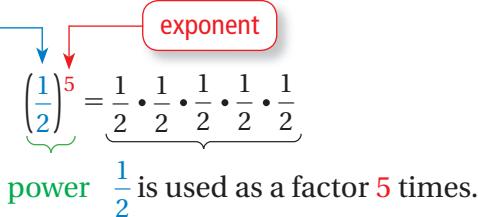


15.1 Lesson

Key Vocabulary

power
base
exponent

A **power** is a product of repeated factors. The **base** of a power is the common factor. The **exponent** of a power indicates the number of times the base is used as a factor.



EXAMPLE 1 Writing Expressions Using Exponents

Study Tip



Use parentheses to write powers with negative bases.

Write each product using exponents.

a. $(-7) \cdot (-7) \cdot (-7)$

Because -7 is used as a factor 3 times, its exponent is 3.

So, $(-7) \cdot (-7) \cdot (-7) = (-7)^3$.

b. $\pi \cdot \pi \cdot r \cdot r \cdot r$

Because π is used as a factor 2 times, its exponent is 2. Because r is used as a factor 3 times, its exponent is 3.

So, $\pi \cdot \pi \cdot r \cdot r \cdot r = \pi^2 r^3$.

On Your Own

Write the product using exponents.



Exercises 3–10

1. $\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}$

2. $0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot x \cdot x$

EXAMPLE 2 Evaluating Expressions

Evaluate the expression.

a. $(-2)^4$

$$(-2)^4 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) \\ = 16$$

Write as repeated multiplication.
Simplify.

The factor is -2 .

b. -2^4

$$-2^4 = -(2 \cdot 2 \cdot 2 \cdot 2) \\ = -16$$

Write as repeated multiplication.
Simplify.

The factor is 2 .

EXAMPLE**3****Using Order of Operations****Evaluate the expression.**

a. $3 + 2 \cdot 3^4$

$$3 + 2 \cdot 3^4 = 3 + 2 \cdot 81 \quad \text{Evaluate the power.}$$

$$= 3 + 162 \quad \text{Multiply.}$$

$$= 165 \quad \text{Add.}$$

b. $3^3 - 8^2 \div 2$

$$3^3 - 8^2 \div 2 = 27 - 64 \div 2 \quad \text{Evaluate the powers.}$$

$$= 27 - 32 \quad \text{Divide.}$$

$$= -5 \quad \text{Subtract.}$$

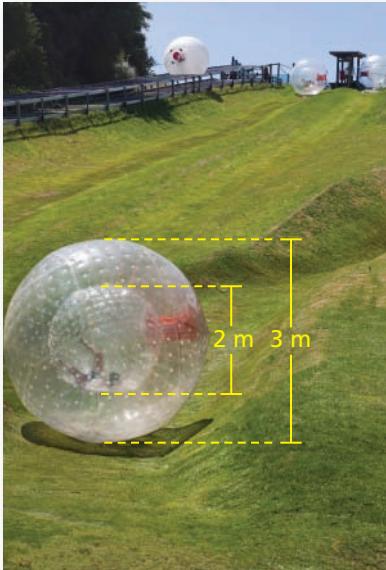
On Your Own**Now You're Ready**Exercises 11–16
and 21–26**Evaluate the expression.**

3. -5^4

4. $\left(-\frac{1}{6}\right)^3$

5. $|-3^3 \div 27|$

6. $9 - 2^5 \cdot 0.5$

EXAMPLE**4****Real-Life Application**

In sphering, a person is secured inside a small, hollow sphere that is surrounded by a larger sphere. The space between the spheres is inflated with air. What is the volume of the inflated space?

(The volume V of a sphere is $V = \frac{4}{3}\pi r^3$. Use 3.14 for π .)

Outer sphere

$$V = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi(1.5)^3$$

$$= \frac{4}{3}\pi(3.375)$$

$$\approx 14.13$$

Write formula.

Substitute.

Evaluate the power.

Multiply.

Inner sphere

$$V = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi(1)^3$$

$$= \frac{4}{3}\pi(1)$$

$$\approx 4.19$$

- ❖ So, the volume of the inflated space is about $14.13 - 4.19$, or 9.94 cubic meters.

On Your Own

7. **WHAT IF?** In Example 4, the diameter of the inner sphere is 1.8 meters. What is the volume of the inflated space?

15.1 Exercises



Vocabulary and Concept Check

- VOCABULARY** Describe the difference between an exponent and a power. Can the two words be used interchangeably?
- WHICH ONE DOESN'T BELONG?** Which one does *not* belong with the other three? Explain your reasoning.

5^3
The exponent is 3.

5^3
The power is 5.

5^3
The base is 5.

5^3
Five is used as a factor 3 times.



Practice and Problem Solving

Write the product using exponents.

1. $3 \cdot 3 \cdot 3 \cdot 3$

4. $(-6) \cdot (-6)$

5. $\left(-\frac{1}{2}\right) \cdot \left(-\frac{1}{2}\right) \cdot \left(-\frac{1}{2}\right)$

6. $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$

7. $\pi \cdot \pi \cdot \pi \cdot x \cdot x \cdot x \cdot x \cdot x$

8. $(-4) \cdot (-4) \cdot (-4) \cdot y \cdot y$

9. $8 \cdot 8 \cdot 8 \cdot 8 \cdot b \cdot b \cdot b$

10. $(-t) \cdot (-t) \cdot (-t) \cdot (-t) \cdot (-t)$

Evaluate the expression.

2. 11^2

12. -11^3

13. $(-1)^6$

14. $\left(\frac{1}{2}\right)^6$

15. $\left(-\frac{1}{12}\right)^2$

16. $-\left(\frac{1}{9}\right)^3$

17. **ERROR ANALYSIS** Describe and correct the error in evaluating the expression.



$$6^3 = 6 \cdot 3 = 18$$

18. **PRIME FACTORIZATION** Write the prime factorization of 675 using exponents.

19. **NUMBER SENSE** Write $-\left(\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}\right)$ using exponents.



20. **RUSSIAN DOLLS** The largest doll is 12 inches tall. The height of each of the other dolls is $\frac{7}{10}$ the height of the next larger doll. Write an expression for the height of the smallest doll. What is the height of the smallest doll?

Evaluate the expression.

3 21. $5 + 3 \cdot 2^3$

22. $2 + 7 \cdot (-3)^2$

23. $(13^2 - 12^2) \div 5$

24. $\frac{1}{2}(4^3 - 6 \cdot 3^2)$

25. $\left| \frac{1}{2}(7 + 5^3) \right|$

26. $\left| \left(-\frac{1}{2} \right)^3 \div \left(\frac{1}{4} \right)^2 \right|$

27. **MONEY** You have a part-time job. One day your boss offers to pay you either $2^h - 1$ or 2^{h-1} dollars for each hour h you work that day. Copy and complete the table. Which option should you choose? Explain.

