# 2.5 Systems of Linear Equations



# **Essential Question** How can you solve a system of linear equations?

**ACTIVITY:** Writing a System of Linear Equations

Work with a partner.

Your family starts a bed-and-breakfast in your home. You spend \$500 fixing up a bedroom to rent. Your cost for food and utilities is \$10 per night. Your family charges \$60 per night to rent the bedroom.



**a.** Write an equation that represents your costs.

Cost, C	_	\$10 per		Number of	+	\$500
(in dollars)		night	·	nights, <i>x</i>	'	<i>ф</i> 300

**b.** Write an equation that represents your revenue (income).

Revenue, R	_	\$60 per		Number of
(in dollars)		night	Ū	nights, <i>x</i>

**c.** A set of two (or more) linear equations is called a **system of linear equations**. Write the system of linear equations for this problem.

2

### **ACTIVITY: Using a Table to Solve a System**

Use the cost and revenue equations from Activity 1 to find how many nights you need to rent the bedroom before you recover the cost of fixing up the bedroom. This is the *break-even point* for your business.

**a.** Copy and complete the table.

x	0	1	2	3	4	5	6	7	8	9	10	11
С												
R												

b. How many nights do you need to rent the bedroom before you break even?

### ACTIVITY: Using a Graph to Solve a System

**a.** Graph the cost equation from Activity 1.

3

- **b.** In the same coordinate plane, graph the revenue equation from Activity 1.
- **c.** Find the point of intersection of the two graphs. The *x*-value of this point is the number of nights you need to rent the bedroom to break even.



### ACTIVITY: Using an Equation to Solve a System

**a.** Write the cost equation from Activity 1.



**b.** Write the revenue equation from Activity 1.



- **c.** The break-even point occurs when C = R. Set the expression for *C* equal to the expression for *R*. You should obtain an equation with *x* on both sides. Solve this equation for *x*. The solution is your break-even point.
- **d.** Did you obtain the same break-even point in Activities 2, 3, and 4? If not, check your work. The break-even point should be the same in all three activities.

### -What Is Your Answer?

- 5. IN YOUR OWN WORDS How can you solve a system of linear equations?
- **6.** When solving a system of linear equations, explain why it is a good idea to use two different ways to find the solution.



Use what you learned about systems of linear equations to complete Exercises 3 and 4 on page 80.

