = |-2x - 6| - 1$

Ⓓ $d(x) = \left|9 - \frac{1}{2}x\right|$

48. THOUGHT PROVOKING
Graph an absolute value function f that represents the route a wide receiver runs in a football game. Let the x-axis represent distance (in yards) across the field horizontally. Let the y-axis represent distance (in yards) down the field. Limit the domain so the route is realistic.



49. USING TOOLS Graph $y = 2|x + 2| - 6$ and $y = -2$ in the same coordinate plane. Use the graph to solve the equation $2|x + 2| - 6 = -2$. Use technology to check your solutions.



50. DIG DEEPER Write the vertex of the absolute value function $f(x) = |ax - h| + k$ in terms of a, h, and k.

4 MTR **51. MAKING AN ARGUMENT** Let p be a positive constant, where the graph of $y = |x| + p$ is a vertical translation in the positive direction of the graph of $y = |x|$. Does this mean that the graph of $y = |x + p|$ is a horizontal translation in the positive direction of the graph of $y = |x|$? Explain.

REVIEW & REFRESH

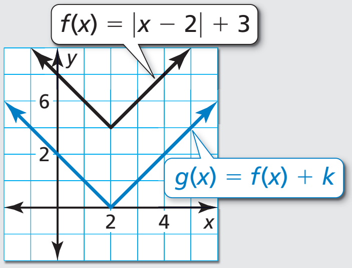
WATCH

In Exercises 52 and 53, solve the equation.

52. $-4|2x - 3| + 12 = -8$

53. $|x + 4| = |5x + 2|$

54. Describe the transformation from the graph of f to the graph of g.



In Exercises 55–58, solve the inequality. Graph the solution, if possible.

55. $2a - 7 \leq -2$

56. $-3(2p + 4) > -6p - 5$

57. $4(3h + 1.5) \geq 6(2h - 2)$

58. $-4(x + 6) < 2(2x - 9)$

In Exercises 59 and 60, use the graphs of f and g to describe the transformation from the graph of f to the graph of g.

59. $f(x) = -\frac{1}{2}x$; $g(x) = f(x + 2)$

60. $f(x) = 3x - 1$; $g(x) = -f(x)$

7 MTR **61. MODELING REAL LIFE** You have \$15 to purchase pecans and walnuts. The equation $12x + 7.5y = 15$ models this situation, where x is the number of pounds of pecans and y is the number of pounds of walnuts.

a. Interpret the terms and coefficients in the equation.

b. Graph the equation. Interpret the intercepts.

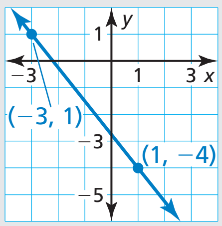
62. Let $f(t)$ be the outside temperature (in degrees Celsius) t hours after 9 A.M. Explain the meaning of each statement.

a. $f(4) = 30$ **b.** $f(m) = 28.9$ **c.** $f(2) = f(9)$ **d.** $f(6.5) > f(0)$

63. Solve $y = 3x - 7$ for x.

64. OPEN-ENDED Draw a graph that does *not* represent a function.

65. Find the slope of the line.



3 Chapter Review WITH CalcChat®

Chapter Learning Target: Understand graphing linear functions.

Chapter Success Criteria:
- ◆ I can identify the graph of a linear function.
- ◆ I can graph linear functions written in different forms.
- ■ I can describe the characteristics of a function.
- ■ I can explain how a transformation affects the graph of a linear function.

◆ Surface
■ Deep

SELF-ASSESSMENT 1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

3.1 Functions (pp. 173–186) WATCH

Learning Target: Understand the concept of a function.

Vocabulary
relation
function
domain
range
independent variable
dependent variable

Determine whether the relation is a function. Explain.

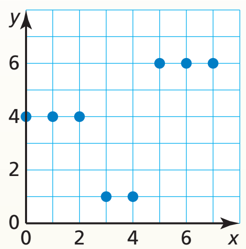
1. (0, 1), (5, 6), (7, 9), (8, 9)

2.

Input, *x*	Output, *y*
5	11
7	19
9	3

Determine whether the graph represents a function. Then find the domain and range.