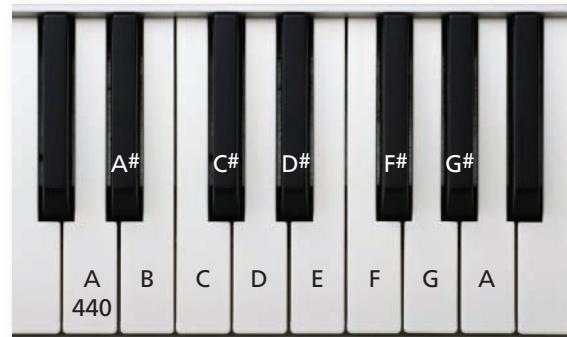
h	1	2	3	4	5
$2^h - 1$					
2^{h-1}					

28. **CARBON-14 DATING** Carbon-14 dating is used by scientists to determine the age of a sample.

- The amount C (in grams) of a 100-gram sample of carbon-14 remaining after t years is represented by the equation $C = 100(0.99988)^t$. Use a calculator to find the amount of carbon-14 remaining after 4 years.
- What percent of the carbon-14 remains after 4 years?

29. **Critical Thinking** The frequency (in vibrations per second) of a note on a piano is represented by the equation $F = 440(1.0595)^n$, where n is the number of notes above A-440. Each black or white key represents one note.

- How many notes do you take to travel from A-440 to A?
- What is the frequency of A?
- Describe the relationship between the number of notes between A-440 and A and the frequency of the notes.



Fair Game Review What you learned in previous grades & lessons

Tell which property is illustrated by the statement.

30. $8 \cdot x = x \cdot 8$

31. $(2 \cdot 10)x = 2(10 \cdot x)$

32. $3(x \cdot 1) = 3x$

33. **MULTIPLE CHOICE** A cone of yarn has a surface area of 16π square inches. What is the slant height of the cone of yarn?

(A) 4 in.

(B) 6 in.

(C) 8 in.

(D) 10 in.



15.2 Lesson



Product of Powers Property

Words To multiply powers with the same base, add their exponents.

Numbers $4^2 \cdot 4^3 = 4^{2+3} = 4^5$

Algebra $a^m \cdot a^n = a^{m+n}$

EXAMPLE 1 Multiplying Powers with the Same Base

a. $2^4 \cdot 2^5 = 2^{4+5}$

$= 2^9$

The base is 2. Add the exponents.

Simplify.

b. $-5 \cdot (-5)^6 = (-5)^1 \cdot (-5)^6$

$= (-5)^{1+6}$

$= (-5)^7$

Rewrite -5 as $(-5)^1$.

The base is -5 . Add the exponents.

Simplify.

c. $x^3 \cdot x^7 = x^{3+7}$

$= x^{10}$

The base is x . Add the exponents.

Simplify.

Study Tip



When a number is written without an exponent, its exponent is 1.

On Your Own

Simplify the expression. Write your answer as a power.

1. $6^2 \cdot 6^4$

2. $\left(-\frac{1}{2}\right)^3 \cdot \left(-\frac{1}{2}\right)^6$

3. $z \cdot z^{12}$

EXAMPLE 2 Raising a Power to a Power

a. $(3^4)^3 = 3^4 \cdot 3^4 \cdot 3^4$

$= 3^{4+4+4}$

$= 3^{12}$

Write as repeated multiplication.

The base is 3. Add the exponents.

Simplify.

b. $(w^5)^4 = w^5 \cdot w^5 \cdot w^5 \cdot w^5$

$= w^{5+5+5+5}$

$= w^{20}$

Write as repeated multiplication.

The base is w . Add the exponents.

Simplify.

On Your Own

Simplify the expression. Write your answer as a power.

4. $(4^4)^3$

5. $(y^2)^4$

6. $(\pi^3)^3$

7. $((-4)^3)^2$



Exercises 3–14

EXAMPLE**3****Raising a Product to a Power**

- a. $(2x)^3 = 2x \cdot 2x \cdot 2x$ Write as repeated multiplication.
 $= (2 \cdot 2 \cdot 2) \cdot (x \cdot x \cdot x)$ Group like bases using properties of multiplication.
 $= 2^{1+1+1} \cdot x^{1+1+1}$ The bases are 2 and x . Add the exponents.
 $= 2^3 \cdot x^3 = 8x^3$ Simplify.
- b. $(xy)^2 = xy \cdot xy$ Write as repeated multiplication.
 $= (x \cdot x) \cdot (y \cdot y)$ Group like bases using properties of multiplication.
 $= x^{1+1} \cdot y^{1+1}$ The bases are x and y . Add the exponents.
 $= x^2y^2$ Simplify.

**On Your Own**

Exercises 17–22

Simplify the expression.

8. $(5y)^4$

9. $(0.5n)^2$

10. $(ab)^5$

EXAMPLE**4****Standardized Test Practice**

Details	
Local Disk (C:)	
Local Disk	
Free Space: 16GB	
Total Space: 64GB	

A gigabyte (GB) of computer storage space is 2^{30} bytes. The details of a computer are shown. How many bytes of total storage space does the computer have?

- (A) 2^{34} (B) 2^{36} (C) 2^{180} (D) 128^{30}

The computer has 64 gigabytes of total storage space. Notice that 64 can be written as a power, 2^6 . Use a model to solve the problem.

$$\begin{aligned} \frac{\text{Total number}}{\text{of bytes}} &= \frac{\text{Number of bytes}}{\text{in a gigabyte}} \cdot \frac{\text{Number of}}{\text{gigabytes}} \\ &= 2^{30} \cdot 2^6 \quad \text{Substitute.} \\ &= 2^{30+6} \quad \text{Add exponents.} \\ &= 2^{36} \quad \text{Simplify.} \end{aligned}$$

- The computer has 2^{36} bytes of total storage space. The correct answer is (B).

**On Your Own**

11. How many bytes of free storage space does the computer have?

15.2 Exercises



Vocabulary and Concept Check

- REASONING** When should you use the Product of Powers Property?
- CRITICAL THINKING** Can you use the Product of Powers Property to multiply powers with different bases? Explain.



Practice and Problem Solving

Simplify the expression. Write your answer as a power.

1. 2. 3. $3^2 \cdot 3^2$

4. $8^{10} \cdot 8^4$

5. $(-4)^5 \cdot (-4)^7$

6. $a^3 \cdot a^3$

7. $h^6 \cdot h$

8. $\left(\frac{2}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^6$

9. $\left(-\frac{5}{7}\right)^8 \cdot \left(-\frac{5}{7}\right)^9$

10. $(-2.9) \cdot (-2.9)^7$

11. $(5^4)^3$

12. $(b^{12})^3$

13. $(3.8^3)^4$

14. $\left(\left(-\frac{3}{4}\right)^5\right)^2$

ERROR ANALYSIS Describe and correct the error in simplifying the expression.

15. $\cancel{\text{X}}$ $5^2 \cdot 5^9 = (5 \cdot 5)^{2+9}$
 $= 25^{11}$

16. $\cancel{\text{X}}$ $(r^6)^4 = r^{6+4}$
 $= r^{10}$

Simplify the expression.

