English Spanish

7.6 Surface Areas and Volumes of Similar Solids

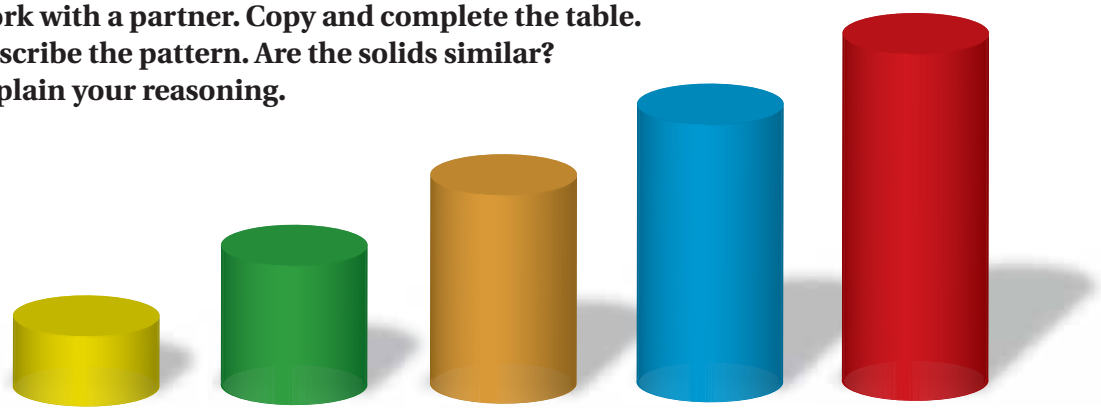
STATE STANDARDS
MA.7.G.4.1

Essential Question When the dimensions of a solid increase by a factor of k, how does the surface area change? How does the volume change?

1 ACTIVITY: Comparing Volumes and Surface Areas

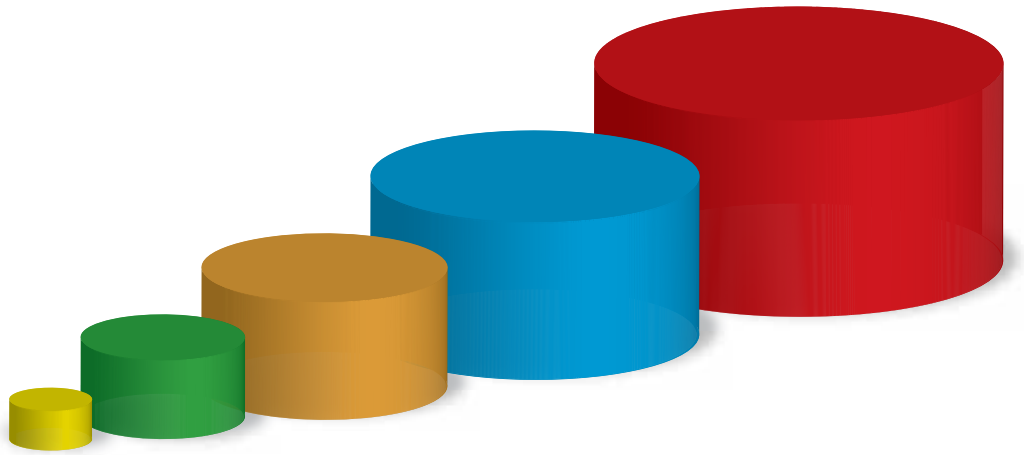
Work with a partner. Copy and complete the table. Describe the pattern. Are the solids similar? Explain your reasoning.

a.



Radius	1	1	1	1	1
Height	1	2	3	4	5
Surface Area					
Volume					

b.