### 2.5 Lesson



				BigId	easMo	ath 🔪	com
Key Vocabulary () system of linear equations, <i>p. 78</i> solution of a system of linear equations, <i>p. 78</i>	A <b>system of linear equ</b> same variables. A <b>solut</b> is an ordered pair that	<b>ations</b> is a set o <b>ion of a syster</b> makes each eq	of two or mor <mark>n of linear eq</mark> juation true.	e linear <mark>Juations</mark>	equa <mark>s</mark> in tw	tions 70 var	in the iables
EXAMPLE	Solving a Syste	m of Linear	Equations	Using	a Tal	ole	
Reading	Solve the system.	y = x - 5 $y = -3x + 7$	Equation 1 Equation 2	2			
A system of linear	<b>Step 1:</b> Make a table	e of values.	x	0	1	2	3
equations is also called a <i>linear system</i> .	<b>Step 2:</b> Find an <i>x</i> -va	lue that	y = x - 5	-5	-4	-3	-2
	for both equ	ations.	y = -3x + 2	7 7	4	1	-2
	∴ The solution is (3)	, -2).		·			
EXAMPLE	2 Solving a Syste	em of Linear	Equations	Using	a Gr	aph	
	Solve the system.	v = 2x + 3	Equation 1				
	,	y = -x + 6	Equation 2	2			
	Step 1: Graph each	y = -x + 6 equation.	Equation 2	2			
	Step 1: Graph each Step 2: Find the poi graphs appe	y = -x + 6 equation. nt of intersecti ar to intersect	Equation 2 fon. The at (1, 5).	2	y = 2x	<u>(+3</u>	
	Step 1: Graph each Step 2: Find the poi graphs appe Step 3: Check your s	y = -x + 6 equation. nt of intersecti ar to intersect solution.	Equation 2 ion. The at (1, 5).	2	y = 2x	(+3)	
	<ul> <li>Step 1: Graph each</li> <li>Step 2: Find the poi graphs appe</li> <li>Step 3: Check your section 1</li> </ul>	y = -x + 6 equation. nt of intersecti ar to intersect solution. Equation 2	Equation 2 fon. The at (1, 5).		y = 2x	(+3)	
	Step 1: Graph each Step 2: Find the poingraphs apper Step 3: Check your s Equation 1 $y = 2x + 3$	y = -x + 6 equation. nt of intersection ar to intersection solution. Equation y = -x + 2	Equation 2 fon. The at (1, 5).		y = 2x	(+3) y = -	-x+6
	Step 1: Graph each Step 2: Find the poingraphs apper Step 3: Check your s Equation 1 y = 2x + 3 $5 \stackrel{?}{=} 2(1) + 3$	y = -x + 6 equation. nt of intersection ar to intersection solution. Equation 2 y = -x + 6 $y = -x + 6$	Equation 2 fon. The at (1, 5).	2	y = 2x	(+3) y = -	-x + 6
	Step 1: Graph each Step 2: Find the poi graphs appe Step 3: Check your s Equation 1 y = 2x + 3 $5 \stackrel{?}{=} 2(1) + 3$ 5 = 5	y = -x + 6 equation. nt of intersection ar to intersection solution. Equation 2 y = -x + 6 $y = -x + 6$ $5 = -1 + 6$ $5 = 5 = 6$	Equation 2 foon. The at $(1, 5)$ .		y = 2x y (1, 5 1 2	(x+3)	-x + 6
	Step 1: Graph each Step 2: Find the poi graphs appe Step 3: Check your s Equation 1 y = 2x + 3 $5 \stackrel{?}{=} 2(1) + 3$ 5 = 5 The solution is (1)	y = -x + 6 equation. nt of intersection ar to intersection solution. Equation 2 y = -x + 6 y = -x + 6 y = -x + 6 5 = -1 + 6 5 = 5 = 5	Equation 2 fon. The at $(1, 5)$ .		y = 2x y (1, 5 1 2	x + 3	-x + 6
	Step 1: Graph each Step 2: Find the poi graphs appe Step 3: Check your s Equation 1 y = 2x + 3 $5 \stackrel{?}{=} 2(1) + 3$ 5 = 5 $\therefore$ The solution is (1) On Your Own	y = -x + 6 equation. nt of intersection ar to intersection solution. Equation 2 $y = -x + \frac{5}{2} = -1 = \frac{2}{5} = 5$ (2, 5).	Equation 2 fon. The at $(1, 5)$ .		y = 2x y (1, 5 1 2	x + 3	
Now You're Rend.	Step 1: Graph each Step 2: Find the poi graphs appe Step 3: Check your s Equation 1 y = 2x + 3 $5 \stackrel{?}{=} 2(1) + 3$ 5 = 5 $\therefore$ The solution is (1) On Your Own Solve the system of	y = -x + 6 equation. nt of intersection ar to intersection solution. Equation 2 y = -x + 6 5 = -1 + 2 5 = 5 = 5 e., 5).	Equation 2 fon. The at $(1, 5)$ .	2 7 7 6 6 5 4 3 7 1 1 7 1 4 3 1	y = 2x	(+3) y =	-x + 6
Now You're Ready Exercises 5-7	Step 1: Graph each Step 2: Find the poi graphs appe Step 3: Check your s Equation 1 y = 2x + 3 $5 \stackrel{?}{=} 2(1) + 3$ 5 = 5 $\therefore$ The solution is (1) On Your Own Solve the system of 1. $y = x - 1$	y = -x + 6 equation. nt of intersection ar to intersection solution. Equation 2 y = -x + 5 $5 = -1 + 5$ $5 = 5$ (1) (1) Equation 2 (1) (2) (2) (3) (4) (4) (4) (4) (4) (4) (4) (4) (4) (4	Equation 2 fon. The at $(1, 5)$ .	2 7 6 6 5 4 3 7 1 1 7 1 4 3 1 1 1 1 9 1 9	y = 2x $y = 1$	(+3) y =	-x + 6



#### Solving a System of Linear Equations Algebraically

- **Step 1** Solve both equations for one of the variables.
- **Step 2** Set the expressions equal to each other and solve for the variable.
- **Step 3** Substitute back into one of the original equations and solve for the other variable.

#### EXAMPLE

3

#### Solving a System of Linear Equations Algebraically

A middle school yearbook committee has 35 members. There are 7 more girls than boys. Use the models to write a system of linear equations. Then solve the system to find the number of boys *x* and the number of girls *y*.