3. 17. $(6g)^3$

18. $(-3v)^5$

19. $\left(\frac{1}{5}k\right)^2$

20. $(1.2m)^4$

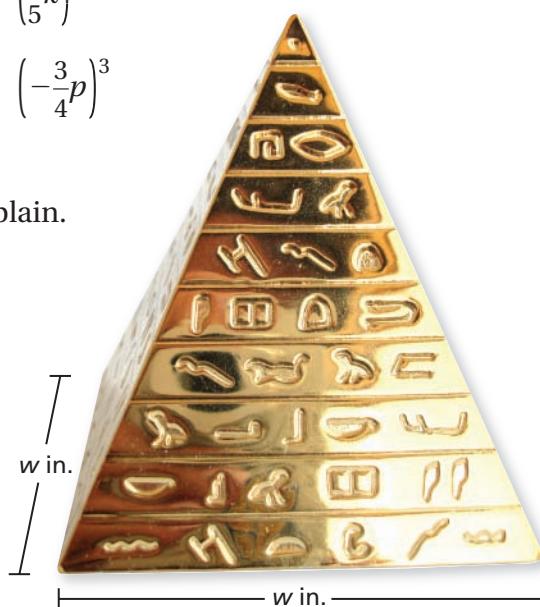
21. $(rt)^{12}$

22. $\left(-\frac{3}{4}p\right)^3$

23. **CRITICAL THINKING** Is $3^2 + 3^3$ equal to 3^5 ? Explain.

24. **ARTIFACT** A display case for the artifact is in the shape of a cube. Each side of the display case is three times longer than the width of the artifact.

- Write an expression for the volume of the case. Write your answer as a power.
- Simplify the expression.



Simplify the expression.

25. $2^4 \cdot 2^5 - (2^2)^2$

26. $16\left(\frac{1}{2}x\right)^4$

27. $5^2(5^3 \cdot 5^2)$

28. **WATER** Florida has about 10 times more miles of rivers, streams, and waterways than miles of coastline. There are approximately 10^3 miles of coastline in Florida. About how many miles of rivers, streams, and waterways are in Florida? Write your answer as a power.

29. **PYTHON EGG** The volume V of a python egg is given by the formula $V = \frac{4}{3}\pi abc$. For the python egg shown, $a = 2$ inches, $b = 2$ inches, and $c = 3$ inches.
- Find the volume of the python egg.
 - Square the dimensions of the python egg. Then evaluate the formula. How does this volume compare to your answer in part (a)?



30. **PYRAMID** The volume of a square pyramid is $V = \frac{1}{3}b^2h$, where b is the length of one side of the base and h is the height of the pyramid. The length of each side of the base increases by 50%. Write a formula for the volume of the new pyramid.
31. **MAIL** The United States Postal Service delivers about $2^6 \cdot 5^3$ pieces of mail each second. There are $2^8 \cdot 3^4 \cdot 5^2$ seconds in 6 days. How many pieces of mail does the United States Postal Service deliver in 6 days? Write your answer as a power.

32. **Critical Thinking** Find the value of x in the equation without evaluating the power.

a. $2^5 \cdot 2^x = 256$

b. $\left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^x = \frac{1}{729}$



Fair Game Review What you learned in previous grades & lessons

Simplify.

33. $\frac{4 \cdot 4}{4}$

34. $\frac{5 \cdot 5 \cdot 5}{5}$

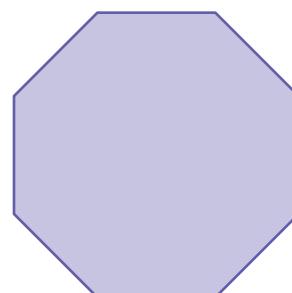
35. $\frac{2 \cdot 3}{2}$

36. $\frac{8 \cdot 6 \cdot 6}{6 \cdot 8}$

37. **MULTIPLE CHOICE** What is the measure of each angle of the regular polygon?

- (A) 45°
(C) 1080°

- (B) 135°
(D) 1440°



15.3 Lesson

Key Idea

Quotient of Powers Property

Words To divide powers with the same base, subtract their exponents.

Numbers $\frac{4^5}{4^2} = 4^{5-2} = 4^3$

Algebra $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

EXAMPLE 1 Dividing Powers with the Same Base

a. $\frac{2^6}{2^4} = 2^{6-4}$
= 2^2

The base is 2. Subtract the exponents.
Simplify.

b. $\frac{(-7)^9}{(-7)^3} = (-7)^{9-3}$
= $(-7)^6$

The base is -7 . Subtract the exponents.
Simplify.

c. $\frac{h^7}{h^6} = h^{7-6}$
= $h^1 = h$

The base is h . Subtract the exponents.
Simplify.

Common Error

When dividing powers, do not divide the bases.

$\frac{2^6}{2^4} = 2^2$, not 1^2 .

Now You're Ready

Exercises 7–14

On Your Own

Simplify the expression. Write your answer as a power.

1. $\frac{9^7}{9^4}$

2. $\frac{4.2^6}{4.2^5}$

3. $\frac{(-8)^8}{(-8)^4}$

4. $\frac{x^8}{x^3}$

EXAMPLE 2 Simplifying an Expression

Simplify $\frac{3^4 \cdot 3^2}{3^3}$. Write your answer as a power.

The numerator is a product of powers.

$$\frac{3^4 \cdot 3^2}{3^3} = \frac{3^{4+2}}{3^3}$$

Add the exponents in the numerator.

$$\begin{aligned} &= \frac{3^6}{3^3} \\ &= 3^{6-3} \\ &= 3^3 \end{aligned}$$

Simplify.

The base is 3. Subtract the exponents.
Simplify.

EXAMPLE

3

Simplifying an Expression**Study Tip**

You can also simplify the expression in Example 3 as follows.

$$\begin{aligned}\frac{a^{10}}{a^6} \cdot \frac{a^7}{a^4} &= \frac{a^{10} \cdot a^7}{a^6 \cdot a^4} \\ &= \frac{a^{17}}{a^{10}} \\ &= a^{17-10} \\ &= a^7\end{aligned}$$

Simplify $\frac{a^{10}}{a^6} \cdot \frac{a^7}{a^4}$. Write your answer as a power.

$$\frac{a^{10}}{a^6} \cdot \frac{a^7}{a^4} = a^{10-6} \cdot a^{7-4}$$

Subtract the exponents.

$$= a^4 \cdot a^3$$

Simplify.

$$= a^{4+3}$$

Add the exponents.

$$= a^7$$

Simplify.

On Your Own

Simplify the expression. Write your answer as a power.

Now You're Ready
Exercises 16–21

5. $\frac{2^{15}}{2^3 \cdot 2^5}$

6. $\frac{d^5}{d} \cdot \frac{d^9}{d^8}$

EXAMPLE

4

Real-Life Application

The projected population of Florida in 2030 is about $4 \cdot 7.2^8$. Predict the average number of people per square kilometer in 2030.

Use a model to solve the problem.

$$\text{People per square kilometer} = \frac{\text{Population in 2030}}{\text{Land area}}$$



$$= \frac{4 \cdot 7.2^8}{7.2^6}$$

Substitute.

$$= 4 \cdot \frac{7.2^8}{7.2^6}$$

Rewrite.

$$= 4 \cdot 7.2^2$$

Subtract the exponents.

$$= 207.36$$

Evaluate.

- There will be about 207 people per square kilometer in Florida in 2030.

On Your Own**On Your Own**

Now You're Ready
Exercises 23–28

- The projected population of Alabama in 2020 is about $2.25 \cdot 2^{21}$. The land area of Alabama is about 2^{17} square kilometers. Predict the average number of people per square kilometer in 2020.