Radius	1	2	3	4	5
Height	1	2	3	4	5
Surface Area					
Volume					

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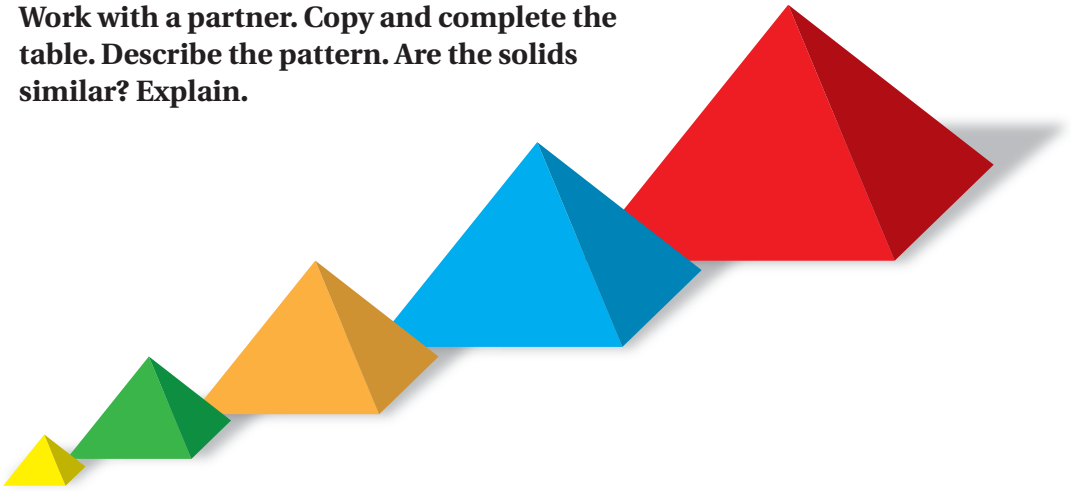
2 ACTIVITY: Comparing Volumes and Surface Areas

Work with a partner. Copy and complete the table. Describe the pattern. Are the solids similar? Explain.



Base Side	6	12	18	24	30
Height	4	8	12	16	20
Slant Height	5	10	15	20	25
Surface Area					
Volume					

What Is Your Answer?

3. **IN YOUR OWN WORDS** When the dimensions of a solid increase by a factor of k, how does the surface area change?

4. **IN YOUR OWN WORDS** When the dimensions of a solid increase by a factor of k, how does the volume change?

5. All the dimensions of a cone increase by a factor of 5.

   a. How many times greater is the surface area? Explain.

   5 10 25 125

   b. How many times greater is the volume? Explain.

   5 10 25 125

Practice Use what you learned about the surface areas and volumes of similar solids to complete Exercises 4–6 on page 335.

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7.6 Lesson

Check It Out Lesson Tutorials BigIdeasMath.com

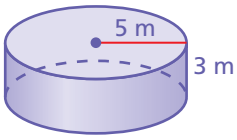
Key Vocabulary
similar solids, *p. 332*

Solids of the same type that have proportional corresponding linear measures are **similar solids**.

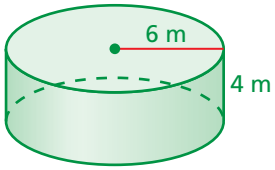
EXAMPLE 1 Identifying Similar Solids

Which cylinder is similar to Cylinder A?

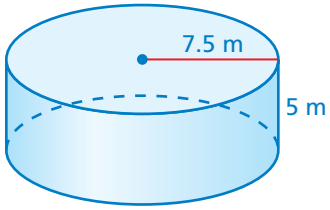
Cylinder B



Cylinder A



Cylinder C



Check to see if corresponding linear measures are proportional.

***Cylinder* A *and Cylinder* B**

$\frac{\text{Height of A}}{\text{Height of B}} = \frac{4}{3}$ $\quad$ $\frac{\text{Radius of A}}{\text{Radius of B}} = \frac{6}{5}$

Not proportional

***Cylinder* A *and Cylinder* C**

$\frac{\text{Height of A}}{\text{Height of C}} = \frac{4}{5}$ $\quad$ $\frac{\text{Radius of A}}{\text{Radius of C}} = \frac{6}{7.5} = \frac{4}{5}$

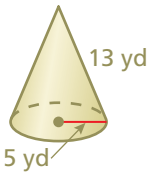
Proportional

So, Cylinder C is similar to Cylinder A.

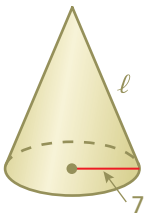
EXAMPLE 2 Finding Missing Measures in Similar Solids

The cones are similar. Find the missing slant height ℓ.

Cone X



Cone Y



$$\frac{\text{Radius of X}}{\text{Radius of Y}} = \frac{\text{Slant height of X}}{\text{Slant height of Y}}$$

$\frac{5}{7} = \frac{13}{\ell}$ Substitute.

$5\ell = 91$ Use Cross Products Property.

$\ell = 18.2$ Divide each side by 5.

The slant height is 18.2 yards.

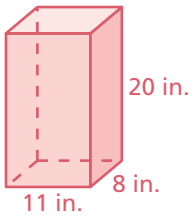
On Your Own

Now You're Ready
Exercises 4–9

1. Cylinder D has a radius of 7.5 meters and a height of 4.5 meters. Which cylinder in Example 1 is similar to Cylinder D?