3.



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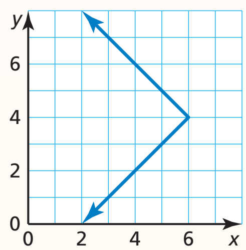

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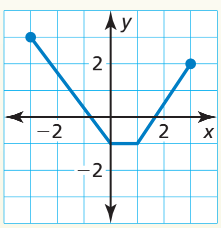

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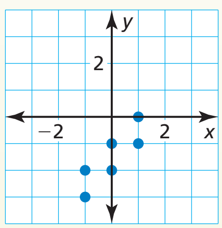


GO DIGITAL

7. You have \$170. You start a part-time job that pays \$8.50 per hour.

a. Does the situation represent a function? If so, identify the independent and dependent variables.

b. You work no more than 4 hours. Find the domain and range.

8. Write a relation consisting of five ordered pairs that satisfies the following conditions.

- The relation is a function.
- Switching the x- and y-coordinates of each ordered pair results in a relation that is *not* a function.

3.2 Characteristics of Functions *(pp. 187–196)* WATCH

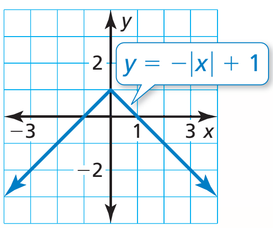
Learning Target: Describe characteristics of functions.

Vocabulary (AZ VOCAB)

x-intercept
y-intercept
increasing
decreasing
end behavior

Approximate when the function is positive, negative, increasing, or decreasing. Then describe the end behavior of the function.

9.



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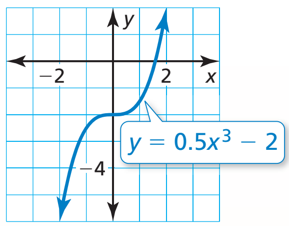
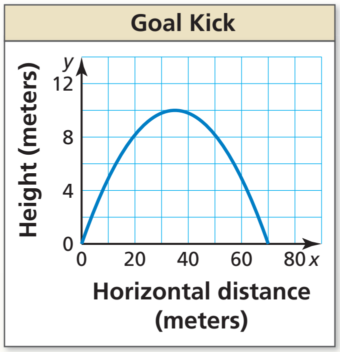
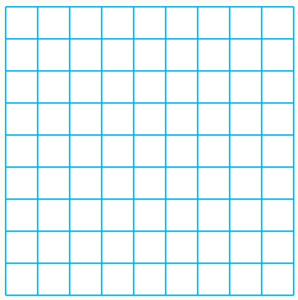


11. A goalie kicks a soccer ball so that it lands about 60 meters away, reaching a maximum height of about 15 meters after traveling a horizontal distance of about 30 meters. The graph shows the path of the second kick. Compare the two kicks.



12. Sketch a graph of a function with the given characteristics.

- The x-intercepts are $-\frac{7}{2}$, $-\frac{1}{2}$, and $\frac{5}{2}$.
- The function is increasing when $x < -\frac{3}{2}$, decreasing when $-\frac{3}{2} < x < 1$, and increasing when $x > 1$.



3.3 Linear Functions (pp. 197–212)

WATCH

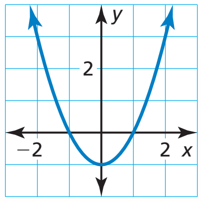
Learning Target: Identify and graph linear functions.

Determine whether the table, graph, or equation represents a *linear* or *nonlinear* function. Explain.

13.

x	2	7	12	17
y	2	−1	−4	−7

14.



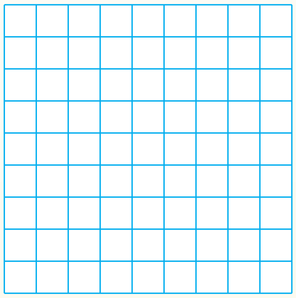
15. $y - 3 = 3x + 2$

16. $y = 2\sqrt{x}$

Graph the function using its domain. Explain your reasoning.