15.3 Exercises



Vocabulary and Concept Check

1. **WRITING** Explain in your own words what it means to divide powers.
2. **WHICH ONE DOESN'T BELONG?** Which quotient does *not* belong with the other three? Explain your reasoning.

$$\frac{(-10)^7}{(-10)^2}$$

$$\frac{6^3}{6^2}$$

$$\frac{(-4)^8}{(-3)^4}$$

$$\frac{5^6}{5^3}$$



Practice and Problem Solving

Simplify the expression. Write your answer as a power.

3. $\frac{6^{10}}{6^4}$

4. $\frac{8^9}{8^7}$

5. $\frac{(-3)^4}{(-3)^1}$

6. $\frac{4.5^5}{4.5^3}$

1 7. $\frac{5^9}{5^3}$

8. $\frac{64^4}{64^3}$

9. $\frac{(-17)^5}{(-17)^2}$

10. $\frac{(-7.9)^{10}}{(-7.9)^4}$

11. $\frac{(-6.4)^8}{(-6.4)^6}$

12. $\frac{\pi^{11}}{\pi^7}$

13. $\frac{b^{24}}{b^{11}}$

14. $\frac{n^{18}}{n^7}$

15. **ERROR ANALYSIS** Describe and correct the error in simplifying the quotient.

$$\frac{6^{15}}{6^5} = \frac{15}{5} = 6^3$$

Simplify the expression. Write your answer as a power.

2 3 16. $\frac{7^5 \cdot 7^3}{7^2}$

17. $\frac{2^{19} \cdot 2^5}{2^{12} \cdot 2^3}$

18. $\frac{(-8.3)^8}{(-8.3)^7} \cdot \frac{(-8.3)^4}{(-8.3)^3}$

19. $\frac{\pi^{30}}{\pi^{18} \cdot \pi^4}$

20. $\frac{c^{22}}{c^8 \cdot c^9}$

21. $\frac{k^{13}}{k^5} \cdot \frac{k^{17}}{k^{11}}$

22. **SOUND INTENSITY** The sound intensity of a normal conversation is 10^6 times greater than the quietest noise a person can hear. The sound intensity of a jet at takeoff is 10^{14} times greater than the quietest noise a person can hear. How many times more intense is the sound of a jet at takeoff than the sound of a normal conversation?



Simplify the expression.

4 23. $\frac{x \cdot 4^8}{4^5}$

26. $\frac{5^{12} \cdot c^{10} \cdot d^2}{5^9 \cdot c^9}$

24. $\frac{6^3 \cdot w}{6^2}$

27. $\frac{x^{15}y^9}{x^8y^3}$

25. $\frac{a^3 \cdot b^4 \cdot 5^4}{b^2 \cdot 5}$

28. $\frac{m^{10}n^7}{m^1n^6}$

29. **MEMORY** The memory capacities and prices of five MP3 players are shown in the table.

MP3 Player	Memory (GB)	Price
A	2^1	\$70
B	2^2	\$120
C	2^3	\$170
D	2^4	\$220
E	2^5	\$270

- a. How many times more memory does MP3 Player D have than MP3 Player B?
- b. Do the differences in price between consecutive sizes reflect a constant rate of change?

30. **CRITICAL THINKING** Consider the equation $\frac{9^m}{9^n} = 9^2$.

- a. Find two numbers m and n that satisfy the equation.
 b. Are there any other pairs of numbers that satisfy the equation? Explain.



Milky Way Galaxy
 $10 \cdot 10^{10}$ stars

31. **STARS** There are about 10^{24} stars in the Universe. Each galaxy has approximately the same number of stars as the Milky Way Galaxy. About how many galaxies are in the Universe?

32. **Number Sense** Find the value of x that makes $\frac{8^{3x}}{8^{2x+1}} = 8^9$ true. Explain how you found your answer.



Fair Game Review What you learned in previous grades & lessons

Subtract.

33. $-4 - 5$

34. $-23 - (-15)$

35. $33 - (-28)$

36. $18 - 22$

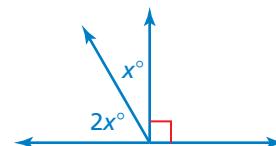
37. **MULTIPLE CHOICE** What is the value of x ?

(A) 20

(B) 30

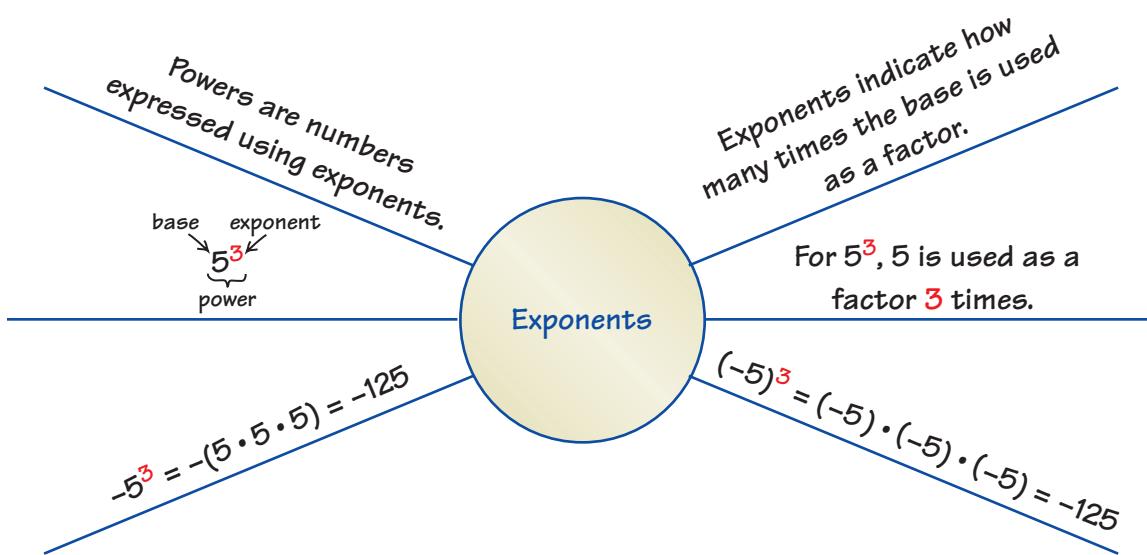
(C) 45

(D) 60



15 Study Help

You can use an **information wheel** to organize information about a topic. Here is an example of an information wheel for exponents.



On Your Own

Make an information wheel to help you study these topics.

1. order of operations
2. Product of Powers Property
3. Quotient of Powers Property

After you complete this chapter, make information wheels for the following topics.

4. zero exponents
5. negative exponents
6. writing numbers in scientific notation
7. writing numbers in standard form
8. Choose three other topics you studied earlier in this course.
Make an information wheel for each topic to summarize what you know about them.



15.1–15.3 Quiz

Write the product using exponents.

1. $(-5) \cdot (-5) \cdot (-5) \cdot (-5)$

2. $\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6}$

3. $(-x) \cdot (-x) \cdot (-x) \cdot (-x) \cdot (-x) \cdot (-x)$

4. $7 \cdot 7 \cdot m \cdot m \cdot m$

Evaluate the expression.

