7.1 Measures of Central Tendency

Essential Question How can you use measures of central tendency to distribute an amount evenly among a group of people?

1 ACTIVITY: Exploring Mean, Median, and Mode

Work with a partner. Forty-five coins are arranged in nine stacks.

a. Record the number of coins in each stack in a table.

Stack	1	2	3	4	5	6	7	8	9
Coins									

b. Find the mean, median, and mode of the number of coins in each stack.

c. By moving coins from one stack to another, can you change the mean? the median? the mode? Explain.

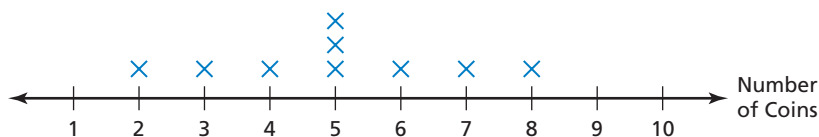d. Is it possible to arrange the coins in stacks so that the median is 6? 8? Explain.

2 EXAMPLE: Drawing a Line Plot

Work with a partner.

a. Draw a number line. Label the tick marks from 1 to 10.

b. Place each stack of coins in Activity 1 above the number of coins in the stack.
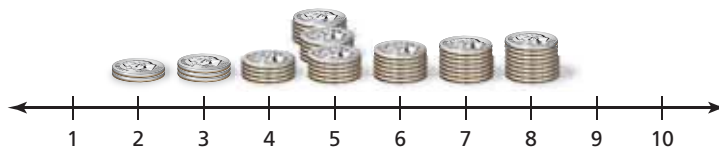
c. Draw an ✕ to represent each stack. This graph is called a *line plot*.

ACTIVITY: Fair and Unfair Distributions

Work with a partner.

A distribution of coins to nine people is considered *fair* if each person has the same number of coins.

● Distribute the 45 coins into 9 stacks using a fair distribution. How is this distribution related to the mean?

● Draw a line plot for each distribution. Which distributions seem most fair? Which distributions seem least fair? Explain your reasoning.

a.

b.

c.

d.

e.

f.

What Is Your Answer?

4. IN YOUR OWN WORDS How can you use measures of central tendency to distribute an amount evenly among a group of people?

5. Use the Internet or some other reference to find examples of mean or median incomes of groups of people. Describe possible distributions that could produce the given means or medians.

Practice

Use what you learned about measures of central tendency to complete Exercise 4 on page 278.

A **measure of central tendency** is a measure that represents the center of a data set. The *mean*, *median*, and *mode* are measures of central tendency.

Key Vocabulary 🔊
measure of central tendency, *p. 276*

🔑 **Key Ideas**

Mean

The *mean* of a data set is the sum of the data divided by the number of data values.

Median

Order the data. For a set with an odd number of values, the *median* is the middle value. For a set with an even number of values, the *median* is the mean of the two middle values.

Mode

The *mode* of a data set is the value or values that occur most often.

Remember 💡

Data can have one mode, more than one mode, or no mode. When each value occurs only once, there is no mode.

EXAMPLE 1   **Finding the Mean, Median, and Mode**

Students' Hourly Wages	
$3.87	$7.25
$8.75	$8.45
$8.25	$7.25
$6.99	$7.99

An amusement park hires students for the summer. The students' hourly wages are given in the table. Find the mean, median, and mode of the hourly wages.

Mean: sum of the data → $\dfrac{58.8}{8} = 7.35$
number of values →

Median: 3.87, 6.99, 7.25, <u>7.25, 7.99</u>, 8.25, 8.45, 8.75   Order the data.

$$\frac{15.24}{2} = 7.62$$ Mean of two middle values

Mode: 3.87, 6.99, 7.25, 7.25, 7.99, 8.25, 8.45, 8.75

The value 7.25 occurs most often.

∴ The mean is $7.35, the median is $7.62, and the mode is $7.25.

