3.1 Algebraic Expressions

**Essential Question** How can you simplify an algebraic expression?

**ACTIVITY: Simplifying Algebraic Expressions**

Work with a partner.

a. Evaluate each algebraic expression when \(x = 0\) and when \(x = 1\). Use the results to match each expression in the left table with its equivalent expression in the right table.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Value When (x = 0)</th>
<th>Value When (x = 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. (3x + 2 - x + 4)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
<tr>
<td>B. (5(x - 3) + 2)</td>
<td>(-x + 1)</td>
<td>(-x + 6)</td>
</tr>
<tr>
<td>C. (x + 3 - (2x + 1))</td>
<td>(4)</td>
<td>(2)</td>
</tr>
<tr>
<td>D. (-4x + 2 - x + 3x)</td>
<td>(2x + 6)</td>
<td>(2x + 1)</td>
</tr>
<tr>
<td>E. (-(1 - x) + 3)</td>
<td>(5x - 13)</td>
<td>(-x - 3)</td>
</tr>
<tr>
<td>F. (2x + x - 3x + 4)</td>
<td>(x + 2)</td>
<td>(x + 2)</td>
</tr>
<tr>
<td>G. (4 - 3 + 2(x - 1))</td>
<td>(2x - 1)</td>
<td>(2x - 1)</td>
</tr>
<tr>
<td>H. (2(1 - x + 4))</td>
<td>(-2x + 2)</td>
<td>(-2x + 2)</td>
</tr>
<tr>
<td>I. (5 - (4 - x + 2x))</td>
<td>(-x + 2)</td>
<td>(-x + 2)</td>
</tr>
<tr>
<td>J. (5x - (2x + 4 - x))</td>
<td>(5x - 2)</td>
<td>(5x - 2)</td>
</tr>
</tbody>
</table>

b. Compare each expression in the left table with its equivalent expression in the right table. In general, how do you think you obtain the equivalent expression in the right column?

Algebraic Expressions

In this lesson, you will

- apply properties of operations to simplify algebraic expressions.
- solve real-life problems.
Work with a partner. Use your results from Activity 1 to write a lesson on simplifying an algebraic expression.

**Key Idea**
Use the following steps to simplify an algebraic expression.

1. 
2. 
3. 

**Examples**

a. 

b. 

c. 

**Exercises**

Simplify the expression.

1. 
2. 
3. 

**What Is Your Answer?**

3. **IN YOUR OWN WORDS** How can you simplify an algebraic expression? Give an example that demonstrates your procedure.

4. **REASONING** Why would you want to simplify an algebraic expression? Discuss several reasons.

Use what you learned about simplifying algebraic expressions to complete Exercises 12–14 on page 84.
3.1 Lesson

Key Vocabulary
like terms, p. 82
simplest form, p. 82

Parts of an algebraic expression are called terms. Like terms are terms that have the same variables raised to the same exponents. Constant terms are also like terms. To identify terms and like terms in an expression, first write the expression as a sum of its terms.

**Example 1**

Identifying Terms and Like Terms

Identify the terms and like terms in each expression.

a. \(9x - 2 + 7 - x\)
   Rewrite as a sum of terms.
   \[9x + (-2) + 7 + (-x)\]
   Terms: \(9x, -2, 7, -x\)
   Like terms: \(9x\) and \(-x\), \(-2\) and \(7\)

b. \(z^2 + 5z - 3z^2 + z\)
   Rewrite as a sum of terms.
   \[z^2 + 5z + (-3z^2) + z\]
   Terms: \(z^2, 5z, -3z^2, z\)
   Like terms: \(z^2\) and \(-3z^2\), \(5z\) and \(z\)

An algebraic expression is in simplest form when it has no like terms and no parentheses. To combine like terms that have variables, use the Distributive Property to add or subtract the coefficients.

**Example 2**

Simplifying an Algebraic Expression

Simplify \(\frac{3}{4}y + 12 - \frac{1}{2}y - 6\).
\[
\frac{3}{4}y + 12 - \frac{1}{2}y - 6 = \frac{3}{4}y + 12 + \left(-\frac{1}{2}y\right) + (-6)
\]
Rewrite as a sum.
\[
= \frac{3}{4}y + \left(-\frac{1}{2}y\right) + 12 + (-6)
\]
Commutative Property of Addition
\[
= \left[\frac{3}{4} + \left(-\frac{1}{2}\right)\right]y + 12 + (-6)
\]
Distributive Property
\[
= \frac{1}{4}y + 6
\]
Combine like terms.

**On Your Own**

Identify the terms and like terms in the expression.

1. \(y + 10 - \frac{3}{2}y\)
2. \(2r^2 + 7r - r^2 - 9\)
3. \(7 + 4p - 5 + p + 2q\)
4. \(14 - 3z + 8 + z\)
5. \(2.5x + 4.3x - 5\)
6. \(\frac{3}{8}b - \frac{3}{4}b\)
EXAMPLE 3  Simplifying an Algebraic Expression

Simplify \(-\frac{1}{2}(6n + 4) + 2n\).

\[-\frac{1}{2}(6n + 4) + 2n = -\frac{1}{2}(6n) + \left(-\frac{1}{2}\right)(4) + 2n\]
\[= -3n + (-2) + 2n\]
\[= -3n + 2n + (-2)\]
\[= (-3 + 2)n + (-2)\]
\[= -n - 2\]

\[= -n - 2\] Simplify.