5. 5^4

6. $(-2)^6$

Simplify the expression. Write your answer as a power.

7. $3^8 \cdot 3$

8. $(a^5)^3$

Simplify the expression.

9. $(3c)^4$

10. $\left(-\frac{2}{7}p\right)^2$

Simplify the expression. Write your answer as a power.

11. $\frac{8^7}{8^4}$

12. $\frac{6^3 \cdot 6^7}{6^2}$

13. $\frac{\pi^{15}}{\pi^3 \cdot \pi^9}$

14. $\frac{t^{13}}{t^5} \cdot \frac{t^8}{t^6}$

15. **SEQUENCE** The n th term of a sequence can be found by evaluating $10^n - 1$. Copy and complete the table to find the first four terms of the sequence.

16. **Critical Thinking** Is $(ab)^2$ equivalent to ab^2 ? Explain.

n	$10^n - 1$
1	
2	
3	
4	



17. **EARTHQUAKES** An earthquake of magnitude 3.0 is 10^2 times stronger than an earthquake of magnitude 1.0. An earthquake of magnitude 8.0 is 10^7 times stronger than an earthquake of magnitude 1.0. How many times stronger is an earthquake of magnitude 8.0 than an earthquake of magnitude 3.0?

15.4 Lesson

Key Ideas

Zero Exponents

Words Any nonzero number to the zero power is equal to 1. Zero to the zero power, 0^0 , is *undefined*.

Numbers $4^0 = 1$

Algebra $a^0 = 1$, where $a \neq 0$

Negative Exponents

Words For any integer n and any number a not equal to 0, a^{-n} is equal to 1 divided by a^n .

Numbers $4^{-2} = \frac{1}{4^2}$

Algebra $a^{-n} = \frac{1}{a^n}$, where $a \neq 0$

EXAMPLE 1 Evaluating Expressions

a. $3^{-4} = \frac{1}{3^4}$ Definition of negative exponent

$$= \frac{1}{81} \quad \text{Evaluate power.}$$

b. $(-8.5)^{-4} \cdot (-8.5)^4 = (-8.5)^{-4+4}$ Add the exponents.

$$= (-8.5)^0 \quad \text{Simplify.}$$

$$= 1 \quad \text{Definition of zero exponent}$$

c. $\frac{2^6}{2^8} = 2^{6-8}$ Subtract the exponents.

$$= 2^{-2} \quad \text{Simplify.}$$

$$= \frac{1}{2^2} \quad \text{Definition of negative exponent}$$

$$= \frac{1}{4} \quad \text{Evaluate power.}$$

On Your Own

Now You're Ready

Exercises 9–16

Evaluate the expression.

1. 4^{-2}

2. $(-2)^{-5}$

3. $6^{-8} \cdot 6^8$

4. $\frac{(-3)^5}{(-3)^6}$

5. $\frac{1}{5^7} \cdot \frac{1}{5^{-4}}$

6. $\frac{4^5 \cdot 4^{-3}}{4^2}$

EXAMPLE**2****Simplifying Expressions**

a. $-5x^0 = -5(1)$ Definition of zero exponent
 $= -5$ Multiply.

b. $\frac{9y^{-3}}{y^5} = 9y^{-3-5}$ Subtract the exponents.
 $= 9y^{-8}$ Simplify.
 $= \frac{9}{y^8}$ Definition of negative exponent

On Your Own**Now You're Ready**
Exercises 20–27**Simplify.** Write the expression using only positive exponents.

7. $8x^{-2}$

8. $b^0 \cdot b^{-10}$

9. $\frac{z^6}{15z^9}$

EXAMPLE**3****Real-Life Application**

A drop of water leaks from a faucet every second. How many liters of water leak from the faucet in 1 hour?

Convert 1 hour to seconds.



Drop of water: 50^{-2} L

$$1 \cancel{\text{ hr}} \times \frac{60 \cancel{\text{ min}}}{1 \cancel{\text{ hr}}} \times \frac{60 \text{ sec}}{1 \cancel{\text{ min}}} = 3600 \text{ sec}$$

Water leaks from the faucet at a rate of 50^{-2} liter per second. Multiply the time by the rate.

$$\begin{aligned} 3600 \cdot 50^{-2} &= 3600 \cdot \frac{1}{50^2} && \text{Definition of negative exponent} \\ &= 3600 \cdot \frac{1}{2500} && \text{Evaluate power.} \\ &= \frac{3600}{2500} && \text{Multiply.} \\ &= 1\frac{11}{25} = 1.44 && \text{Simplify.} \end{aligned}$$

So, 1.44 liters of water leak from the faucet in 1 hour.

On Your Own

10. **WHAT IF?** In Example 4, the faucet leaks water at a rate of 5^{-5} liter per second. How many liters of water leak from the faucet in 1 hour?

15.4 Exercises



Vocabulary and Concept Check

- VOCABULARY** If a is a nonzero number, does the value of a^0 depend on the value of a ? Explain.
- WRITING** Explain how to evaluate 10^{-3} .
- NUMBER SENSE** Without evaluating, order 5^0 , 5^4 , and 5^{-5} from least to greatest.
- DIFFERENT WORDS, SAME QUESTION** Which is different? Find “both” answers.

Rewrite $\frac{1}{3 \cdot 3 \cdot 3}$ using a negative exponent.

Write 3 to the negative third power.

Write $\frac{1}{3}$ cubed as a power.

Write $(-3) \cdot (-3) \cdot (-3)$ as a power.



Practice and Problem Solving

- Use the Quotient of Powers Property to copy and complete the table.
- What patterns do you see?
- How would you define 5^0 ? Why?
- How can you rewrite 5^{-1} as a fraction?

n	4	3	2	1
5^n				
5^2				

Evaluate the expression.

1. 9. 6^{-2}

10. 158^0

11. $\frac{4^3}{4^5}$

12. $\frac{-3}{(-3)^2}$

13. $(-2)^{-8} \cdot (-2)^8$

14. $3^{-3} \cdot 3^{-2}$

15. $\frac{1}{5^{-3}} \cdot \frac{1}{5^6}$

16. $\frac{(1.5)^2}{(1.5)^{-2} \cdot (1.5)^4}$

17. **ERROR ANALYSIS** Describe and correct the error in evaluating the expression.

$(4)^{-3} = (-4)(-4)(-4)$
 $= -64$



18. **SAND** The mass of a grain of sand is about 10^{-3} gram. About how many grains of sand are in the bag of sand?

19. **CRITICAL THINKING** How can you write the number 1 as 2 to a power? 10 to a power?

Simplify. Write the expression using only positive exponents.

20. $6y^{-4}$

21. $8^{-2} \cdot a^7$

22. $\frac{9c^3}{c^{-4}}$

23. $\frac{5b^{-2}}{b^{-3}}$

24. $\frac{8x^3}{2x^9}$

25. $3d^{-4} \cdot 4d^4$

26. $m^{-2} \cdot n^3$

27. $\frac{3^{-2} \cdot k^0 \cdot w^0}{w^{-6}}$

METRIC UNITS In Exercises 28–31, use the table.

28. How many millimeters are in a decimeter?
29. How many micrometers are in a centimeter?
30. How many nanometers are in a millimeter?
31. How many micrometers are in a meter?