The system is x + y = 35 and y = x + 7.

y = 35 - x

**Step 1:** Solve x + y = 35 for *y*.

Subtract *x* from each side.

**Step 2:** Set the expressions equal to each other and solve for *x*.

$\frac{35-x}{x} = x+7$	Set expressions equal to each other.
28 = 2x	Subtract 7 from each side. Add x to each side.
14 = x	Divide each side by 2.



ow You're Ready

Exercises 13–15

**Step 3:** Substitute x = 14 into one of the original equations and solve for *y*.

y = x + 7	Write one of the original equations.
= 14 + 7	Substitute 14 for <i>x</i> .
= 21	Add.

There are 14 boys and 21 girls on the yearbook committee.

#### On Your Own

**4. WHAT IF?** In Example 3, the yearbook committee has 45 members. Use the models to write a system of linear equations. Then solve the system to find the number of boys *x* and the number of girls *y*.



## 2.5 Exercises



### Vocabulary and Concept Check

- **1. VOCABULARY** Do the equations 4a 3b = 5 and 7b + 2a = -8 form a system of linear equations? Explain.
- **2. REASONING** Can a point in Quadrant II be a break-even point for a system? Explain.

### Practice and Problem Solving

#### Use the table to find the break-even point. Check your solution.

**3.** C = 15x + 150R = 45x **4.** C = 24x + 80

R = 44x

x	0	1	2	3	4	5	6
С							
R							

x	0	1	2	3	4	5	6
С							
R							

0

#### Solve the system of linear equations using a table.

**1 5.** 
$$y = x + 4$$
  
 $y = 3x - 1$ 
**6.**  $y = 1.5x - 2$ 
**7.**  $y = \frac{2}{3}x - 3$   
 $y = -x + 13$ 
**7.**  $y = \frac{2}{3}x - 3$   
 $y = -2x + 5$ 

**8. ERROR ANALYSIS** Describe and correct the error in solving the system of linear equations.

x	0	1	2	3	
y=-2x-1	-1	-3	-5	-7	
y = x - 7	-7	-6	-5	-4	
The solution is $(-5, -5)$ .					

- **9. CARRIAGE RIDES** The cost *C* (in dollars) for the care and maintenance of a horse and carriage is C = 15x + 2000, where *x* is the number of rides.
  - **a.** Write an equation for the revenue *R* in terms of the number of rides.
  - **b.** How many rides are needed for the business to break even?



Solve the system of linear equations using a graph.

2 10. 
$$y = 2x + 9$$
  
 $y = 6 - x$   
11.  $y = -x - 4$   
 $y = \frac{3}{5}x + 4$   
12.  $y = 2x + 5$   
 $y = \frac{1}{2}x - 1$ 

Solve the system of linear equations algebraically.

<b>3 13.</b> $x + y = 27$	<b>14.</b> $y - x = 17$	<b>15.</b> $x - y = 7$
y = x + 3	y = 4x + 2	0.5x + y = 5

**16. HOMEWORK** You have 42 math and science problems for homework. You have 10 more math problems than science problems. Use the model to write a system of linear equations. How many problems do you have in each subject?



- **17. CANOEING** You and your friend are canoeing on the Withlacoochee River Canoe Trail. Your position on the trail *y* (in miles) is represented by y = 3.5x + 28, where *x* is in hours. Your friend's position is represented by y = 2x + 37.
  - **a.** How long will it take you to catch up with your friend?
  - **b.** How far will you each have traveled when you catch up with your friend?

**18.** You buy *x* bottles of face paint and *y* brushes at two stores. The amounts you spend are represented by 10x + 7.5y = 42.5 and 8x + 6y = 34. How many bottles of face paint and brushes did you buy?



### Fair Game Review What you learned in previous grades & lessons

#### Decide whether the two equations are equivalent.

**19.** 4n + 1 = n - 8**20.** 2a + 6 = 12**21.**  $7v - \frac{3}{2} = 5$ 3n = -9a + 3 = 614v - 3 = 15

**22.** MULTIPLE CHOICE Which line has the same slope as  $y = \frac{1}{2}x - 3$ ?

(A) y = -2x + 4 (B) y = 2x + 3 (C) y - 2x = 5 (D) 2y - x = 7