On Your Own

Simplify the expression.

7. \(3(q + 1) - 4\) 
8. \(-2(g + 4) + 7g\) 
9. \(7 - 4\left(\frac{3}{4}x - \frac{1}{4}\right)\)

EXAMPLE 4  Real-Life Application

Each person in a group buys a ticket, a medium drink, and a large popcorn. Write an expression in simplest form that represents the amount of money the group spends at the movies. Interpret the expression.

Words

Each ticket is \$7.50, each medium drink is \$2.75, and each large popcorn is \$4.

Variable

The same number of each item is purchased. So, \(x\) can represent the number of tickets, the number of medium drinks, and the number of large popcorns.

Expression

\[7.50x + 2.75x + 4x = (7.50 + 2.75 + 4)x\]
\[= 14.25x\]

The expression 14.25\(x\) indicates that the total cost per person is \$14.25.

On Your Own

10. WHAT IF? Each person buys a ticket, a large drink, and a small popcorn. How does the expression change? Explain.

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3.1 Exercises

Vocabulary and Concept Check

1. WRITING Explain how to identify the terms of $3y - 4 - 5y$.
2. WRITING Describe how to combine like terms in the expression $3n + 4n - 2$.
3. VOCABULARY Is the expression $3x + 2x - 4$ in simplest form? Explain.
4. REASONING Which algebraic expression is in simplest form? Explain.

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Practice and Problem Solving

Identify the terms and like terms in the expression.

5. $t + 8 + 3t$
6. $3z + 4 + 2 + 4z$
7. $2n - n - 4 + 7n$
8. $-x - 9x^2 + 12x^2 + 7$
9. $1.4y + 5 - 4.2 - 5y^2 + z$
10. $\frac{1}{2}s - 4 + \frac{3}{4}s + \frac{1}{8} - s^3$

11. ERROR ANALYSIS Describe and correct the error in identifying the like terms in the expression.

$3x - 5 + 2x^2 + 9x = 3x + 2x^2 + 9x - 5$
Like Terms: $3x, 2x^2$, and $9x$

Simplify the expression.

12. $12g + 9g$
13. $11x + 9 - 7$
14. $8s - 11s + 6$
15. $4.2v - 5 - 6.5v$
16. $8 + 4a + 6.2 - 9a$
17. $\frac{2}{3}y - 4 + 7 - \frac{9}{10}y$
18. $4(b - 6) + 19$
19. $4p - 5(p + 6)$
20. $\frac{2}{3}(12c - 9) + 14c$

21. HIKING On a hike, each hiker carries the items shown. Write an expression in simplest form that represents the weight carried by $x$ hikers. Interpret the expression.

4.6 lb
3.4 lb
2.2 lb

Chapter 3 Expressions and Equations
22. **STRUCTURE** Evaluate the expression \(-8x + 5 - 2x - 4 + 5x\) when \(x = 2\) before and after simplifying. Which method do you prefer? Explain.

23. **REASONING** Are the expressions \(8x^2 + 3(x^2 + y)\) and \(7x^2 + 7y + 4x^2 - 4y\) equivalent? Explain your reasoning.

24. **CRITICAL THINKING** Which solution shows a correct way of simplifying \(6 - 4(2 - 5x)\)? Explain the errors made in the other solutions.

\[
\begin{align*}
A & : 6 - 4(2 - 5x) = 6 - 4(-3x) = 6 + 12x \\
B & : 6 - 4(2 - 5x) = 6 - 8 + 20x = -2 + 20x \\
C & : 6 - 4(2 - 5x) = 2(2 - 5x) = 4 - 10x \\
D & : 6 - 4(2 - 5x) = 6 - 8 - 20x = -2 - 20x
\end{align*}
\]

25. **BANNER** Write an expression in simplest form that represents the area of the banner.

26. **CAR WASH** Write an expression in simplest form that represents the earnings for washing and waxing \(x\) cars and \(y\) trucks.

27. \(5(2 + x + 3)\)

28. \((4 + 1)(x + 2x)\)

29. **Critical Thinking** You apply gold foil to a piece of red poster board to make the design shown.
   a. Write an expression in simplest form that represents the area of the gold foil.
   b. Find the area of the gold foil when \(x = 3\).
   c. The pattern at the right is called “St. George’s Cross.” Find a country that uses this pattern as its flag.

30. Order the lengths from least to greatest. *(Skills Review Handbook)*
   - 15 in., 14.8 in., 15.8 in., 14.5 in., 15.3 in.
   - 0.65 m, 0.6 m, 0.52 m, 0.55 m, 0.545 m

31. **MULTIPLE CHOICE** A bird’s nest is 12 feet above the ground. A mole’s den is 12 inches below the ground. What is the difference in height of these two positions? *(Section 1.3)*

\[
\begin{align*}
A & : 24 \text{ in.} \\
B & : 11 \text{ ft} \\
C & : 13 \text{ ft} \\
D & : 24 \text{ ft}
\end{align*}
\]