Unit of Length	Length
decimeter	10^{-1} m
centimeter	10^{-2} m
millimeter	10^{-3} m
micrometer	10^{-6} m
nanometer	10^{-9} m

32. **MICROBES** A species of bacteria is 10 micrometers long. A virus is 10,000 times smaller than the bacteria.

- Using the table above, find the length of the virus in meters.
- Is the answer to part (a) *less than*, *greater than*, or *equal to* one nanometer?

33. **BLOOD DONATION** Every 2 seconds, someone in the United States needs blood. A sample blood donation is shown. ($1 \text{ mm}^3 = 10^{-3} \text{ mL}$)

- One cubic millimeter of blood contains about 10^4 white blood cells. How many white blood cells are in the donation? Write your answer in words.
- One cubic millimeter of blood contains about 5×10^6 red blood cells. How many red blood cells are in the donation? Write your answer in words.
- Compare your answers for parts (a) and (b).

34. **OPEN-ENDED** Write two different powers with negative exponents that have the same value.

35. **Reasoning** The rule for negative exponents states that $a^{-n} = \frac{1}{a^n}$. Explain why this rule does not apply when $a = 0$.



Fair Game Review What you learned in previous grades & lessons

Simplify the expression.

36. $10^3 \cdot 10^6$

37. $10^2 \cdot 10$

38. $\frac{10^8}{10^4}$

39. **MULTIPLE CHOICE** Which data display best shows the variability of a data set?

(A) bar graph

(B) circle graph

(C) scatter plot

(D) box-and-whisker plot

15.5 Lesson

Key Vocabulary
scientific notation

Study Tip

Scientific notation is used to write very small and very large numbers.

Key Idea

Scientific Notation

A number is written in **scientific notation** when it is represented as the product of a factor and a power of 10. The factor must be at least 1 and less than 10.

The factor is at least 1 and less than 10.

8.3×10^{-7}

The power of 10 has an integer exponent.

EXAMPLE

1

Identifying Numbers Written in Scientific Notation

Tell whether the number is written in scientific notation. Explain.

a. 5.9×10^{-6}

- The factor is at least 1 and less than 10. The power of 10 has an integer exponent. So, the number is written in scientific notation.

b. 0.9×10^8

- The factor is less than 1. So, the number is not written in scientific notation.

Key Idea

Writing Numbers in Standard Form

When writing a number from scientific notation to standard form, the absolute value of the exponent tells you how many places to move the decimal point.

- If the exponent is **negative**, move the decimal point to the **left**.
- If the exponent is **positive**, move the decimal point to the **right**.

EXAMPLE

2

Writing Numbers in Standard Form

- a. Write 3.22×10^{-4} in standard form.

$$3.22 \times 10^{-4} = 0.\underbrace{000}_{4}322 \quad \text{Move decimal point } |-4| = 4 \text{ places to the left.}$$

- b. Write 7.9×10^5 in standard form.

$$7.9 \times 10^5 = 790,\underbrace{000}_{5} \quad \text{Move decimal point } |5| = 5 \text{ places to the right.}$$

● **On Your Own**

1. Is 12×10^4 written in scientific notation? Explain.

Write the number in standard form.

2. 6×10^7 3. 9.9×10^{-5} 4. 1.285×10^4

EXAMPLE
3
Comparing Numbers in Scientific Notation

An object with a lesser density than water will float. An object with a greater density than water will sink. Use each given density (in kilograms per cubic meter) to explain what happens when you place a brick and an apple in water.

Water: 1.0×10^3



Brick: 1.84×10^3



Apple: 6.41×10^2



Write each density in standard form.

Water

$$1.0 \times 10^3 = 1000$$

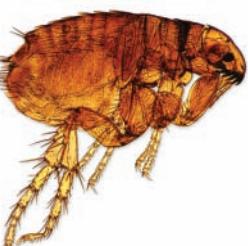
Brick

$$1.84 \times 10^3 = 1840$$

Apple

$$6.41 \times 10^2 = 641$$

- The apple is less dense than water, so it will float. The brick is denser than water, so it will sink.

EXAMPLE
4
Real-Life Application


A female flea consumes about 1.4×10^{-5} liter of blood per day.

A dog has 100 female fleas. How many milliliters of blood do the fleas consume per day?

$$\begin{aligned} 1.4 \times 10^{-5} \cdot 100 &= 0.000014 \cdot 100 \\ &= 0.0014 \end{aligned}$$

Write in standard form.

Multiply.

- The fleas consume about 0.0014 liter, or 1.4 milliliters of blood per day.

● **On Your Own**

5. **WHAT IF?** In Example 3, the density of lead is 1.14×10^4 kilograms per cubic meter. What happens when lead is placed in water?
6. **WHAT IF?** In Example 4, a dog has 75 female fleas. How many milliliters of blood do the fleas consume per day?

15.5 Exercises



Vocabulary and Concept Check

1. **WRITING** Describe the difference between scientific notation and standard form.
2. **WHICH ONE DOESN'T BELONG?** Which number does *not* belong with the other three? Explain.

$$2.8 \times 10^{15}$$

$$4.3 \times 10^{-30}$$

$$1.05 \times 10^{28}$$

$$10 \times 9.2^{-13}$$



Practice and Problem Solving

Write your answer in standard form.

3. A micrometer is 1.0×10^{-6} meter. How long is 150 micrometers in meters?
4. An acre is about 4.05×10^7 square centimeters. How many square centimeters are in 4 acres?
5. A cubic millimeter is about 6.1×10^{-5} cubic inches. How many cubic millimeters are in 1.22 cubic inches?

Tell whether the number is written in scientific notation. Explain.

6. 1.8×10^9
7. 3.45×10^{14}
8. 0.26×10^{-25}
9. 10.5×10^{12}
10. 46×10^{-17}
11. 5×10^{-19}
12. 7.814×10^{-36}
13. 0.999×10^{42}
14. 6.022×10^{23}

Write the number in standard form.

15. 7×10^7
16. 8×10^{-3}
17. 5×10^2
18. 2.7×10^{-4}
19. 4.4×10^{-5}
20. 2.1×10^3
21. 1.66×10^9
22. 3.85×10^{-8}
23. 9.725×10^6

24. **ERROR ANALYSIS** Describe and correct the error in writing the number in standard form.



$$4.1 \times 10^{-6} = 4100000$$



2.7×10^8 platelets per milliliter

25. **PLATELETS** Platelets are cell-like particles in the blood that help form blood clots.
- How many platelets are in 3 milliliters of blood? Write your answer in standard form.
 - An adult body contains about 5 liters of blood. How many platelets are in an adult body?

26. REASONING A googol is 1.0×10^{100} . How many zeros are in a googol?

3 27. STARS The table shows the surface temperatures of five stars.

- Which star has the highest surface temperature?
- Which star has the lowest surface temperature?

Star	Betelgeuse	Bellatrix	Sun	Aldebaran	Rigel
Surface Temperature (°F)	6.2×10^3	3.8×10^4	1.1×10^4	7.2×10^3	2.2×10^4



28. CORAL REEF The area of the Florida Keys National Marine Sanctuary is about 9.6×10^3 square kilometers. The area of the Florida Reef Tract is about 16.2% of the area of the sanctuary. What is the area of the Florida Reef Tract in square kilometers?