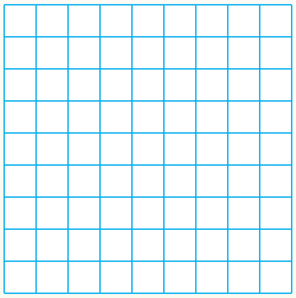
17.

Games, *x*	1	2	3	4	5
Tokens, *y*	8	6	4	2	0

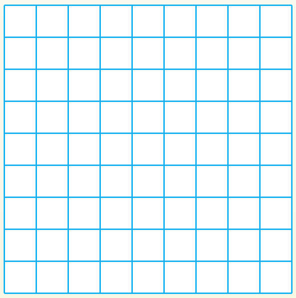


18.

Time (minutes), *x*	1.5	3.0	4.5
Elevation (feet), *y*	1500	3000	4500



19. The function $y = 60 - 8x$ represents the amount y (in dollars) of money you have after buying x movie tickets. (a) Find the domain of the function. Is the domain *discrete* or *continuous*? Explain. (b) Graph the function using its domain.



Vocabulary

AZ VOCAB

- linear equation in two variables
- linear function
- nonlinear function
- solution of a linear equation in two variables
- discrete domain
- continuous domain

3.4 Function Notation (pp. 213–222)

WATCH

Learning Target: Understand and use function notation.

Vocabulary AZ VOCAB

function notation

Evaluate the function when $x = -3$, 0, and 5.

20. $f(x) = x + 8$

21. $h(x) = 3x - 9$

22. Let $p(t)$ be the number of people in a stadium t hours after 11:00 A.M. Explain the meaning of each statement.

a. $p(3) = p(5)$

b. $p(6) < p(2.5)$

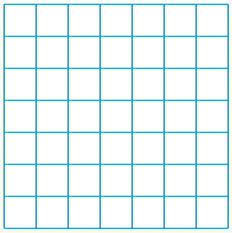
Find the value of x so that the function has the given value.

23. $k(x) = 7x$; $k(x) = 49$

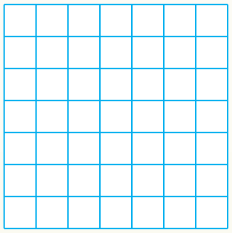
24. $r(x) = -5x - 1$; $r(x) = 19$

Graph the linear function.

25. $g(x) = -2x - 3$



26. $h(x) = \frac{2}{3}x + 4$



27. The function $d(x) = 1375 - 110x$ represents the distance (in miles) a high-speed train is from its destination after x hours.

a. How far is the train from its destination after 8 hours?

b. How long does the train travel before reaching its destination?

3.5 Graphing Linear Equations in Standard Form (pp. 223–232)

WATCH

Learning Target: Graph and interpret linear equations written in standard form.

Vocabulary
standard form

Graph the linear equation.

28. $x = 6$

29. $y = \frac{3}{2}$

Use intercepts to graph the linear equation. Label the points corresponding to the intercepts.

30. $2x + 3y = 6$

31. $8x - 4y = 16$

32. $-12x - 3y = 36$

33. $\frac{1}{2}x - y = -2$

34. Graph the equations $x = -2$, $y = 2$, and $-x + y = 2$. Find the area of the enclosed figure.

35. You lose track of how many 2-point baskets and 3-point baskets a team makes in a basketball game. The team misses all the 1-point baskets and still scores 54 points. The equation $2x + 3y = 54$ models the total points scored.

a. What do the terms and coefficients of the equation represent?

b. Can the number of 3-point baskets made be odd? Explain your reasoning.

c. Graph the equation. Interpret the intercepts.

d. Find four possible solutions in the context of the problem.

3.6 Graphing Linear Equations in Slope-Intercept Form
(pp. 233–250)

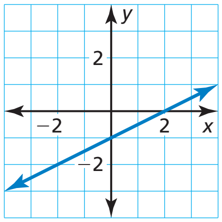
WATCH

Learning Target: Find the slope of a line and use slope-intercept form.

Vocabulary
AZ VOCAB

slope
rise
run
slope-intercept form
constant function

36. Describe the slope of the line. Then find the slope.



The points represented by the table lie on a line. Find the slope of the line.