2. The prisms are similar. Find the missing width and length.

Prism A



Prism B

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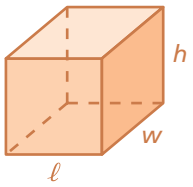
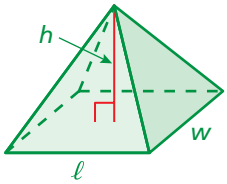
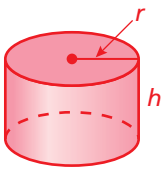
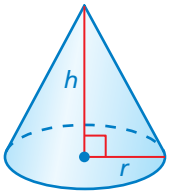
Key Ideas

Linear Measures





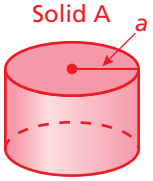


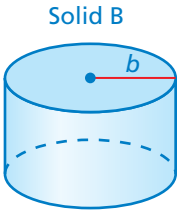


Surface Areas of Similar Solids

If two solids are similar, then the ratio of their surface areas is equal to the square of the ratio of their corresponding linear measures.

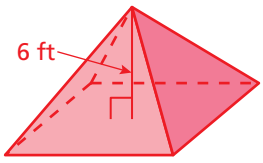
$$\frac{\text{Surface Area of A}}{\text{Surface Area of B}} = \left(\frac{a}{b}\right)^2$$

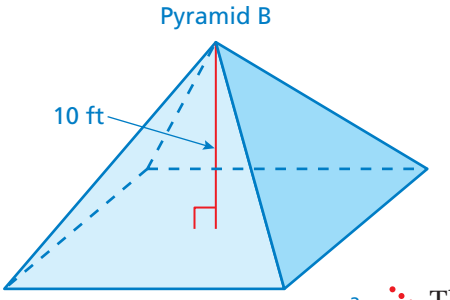




EXAMPLE 3 Finding Surface Area

The pyramids are similar. What is the surface area of Pyramid A?





$$\frac{\text{Surface Area of A}}{\text{Surface Area of B}} = \left(\frac{\text{Height of A}}{\text{Height of B}}\right)^2$$

$$\frac{S}{600} = \left(\frac{6}{10}\right)^2 \quad \text{Substitute.}$$

$$\frac{S}{600} = \frac{36}{100} \quad \text{Evaluate power.}$$

$$\frac{S}{600} \cdot 600 = \frac{36}{100} \cdot 600 \quad \text{Multiply each side by 600.}$$

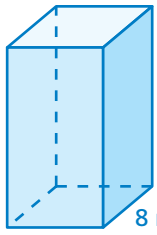
$$S = 216 \quad \text{Simplify.}$$

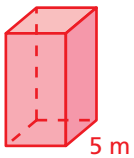
∴ The surface area of Pyramid A is 216 square feet.

On Your Own

The solids are similar. Find the surface area of the red solid. Round your answer to the nearest tenth.

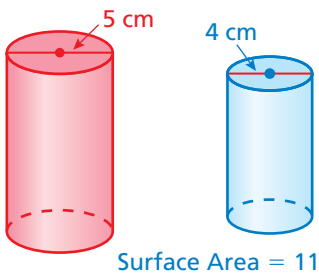
3.





Surface Area = 608 m²

4.



Surface Area = 110 cm²

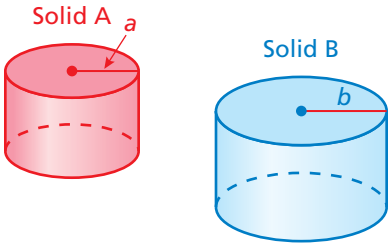
English Spanish

Key Idea

Volumes of Similar Solids

If two solids are similar, then the ratio of their volumes is equal to the cube of the ratio of their corresponding linear measures.

$$\frac{\text{Volume of A}}{\text{Volume of B}} = \left(\frac{a}{b}\right)^3$$



EXAMPLE 4 Standardized Test Practice

Original Tank

Volume = 2000 ft^3

The dimensions of the touch tank at an aquarium are doubled. What is the volume of the new touch tank?

Ⓐ 150 ft^3 Ⓑ 4000 ft^3

Ⓒ 8000 ft^3 Ⓓ 16,000 ft^3

The dimensions are doubled, so the ratio of the dimensions in the original tank to the dimensions in the new tank is 1 : 2.

$$\frac{\text{Original volume}}{\text{New volume}} = \left(\frac{\text{Original dimension}}{\text{New dimension}}\right)^3$$

$$\frac{2000}{V} = \left(\frac{1}{2}\right)^3 \quad \text{Substitute.}$$

$$\frac{2000}{V} = \frac{1}{8} \quad \text{Evaluate power.}$$

$$16{,}000 = V \quad \text{Use Cross Products Property.}$$

∴ The volume of the new tank is 16,000 cubic feet. The correct answer is Ⓓ.