29. REASONING A gigameter is 1.0×10^6 kilometers. How many square kilometers are in 5 square gigameters?

30. WATER There are about 1.4×10^9 cubic kilometers of water on Earth. About 2.5% of the water is fresh water. How much fresh water is on Earth?

31. Critical Thinking The table shows the speed of light through five media.

- In which medium does light travel the fastest?
- In which medium does light travel the slowest?

Medium	Speed
Air	6.7×10^8 mi/h
Glass	6.6×10^8 ft/sec
Ice	2.3×10^5 km/sec
Vacuum	3.0×10^8 m/sec
Water	2.3×10^{10} cm/sec



Fair Game Review

What you learned in previous grades & lessons

Simplify. Write the expression using only positive exponents.

32. $6^3 \cdot 6^5$

33. $10^2 \cdot 10^5$

34. $8^{-1} \cdot 8^{-11}$

35. $10^{-6} \cdot 10^{-10}$

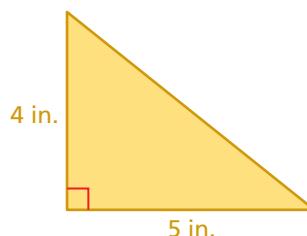
36. MULTIPLE CHOICE What is the length of the hypotenuse of the right triangle?

(A) $\sqrt{18}$ in.

(B) $\sqrt{41}$ in.

(C) 18 in.

(D) 41 in.



15.6 Lesson

Key Idea

Writing Numbers in Scientific Notation

Step 1: Move the decimal point to the right of the first nonzero digit.

Step 2: Count the number of places you moved the decimal point.
This determines the exponent of the power of 10.

Number greater than or equal to 10

Use a positive exponent when you move the decimal point to the left.

$$8600 = 8.6 \times 10^3$$

3

Number between 0 and 1

Use a negative exponent when you move the decimal point to the right.

$$0.0024 = 2.4 \times 10^{-3}$$

3

EXAMPLE 1

Writing Large Numbers in Scientific Notation



Google purchased YouTube for \$1,650,000,000.
Write this number in scientific notation.



The number is greater than 1. So, move the decimal point 9 places to the left.

$$1,650,000,000 = 1.65 \times 10^9$$

9

The exponent is positive.

EXAMPLE 2

Writing Small Numbers in Scientific Notation

The 2004 Indonesian earthquake slowed the rotation of Earth, making the length of a day 0.00000268 second shorter. Write this number in scientific notation.

The number is between 0 and 1. So, move the decimal point 6 places to the right.

$$0.00000268 = 2.68 \times 10^{-6}$$

6

The exponent is negative.

On Your Own

Now You're Ready

Exercises 3–11

Write the number in scientific notation.

1. 50,000

2. 25,000,000

3. 683

4. 0.005

5. 0.00000033

6. 0.000506

EXAMPLE

3

Standardized Test Practice

An album receives an award when it sells 10,000,000 copies.

An album has sold 8,780,000 copies. How many more copies does it need to sell to receive the award?

(A) 1.22×10^{-7}

(B) 1.22×10^{-6}

(C) 1.22×10^6

(D) 1.22×10^7

Use a model to solve the problem.

$$\begin{aligned}\text{Remaining sales needed for award} &= \frac{\text{Sales required for award}}{\text{total}} - \frac{\text{Current sales total}}{\text{total}} \\ &= \frac{10,000,000}{10,000,000} - \frac{8,780,000}{10,000,000} \\ &= 1,220,000 \\ &= 1.22 \times 10^6\end{aligned}$$

❖ The album must sell 1.22×10^6 more copies to receive the award. The correct answer is (C).

4

On Your Own

7. An album has sold 955,000 copies. How many more copies does it need to sell to receive the award? Write your answer in scientific notation.

EXAMPLE

4

Multiplying Numbers in Scientific Notation

Find $(3 \times 10^{-5}) \times (5 \times 10^{-2})$. Write your answer in scientific notation.

Study Tip

You can check your answer using standard form.

$$\begin{aligned}(3 \times 10^{-5}) \\ \times (5 \times 10^{-2}) \\ = 0.00003 \times 0.05 \\ = 0.0000015 \\ = 1.5 \times 10^{-6}\end{aligned}$$

$$(3 \times 10^{-5}) \times (5 \times 10^{-2})$$

$$= 3 \times 5 \times 10^{-5} \times 10^{-2}$$

Commutative Property of Multiplication

$$= (3 \times 5) \times (10^{-5} \times 10^{-2})$$

Associative Property of Multiplication

$$= 15 \times 10^{-7}$$

Simplify.

$$= 1.5 \times 10^1 \times 10^{-7}$$

Write factor in scientific notation.

$$= 1.5 \times 10^{-6}$$

Simplify.

4

On Your Own

Multiply. Write your answer in scientific notation.

8. $(2.5 \times 10^8) \times (2 \times 10^3)$ 9. $(2 \times 10^{-4}) \times (1 \times 10^{-4})$

10. $(5 \times 10^{-4}) \times (5.4 \times 10^{-9})$ 11. $(7 \times 10^2) \times (3 \times 10^5)$



Exercises 14–19

15.6 Exercises



Vocabulary and Concept Check

- REASONING** How do you know whether a number written in standard form will have a positive or negative exponent when written in scientific notation?
- WRITING** Describe how to write a number in scientific notation.



Practice and Problem Solving

Write the number in scientific notation.

1 2

3. 0.0021
4. 5,430,000
5. 321,000,000
6. 0.00000625
7. 0.00004
8. 10,700,000
9. 45,600,000,000
10. 0.0000000009256
11. 840,000

ERROR ANALYSIS Describe and correct the error in writing the number in scientific notation.

12.

0.000036
5
 3.6×10^5

13.

72,500,000
6
 72.5×10^6

Multiply. Write your answer in scientific notation.

4

14. $(4 \times 10^4) \times (2 \times 10^6)$
15. $(3 \times 10^{-8}) \times (3 \times 10^{-2})$
16. $(5 \times 10^{-7}) \times (3 \times 10^6)$
17. $(8 \times 10^3) \times (2 \times 10^4)$
18. $(6 \times 10^8) \times (1.4 \times 10^{-5})$
19. $(7.2 \times 10^{-1}) \times (4 \times 10^{-7})$
20. **HAIR** What is the diameter of a human hair in scientific notation?
21. **EARTH** What is the circumference of Earth in scientific notation?



Diameter: 0.000099 meter

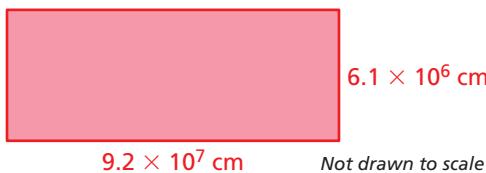


Circumference at the equator:
about 40,100,000 meters

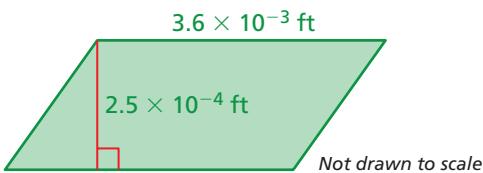
22. **SPRINGS** Every day, more than 250,000,000 gallons of water flow from Wakulla Spring near Tallahassee, Florida. Write this number in scientific notation.