🔴 **On Your Own**

Now You're Ready
Exercises 5–8

1. **WHAT IF?** In Example 1, the park hires another student at an hourly wage of $6.99. How does this additional value affect the mean, median, and mode? Explain.

EXAMPLE **2** **Removing an Outlier**

Identify the outlier in Example 1. How does the outlier affect the mean, median, and mode?

The value $3.87 is low compared to the other wages. It is the outlier.

Find the mean, median, and mode without the outlier.

Mean: $\dfrac{54.93}{7} \approx 7.85$

Median: 6.99, 7.25, 7.25, 7.99, 8.25, 8.45, 8.75   The middle value, 7.99, is the median.

Mode: 6.99, 7.25, 7.25, 7.99, 8.25, 8.45, 8.75   The mode is 7.25.

⋰ By removing the outlier, the mean increases $7.85 − $7.35 = $0.50, the median increases $7.99 − $7.62 = $0.37, and the mode is the same.

EXAMPLE **3** **Changing the Values of a Data Set**

In Example 1, each hourly wage increases $0.40. How does this increase affect the mean, median, and mode?

Students' Hourly Wages	
$4.27	$7.65
$9.15	$8.85
$8.65	$7.65
$7.39	$8.39

Make a new table by adding $0.40 to each hourly wage.

Mean: $\dfrac{62}{8} = 7.75$

Median: 4.27, 7.39, 7.65, 7.65, 8.39, 8.65, 8.85, 9.15   Order the data.

$\dfrac{16.04}{2} = 8.02$ Mean of two middle values

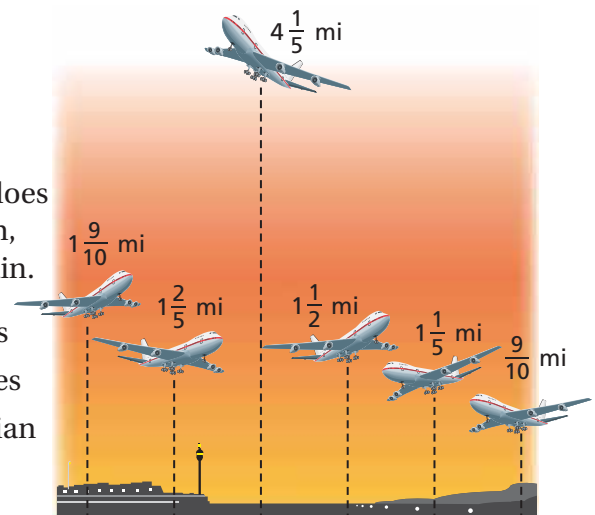**Mode:** 4.27, 7.39, 7.65, 7.65, 8.39, 8.65, 8.85, 9.15   The mode is 7.65.

⋰ By increasing each hourly wage $0.40, the mean, median, and mode all increase $0.40.

🔴 **On Your Own**

The figure shows the altitudes of several airplanes.

2. Identify the outlier. How does the outlier affect the mean, median, and mode? Explain.

3. Each airplane increases its altitude $1\dfrac{1}{2}$ miles. How does this affect the mean, median and mode? Explain.

$4\dfrac{1}{5}$ mi

$1\dfrac{9}{10}$ mi

$1\dfrac{2}{5}$ mi

$1\dfrac{1}{2}$ mi

$1\dfrac{1}{5}$ mi

$\dfrac{9}{10}$ mi

✓ Vocabulary and Concept Check

1. **VOCABULARY** Can a data value be an outlier *and* a measure of central tendency of the same data set? Explain.

2. **OPEN-ENDED** Create a data set that has more than one mode.

3. **WRITING** Describe how removing an outlier from a data set affects the mean of the data set.

Practice and Problem Solving

4. Draw a line plot of the data. Then find the mean, median, and mode of the data.

Bag	1	2	3	4	5	6	7	8	9
Strawberries	10	13	11	15	8	14	7	11	12

Find the mean, median, and mode of the data.

5.

Golf Scores		
3	−2	1
6	4	−1
−3	−1	2

6.