37.

x	*y*
6	9
11	15
16	21
21	27

38.

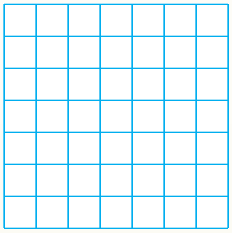
x	*y*
3	−5
3	−2
3	5
3	8

39.

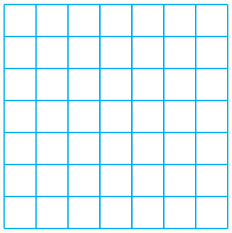
x	*y*
−4	−1.6
−3	−1.6
1	−1.6
9	−1.6

Graph the linear equation. Identify the *x*-intercept.

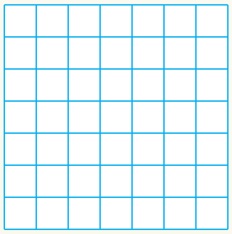
40. $y = 2x + 4$



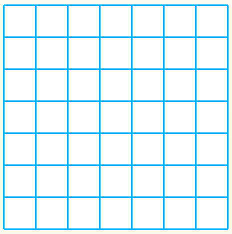
41. $-5x + y = -10$



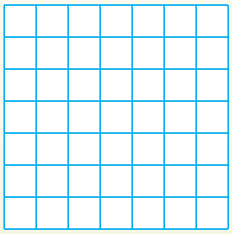
42. $-\frac{1}{2}x + y = 1$



43. $x + 3y = 9$



44. A linear function h models a relationship in which the dependent variable decreases 2 units for every 3 units the independent variable increases. The value of the function at 0 is 2. Graph h. Identify the slope and the intercepts of the graph.



3.7 Transformations of Linear Functions (pp. 251–266)

Learning Target: Graph transformations of linear functions.

Let $f(x) = 3x + 4$. Graph f and h. Describe the transformation from the graph of f to the graph of h.

45. $h(x) = f(x + 3)$

46. $h(x) = f(x) + 1$

47. $h(x) = f(-x)$

48. $h(x) = -f(x)$

49. $h(x) = 3f(x)$

50. $h(x) = f(6x)$

Vocabulary

family of functions
parent function
transformation
translation
reflection
horizontal shrink
horizontal stretch
vertical stretch
vertical shrink

51. Graph $f(x) = -5x - 1$ and $g(x) = 5x + 1$. Describe the transformation from the graph of f to the graph of g.

52. The table represents two linear functions f and g. Describe the transformation from the graph of f to the graph of g.

x	-1	0	1	2	3
$f(x)$	-1	2	5	8	11
$g(x)$	1	2	3	4	5

53. The regular cost (in dollars) of a streaming service for x months is represented by $R(x) = 14x + 3$. The function $D(x) = R(n \cdot x)$ represents the cost of the deluxe plan for the same streaming service. The cost of the deluxe plan for 4 months is \$93. Find the value of n.

3.8 Graphing Absolute Value Functions (pp. 267–280)

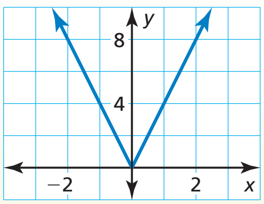
Learning Target: Graph absolute value functions.

Vocabulary

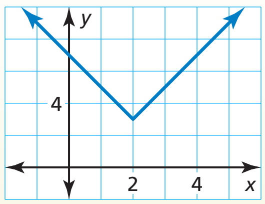
absolute value function
vertex
vertex form

Determine when the function is positive, negative, increasing, or decreasing. Then describe the end behavior of the function.

54.

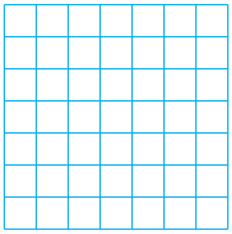


55.

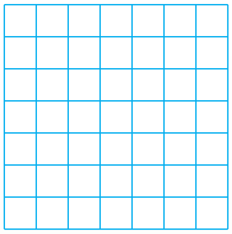


Graph the function. Compare the graph to the graph of $f(x) = |x|$. Find the domain and range.