Find the area of the figure. Write your answer in scientific notation.

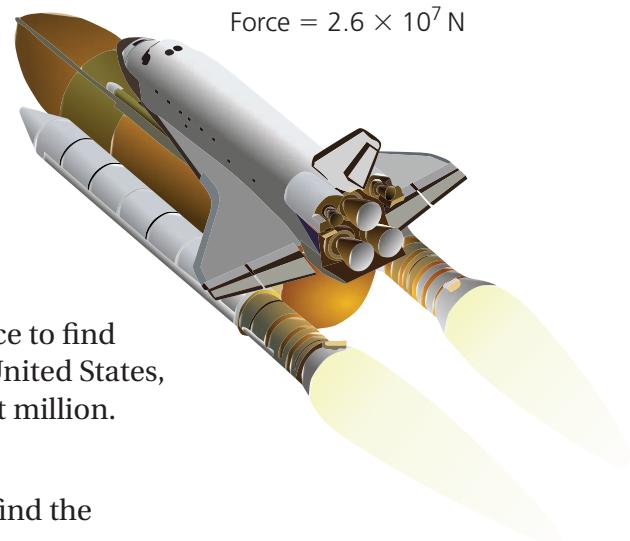
23.



24.



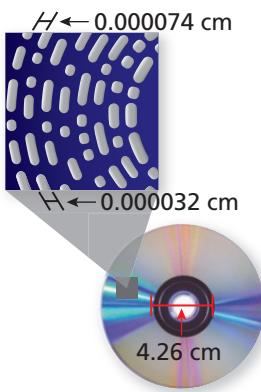
25. **SPACE SHUTTLE** The power of a space shuttle during launch is the force of the solid rocket boosters multiplied by the velocity. The velocity is 3.75×10^2 meters per second. What is the power (in newton-meters per second) of the shuttle shown during launch?



26. **NUMBER SENSE** Write 670 million in three ways.

27. **PROJECT** Use the Internet or some other reference to find the populations of India, China, Argentina, the United States, and Egypt. Round each population to the nearest million.

- Write each population in scientific notation.
- Use the Internet or some other reference to find the population density for each country.
- Use the results of parts (a) and (b) to find the area of each country.



28. **DVDS** On a DVD, information is stored on bumps that spiral around the disk. There are 73,000 ridges (with bumps) and 73,000 valleys (without bumps) across the diameter of the DVD. What is the diameter of the DVD in centimeters?

29. **Number Sense** Simplify. Write your answer in scientific notation.

a.
$$\frac{(53,000,000)(0.002)}{(0.0004)}$$

b.
$$\frac{(0.33)(60,000)}{(90,000,000)}$$



Fair Game Review What you learned in previous grades & lessons

Write and solve an equation to answer the question.

30. 15% of 60 is what number?

31. 85% of what number is 170?

32. **MULTIPLE CHOICE** What is the domain of the function represented by the table?

x	-2	-1	0	1	2
y	-6	-2	2	6	10

(A) -2, -1, 0, 1, 2

(B) -6, -2, 2, 6, 10

(C) all integers

(D) all whole numbers

15.4–15.6 Quiz

Evaluate the expression.

1. $(-4.8)^{-9} \cdot (-4.8)^9$

2. $\frac{5^4}{5^7}$

Simplify. Write the expression using only positive exponents.

3. $8d^{-6}$

4. $\frac{12x^5}{4x^7}$

Tell whether the number is written in scientific notation. Explain.

5. 23×10^9

6. 0.6×10^{-7}

Write the number in standard form.

7. 8×10^6

8. 1.6×10^{-2}

Write the number in scientific notation.

9. 0.00524

10. 892,000,000

Multiply. Write your answer in scientific notation.

11. $(9 \times 10^3) \times (4 \times 10^4)$

12. $(2 \times 10^{-5}) \times (3.1 \times 10^{-2})$

13. **PLANETS** The table shows the equatorial radii of the eight planets in our solar system.

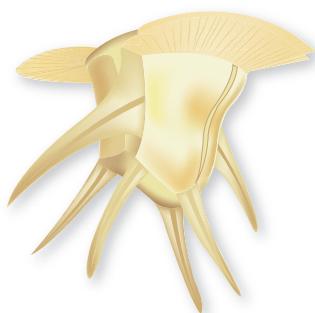
- a. Which planet has the second smallest equatorial radius?
b. Which planet has the second greatest equatorial radius?

14. **OORT CLOUD** The Oort cloud is a spherical cloud that surrounds our solar system. It is about 2×10^5 astronomical units from the Sun.

An astronomical unit is about 1.5×10^8 kilometers.

How far is the Oort cloud from the Sun in kilometers?

Planet	Equatorial Radius (km)
Mercury	2.44×10^3
Venus	6.05×10^3
Earth	6.38×10^3
Mars	3.4×10^3
Jupiter	7.15×10^4
Saturn	6.03×10^4
Uranus	2.56×10^4
Neptune	2.48×10^4



15. **ORGANISM** A one-celled, aquatic organism called a dinoflagellate is 1000 micrometers long.

- a. One micrometer is 10^{-6} meter. What is the length of the dinoflagellate in meters?
b. Is the length of the dinoflagellate equal to 1 millimeter or 1 kilometer? Explain.

15 Chapter Test

Write the product using exponents.

1. $(-15) \cdot (-15) \cdot (-15)$

2. $\left(\frac{1}{12}\right) \cdot \left(\frac{1}{12}\right) \cdot \left(\frac{1}{12}\right) \cdot \left(\frac{1}{12}\right) \cdot \left(\frac{1}{12}\right)$

Evaluate the expression.

3. -2^3

4. $10 + 3^3 \div 9$

Simplify the expression. Write your answer as a power.

5. $9^{10} \cdot 9$

6. $\frac{(-3.5)^{13}}{(-3.5)^9}$

Evaluate the expression.

7. $5^{-2} \cdot 5^2$

8. $\frac{-8}{(-8)^3}$

Write the number in standard form.

9. 3×10^7

10. 9.05×10^{-3}

Multiply. Write your answer in scientific notation.

11. $(7 \times 10^3) \times (5 \times 10^2)$

12. $(3 \times 10^{-5}) \times (2 \times 10^{-3})$



13. **HAMSTER** A hamster toy is in the shape of a sphere. The volume V of a sphere is represented by $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere. What is the volume of the toy? Round your answer to the nearest cubic centimeter. Use 3.14 for π .
14. **CRITICAL THINKING** Is $(xy^2)^3$ the same as $(xy^3)^2$? Explain.

15. **RICE** A grain of rice weighs about 3^3 milligrams. About how many grains of rice are in one scoop?



16. **TASTE BUDS** There are about 10,000 taste buds on a human tongue. Write this number in scientific notation.

17. **LEAD** From 1978 to 2008, the amount of lead allowed in the air in the United States was 1.5×10^{-6} gram per cubic meter. In 2008, the amount allowed was reduced by 90%. What is the new amount of lead allowed in the air?

One scoop of rice weighs about 3^9 milligrams.