Study Tip

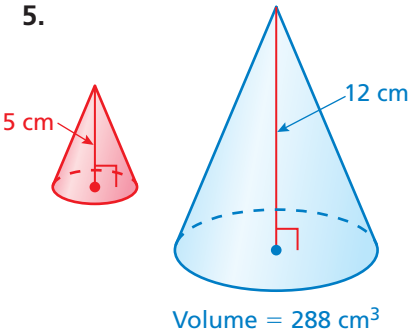
When the dimensions of a solid are multiplied by k, the surface area is multiplied by k^2 and the volume is multiplied by k^3.

On Your Own

Now You're Ready
Exercises 10–13

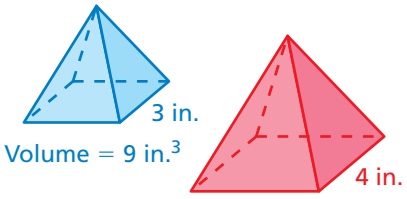
The solids are similar. Find the volume of the red solid. Round your answer to the nearest tenth.

5.



Volume = 288 cm^3

6.



Volume = 9 in.^{3}

English Spanish

7.6 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

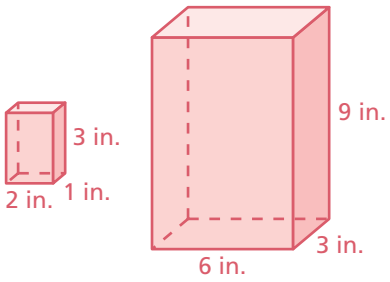
1. **VOCABULARY** What are similar solids?
2. **OPEN-ENDED** Draw two similar solids and label their corresponding linear measures.
3. **REASONING** The ratio of the corresponding linear measures of Cube A to Cube B is $\frac{2}{3}$.
   - **a.** Find the ratio of the area of one face of Cube A to the area of one face of Cube B.
   - **b.** Find the ratio of the volume of Cube A to the volume of Cube B.

Practice and Problem Solving

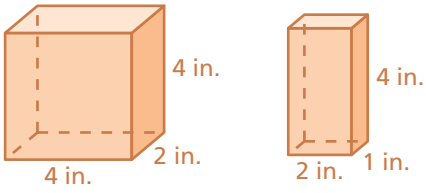
Determine whether the solids are similar.

1

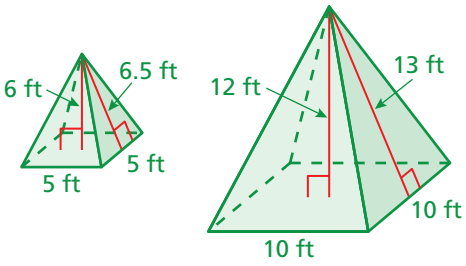
4.



5.



6.



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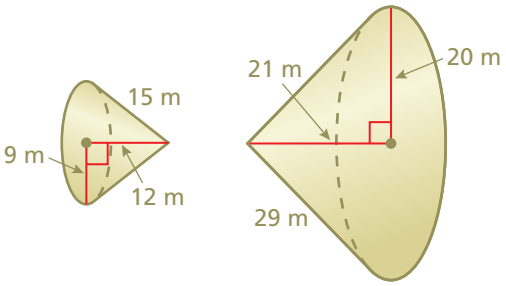
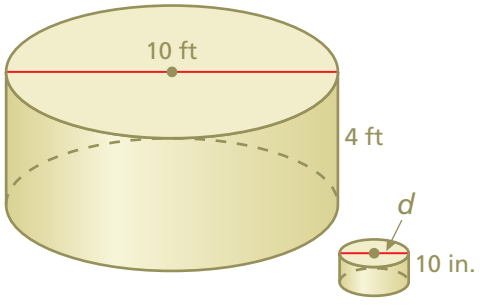


The solids are similar. Find the missing dimension(s).

2

8.