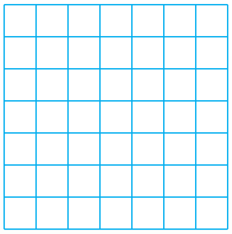
56. $m(x) = |x| + 6$



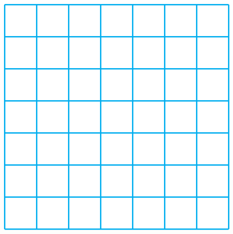
57. $p(x) = |x - 4|$



58. $q(x) = 4|x|$



59. $r(x) = -\frac{1}{4}|x|$



Write an equation that represents the given transformation(s) of the graph of $g(x) = |x|$.

60. horizontal translation 9 units right

61. vertical shrink by a factor of $\frac{1}{6}$

Describe the transformation from the graph of f to the graph of g.

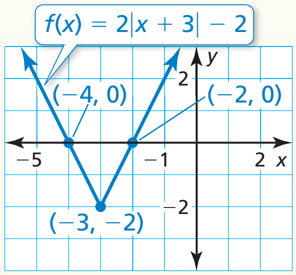
62.

x	-3	-2	-1	0	1
$f(x)$	2	0	2	4	6
$g(x)$	4	0	4	8	12

63.

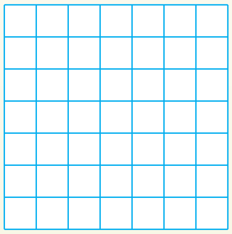
x	-4	-3	-2	-1	0
$f(x)$	-1.5	-1	-0.5	0	-0.5
$g(x)$	-0.5	0	-0.5	-1	-1.5

64. Write a function g whose graph is a horizontal shrink by a factor of $\frac{1}{3}$ of the graph of f shown.



65. Your distance (in miles) from an exit after t hours on a highway is represented by $d(t) = |65t - 13|$.

a. Graph the function and interpret the intercepts.



b. A sign says that after this exit, you will need to travel an additional 52 miles before reaching a second exit. How long does it take to reach the second exit?

Mathematical Thinking and Reasoning

3 MTR **COMPLETE TASKS WITH MATHEMATICAL FLUENCY**

Mathematicians who complete tasks with mathematical fluency select efficient and appropriate methods for solving problems within the given context.

1. In Exercise 24(c) on page 230, explain why using a graph may not be the most appropriate method to solve the problem. Then describe a more appropriate method.

2. Your friend graphs a linear equation to find the x-intercept. What is a possible method your friend could have used? Explain how your friend could find the x-intercept without graphing.

3 Practice Test WITH CalcChat®

Determine whether the relation is a function. If the relation is a function, determine whether the function is *linear* or *nonlinear*. Explain.

1.

x	−1	0	1	2
y	6	5	9	14

2. $y = -2x + 3$

3. $x = -2$

Graph the equation and identify the intercept(s). If the equation is linear, find the slope.

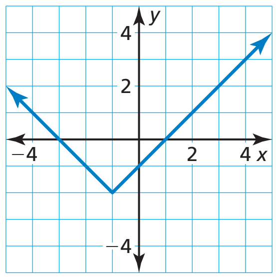
4. $2x - 3y = 6$

5. $y = 4.5$

6. $y = \frac{4}{5}x + 3$

7. $y = |x - 1| - 2$

Use the graph of the function shown.



8. Find the domain and range. Is the domain *discrete* or *continuous*? Explain.

9. Approximate when the function is positive, negative, increasing, or decreasing. Then describe the end behavior of the function.

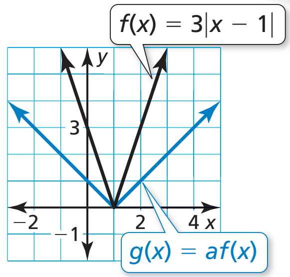
Graph *f* and *g*. Describe the transformations from the graph of *f* to the graph of *g*.