Changes in Stock Value (dollars)			
1.05	2.03	−1.78	−2.41
−2.64	0.67	4.02	1.39
0.66	−0.38	−3.01	2.20

7.

Movie lengths (hours)
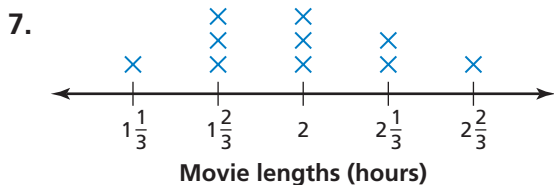
8. **Available Memory**

Stem	Leaf
6	5
7	0 5 5
8	0 4 5
9	4

Key: 7│5 = 75 megabytes

9. **ERROR ANALYSIS** Describe and correct the error in finding the median.

✗ Test scores: 98, 90, 80, 80, 90, 90

The median is $\dfrac{528}{6} = 88$.

10. **POLAR BEARS** The table shows the masses of eight polar bears. Find the mean, median, and mode of the masses.

Masses (kilograms)			
455	262	471	358
364	553	352	467

Find the value of *x*.

11. Mean is 6; 2, 8, 9, 7, 6, *x*

12. Mean is 0; 11.5, 12.5, -10, -7.5, *x*

13. Median is 14; 9, 10, 12, *x*, 20, 25

14. Median is 51; 30, 45, *x*, 100

2 **15.** **TEMPERATURES** An environmentalist records the average temperatures of five regions.

    a. Identify the outlier.

    b. Which measure of central tendency will be most affected by removing the outlier?
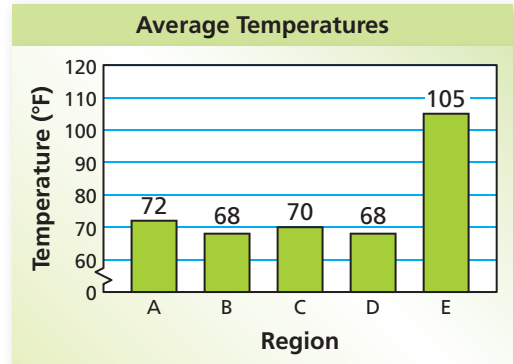
16. **RIVER** The Suwannee River Trail has 11 segments in Florida. The lengths (in miles) of the segments are 8.9, 9.7, 6, 2.2, 19.6, 15, 7.6, 0.5, 14.2, 14.6, and 22.4.

    a. Find the mean, median, and mode of the lengths.

    b. A canoeist skips the first segment of the trail. How does this affect the mean, median, and mode of the remaining lengths? Explain.

3 **17.** **REASONING** The value of each stock in Exercise 6 decreases $0.05. How does this affect the mean, median, and mode? Explain.

18. **Critical Thinking** The circle graph shows the ages of 200 students in a college psychology class.

    a. Find the mean, median, and mode of the students' ages.

    b. Identify the outliers. How do the outliers affect the mean, median, and mode?

College Student Ages

20 yrs 14% *21 yrs 20%* *19 yrs 30%* *37 yrs 1%* *18 yrs 35%*
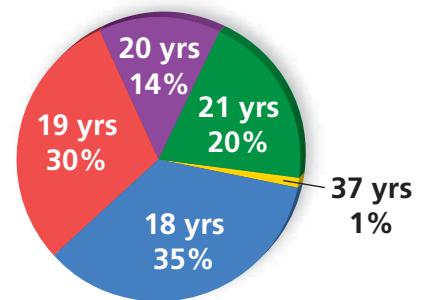
Fair Game Review *What you learned in previous grades & lessons*

Order the values from least to greatest.

19. 1, -3, -8, 4, 7, -5

20. 1.2, -2.8, $\dfrac{3}{2}$, 5.4, -4.7, $-\dfrac{2}{3}$

21. **MULTIPLE CHOICE** Which equation represents a linear function?

    A $y = x^2$ **B** $y = 2x$ **C** $y = \dfrac{2}{x}$ **D** $xy = 2$