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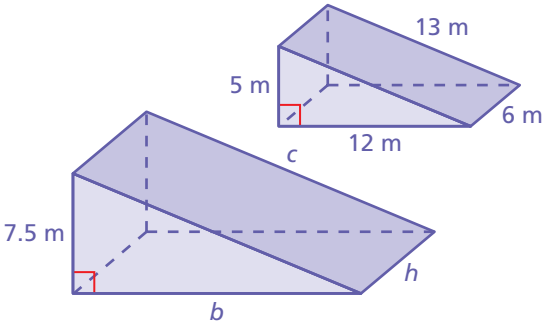
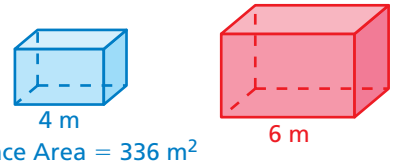


English Spanish

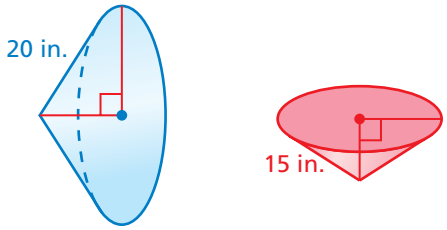
The solids are similar. Find the surface area S or volume V of the red solid. Round your answer to the nearest tenth.

3 4 **10.**



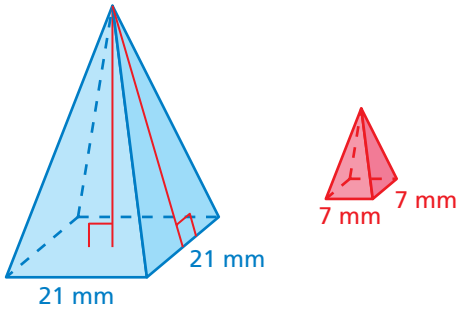
Surface Area = 336 m^2

11.



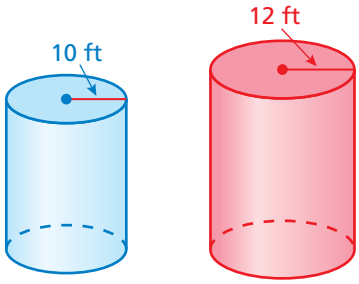
Surface Area = 1800 $in.^2$

12.



Volume = 5292 mm^3

13.



Volume = 7850 ft^3

14. ERROR ANALYSIS The ratio of the corresponding linear measures of two similar solids is 3 : 5. The volume of the smaller solid is 108 cubic inches. Describe and correct the error in finding the volume of the larger solid.

✗

$$\frac{108}{V} = \left(\frac{3}{5}\right)^2$$

$$\frac{108}{V} = \frac{9}{25}$$

$$300 = V$$

The volume of the larger solid is 300 cubic inches.

15. MIXED FRUIT The ratio of the corresponding linear measures of two similar cans of fruit is 4 to 7. The smaller can has a surface area of 220 square centimeters. Find the surface area of the larger can.

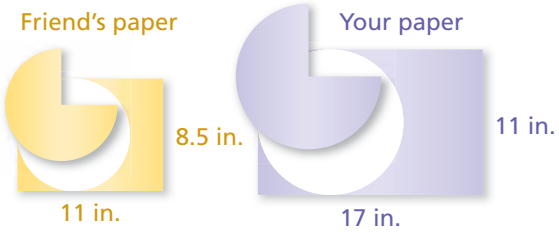
16. CLASSIC MUSTANG The volume of a 1968 Ford Mustang GT engine is 390 cubic inches. Which scale model of the Mustang has the greater engine volume, a 1 : 18 scale model or a 1 : 24 scale model? How much greater?

English Spanish

17. Critical Thinking You and a friend make paper cones to collect beach glass. You cut out the largest possible three-fourths circle from each piece of paper.

a. Are the cones similar? Explain your reasoning.

b. Your friend says that because your sheet of paper is twice as large, your cone will hold exactly twice the volume of beach glass. Is this true? Explain your reasoning.



18. **MARBLE STATUE** You have a small marble statue of Wolfgang Mozart that is 10 inches tall and weighs 16 pounds. The original statue in Vienna is 7 feet tall.

a. Estimate the weight of the original statue. Explain your reasoning.

b. If the original statue were 20 feet tall, how much would it weigh?

Wolfgang Mozart

19. **RUSSIAN DOLLS** The largest doll is 7 inches tall. Each of the other dolls is 1 inch shorter than the next larger doll. Make a table that compares the surface areas and volumes of the seven dolls.

Fair Game Review What you learned in previous grades & lessons

Add. SECTION 1.2

20. $69 + (-31) + 7 + (-6)$ **21.** $-2 + (-5) + (-12) + 20$ **22.** $10 + (-6) + (-5) + 1$

23. **MULTIPLE CHOICE** What is the mean of the numbers below? SKILLS REVIEW HANDBOOK

14, 6, 21, 8, 14, 19, 30

(A) 6 (B) 15 (C) 16 (D) 56