10. $f(x) = x;\ g(x) = -x + 3$

11. $f(x) = 5x;\ g(x) = 5x + 3$

GO DIGITAL

12. Describe the transformation from the graph of f to the graph of g.

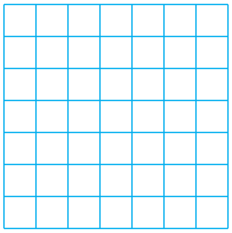


13. The function $m = 30 - 3r$ represents the amount m (in dollars) of money you have after renting r video games.

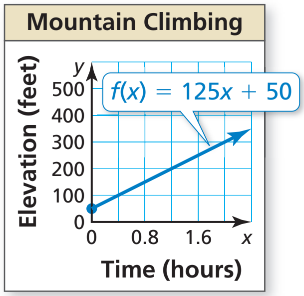
a. Identify the independent and dependent variables.

b. Find the domain and range of the function. Is the domain *discrete* or *continuous*? Explain.

c. Graph the function using its domain.



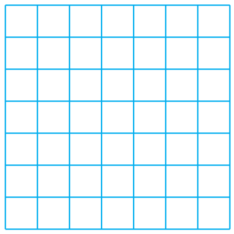
14. A mountain climber is scaling a cliff that is 500 feet above sea level. The graph shows the elevation of the climber over time.



a. Interpret the slope and the y-intercept.

b. How long does it take the climber to reach the top of the cliff? Explain.

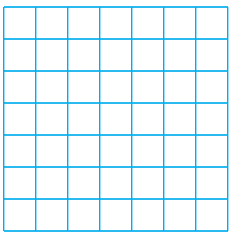
15. Graph the equations $y = 2$, $y = -3$, $y = -\frac{5}{2}x + 12$, and $-5x + 2y = -6$ in the same coordinate plane. What is the area of the enclosed shape?



16. A rock band releases a new single. The number (in thousands) of times the song is downloaded at an online store increases and then decreases as described by the function $n(t) = -2|t - 20| + 40$, where t is time (in weeks).

a. Identify the independent and dependent variables.

b. Graph n. Describe the transformation from the graph of $f(t) = -\frac{1}{2}|t - 20| + 10$ to the graph of n.



3 Performance Task

Into the Deep!

A 12-liter tank compressed to 200 bar can hold 2400 liters of air, which lasts a beginner scuba diver about 1 hour when diving at a depth of 10 meters.

Scuba tanks come in several sizes and are filled with compressed air. The table shows the capacities of several tanks compressed to a pressure of 200 bar or 300 bar.

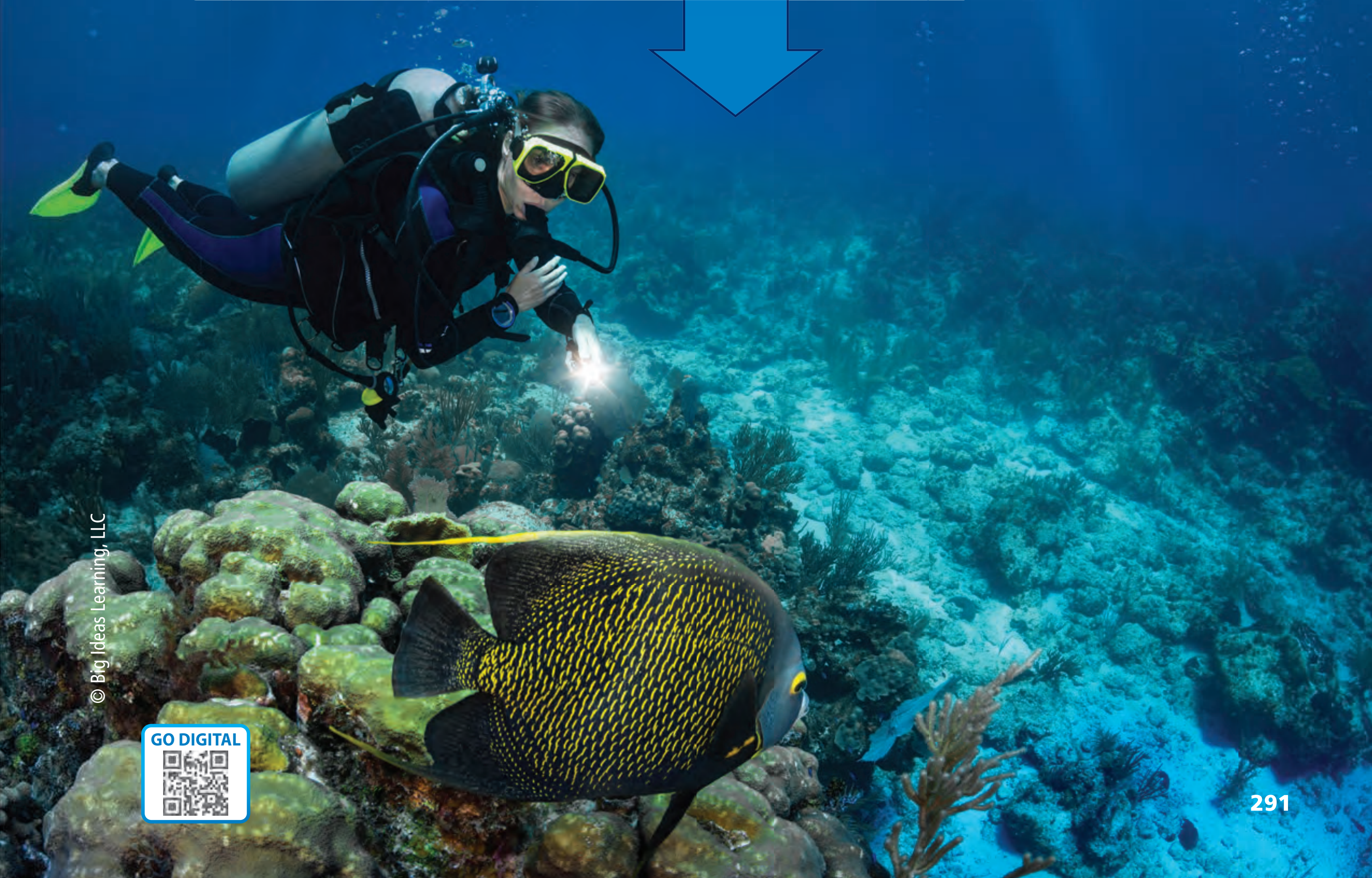
		TANK SIZE			
		10 L	12 L	15 L	18 L
COMPRESSION	200 bar	2000 L	**2400 L**	3000 L	3600 L
	300 bar	3000 L	3600 L	4500 L	5400 L

Pressure increases 1 atmosphere (atm) for every 10-meter increase in depth.

As pressure doubles, the amount of air that you inhale with each breath also doubles.

Pressure	Depth
1 atm	0 m
2 atm	10 m
3 atm	20 m
4 atm	30 m
5 atm	40 m

The maximum depth for recreational divers is about 40 meters.



GO DIGITAL

Scuba stands for:
Self
Contained
Underwater
Breathing
Apparatus.

DIVE PLANNING

Plan a dive. Make sure to choose a tank size, a depth that you will dive to, and the amount of time that you will spend underwater. Include a graph of the amount of oxygen you will have remaining over time.

Explain how you can increase the duration of your dive. Describe how to model this change using transformations.

3 B.E.S.T. Test Prep WITH CalcChat®
Cumulative Practice

WATCH Tutorial videos are available for each exercise.

1. A car rental company charges an initial fee of \$42 and a daily fee of \$12. A customer pays a total of \$138. How many days did the customer rent the car?

GRIDDED RESPONSE

2. Which of the following numbers is *not* a solution of $6x - 11 \geq 4x - 13$?

Ⓐ -2 Ⓑ -1 Ⓒ 0 Ⓓ 1

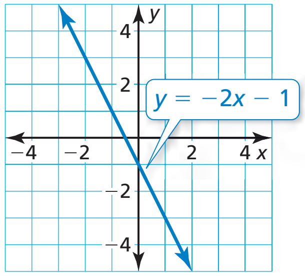
3. Solve $8x - 7 = 4(x + 1) + 7$.

GRIDDED RESPONSE

$x =$ ____

GO DIGITAL

4. Which ordered pair is a solution of the equation represented by the graph?



Ⓐ $(-1.5, 4)$ Ⓒ $(2, 3)$

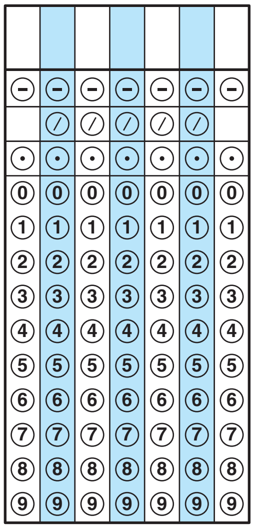
Ⓑ $(-1, 0)$ Ⓓ $(0, -1)$

5. Determine whether each function is *linear* or *nonlinear*.

Function	Linear	Nonlinear
x: 1, 2, 3, 4; y: 18, 15, 11, 6	Ⓐ	Ⓑ
$-x + y = 12$	Ⓒ	Ⓓ
x: 3, 6, 9, 12; y: 4, 11, 18, 25	Ⓔ	Ⓕ
$y = \frac{6}{x}$	Ⓖ	Ⓗ

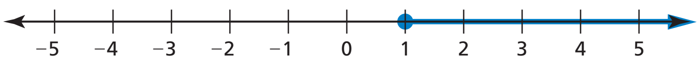
6. The expression $18 + 1.5x$ is 8 less than 36.5. What is the value of $4x$?

GRIDDED RESPONSE



7. Which inequality symbol completes the inequality so that its solution is represented by the graph?

$$-3(x + 7) \;\square\; -24$$



Ⓐ < Ⓑ ≤ Ⓒ > Ⓓ ≥

8. The table shows the cost (in dollars) of shrimp at a seafood market. Is this situation represented by a *linear* or *nonlinear* function? Is the domain *discrete* or *continuous*?

Pounds, x	0.5	1	1.5	2
Cost, y	6	12	18	24

	Discrete	**Continuous**
Linear	Ⓐ	Ⓑ
Nonlinear	Ⓒ	Ⓓ

9. Which equations have exactly one solution? Select all that apply.

Ⓐ $2x - 9 = 5x - 33$

Ⓑ $5x - 6 = 10x + 10$

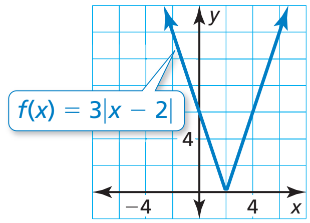
Ⓒ $2(8x - 3) = 4(4x + 7)$

Ⓓ $-7x + 5 = 2(x - 10.1)$

Ⓔ $6(2x + 4) = 4(4x + 10)$

Ⓕ $8(3x + 4) = 2(12x + 16)$

10. The graph of which function g is a vertical shrink by a factor of $\frac{1}{4}$ of the graph of f?



Ⓐ $g(x) = 3|x - 2| + \frac{1}{4}$

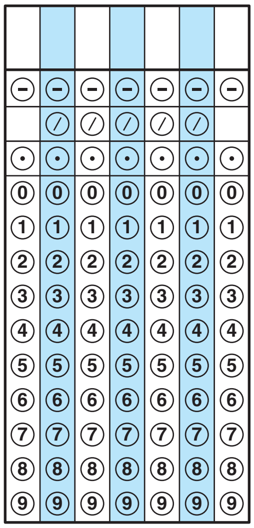
Ⓑ $g(x) = 12|x - 2|$

Ⓒ $g(x) = 3\left|\frac{1}{4}x - 2\right|$

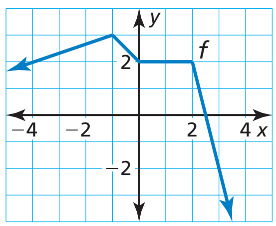
Ⓓ $g(x) = \frac{3}{4}|x - 2|$

11. What is the sum of the integer solutions of $2|x - 5| < 16$??

GRIDDED RESPONSE



12. The graph of the function f is shown. Which of the following equal 2?



I. $f(-4)$ II. $f\left(\frac{5}{4}\right)$ III. $f(2)$

Ⓐ I only

Ⓑ I and III only

Ⓒ II and III only

Ⓓ I, II, and III