English Spanish

B.2 Formula for Compound Interest

STATE STANDARDS
MA.8.A.6.3
MA.8.A.6.4

Essential Question How can you use a formula to find the balance in an account that earns compound interest?

1 ACTIVITY: Finding a Formula for Compound Interest

Work with a partner. You deposit $1000 in a savings account that earns 6% interest compounded annually. Explain why parts (a)–(d) in the table are true.

a. Sample:

$$\text{Balance at End of Year} = \text{Principal} + \text{Annual Interest}$$
$$= \$1000 + \$1000(0.06)$$
$$= \$1000(1 + 0.06)$$
$$= \$1000(1.06)$$

t	Principal and Interest	Annual Interest	Balance at End of Year
1	$1000.00	$1000(0.06)	**a.** $B = \$1000(1.06)$
2	$1000(1.06)	**b.** $1000(1.06)(0.06)	**c.** $B = \$1000(1.06)^2$
3	$1000(1.06)^2$	$1000(1.06)^2(0.06)$	$B = \$1000(1.06)^3$
4	$1000(1.06)^3$	$1000(1.06)^3(0.06)$	$B = \$1000(1.06)^4$
5	$1000(1.06)^4$	$1000(1.06)^4(0.06)$	**d.** $B = \$1000(1.06)^5$
6	$1000(1.06)^5$	$1000(1.06)^5(0.06)$	$B = \$1000(1.06)^6$
7	$1000(1.06)^6$	$1000(1.06)^6(0.06)$	$B = \$1000(1.06)^7$
8	$1000(1.06)^7$	$1000(1.06)^7(0.06)$	$B = \$1000(1.06)^8$
9	$1000(1.06)^8$	$1000(1.06)^8(0.06)$	$B = \$1000(1.06)^9$
10	$1000(1.06)^9$	$1000(1.06)^9(0.06)$	$B = \$1000(1.06)^{10}$

e. Use the pattern in the table to find the balance after 20 years.

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2 ACTIVITY: Writing a Formula

Work with a partner. Use the pattern in Activity 1 to write a formula for the balance in an account that earns interest compounded annually after t years.

P = Principal (initial deposit)

r = Annual interest rate (in decimal form)

t = Time (in years)

B = Balance after t years

$B =$

3 ACTIVITY: A Penny Saved

Work with a partner. In his will, Benjamin Franklin gave $4000 to the state of Pennsylvania. He instructed that the money be invested for 200 years. After 200 years, the money should be used to do good. Franklin died in 1790. In 1990, his gift had grown to over $2 million.

Use your formula from Activity 2 to copy and complete the table. Then approximate the annual interest rate that Benjamin Franklin's gift earned.

"A penny saved is a penny earned." Benjamin Franklin

Annual Interest Rate	Balance After 200 years
3.0%	
3.1%	
3.2%	
3.3%	
3.4%	

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use a formula to find the balance in an account that earns compound interest? How much would Benjamin Franklin's account have been worth if it had earned 6% interest each year?

Practice Use what you learned about the formula for compound interest to complete Exercises 3 and 4 on page A20.

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B.2 Lesson

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Key Idea

Interest Compounded Annually

$$B = P(1 + r)^t$$

- B: Balance
- P: Principal
- r: Annual interest rate (in decimal form)
- t: Time (in years)

EXAMPLE 1 Finding an Account Balance

You deposit $1000 in a savings account. The account earns 3% interest compounded annually. (a) What is the balance after 2 years? (b) What is the interest earned?

a. $B = P(1 + r)^t$ Write compound interest formula.

$= 1000(1 + 0.03)^2$ Substitute.

$= 1000(1.03)^2$ Add.

$= 1060.9$ Simplify.

∴ The balance is $1060.90 after 2 years.

b. The interest earned is the difference of the balance and the principal.

∴ So, the interest earned is $1060.90 − $1000.00 = $60.90.

Study Tip

For small values of t, you can use the simple interest formula to estimate the interest.

$I = Prt$
$= 1000(0.03)(2)$
$= 60$

EXAMPLE 2 Finding a Principal

An account opened on January 1, 2010 earns 5% interest compounded annually. The only change in the account is interest earned. What is the principal?

Descartes the Cat
9 Lives Lane
Meow Town, FL

Account No. 3-141592654

Date	Description	Deposits	Balance
01/01/2012	Previous Balance		$8820.00
12/31/2012	Interest Earned	$441.00	$9261.00

The balance is $9261.00 after 3 years.
Use the formula to find the principal.

$B = P(1 + r)^t$ Write formula.

$9261.00 = P(1 + 0.05)^3$ Substitute.

$9261.00 = P(1.05)^3$ Add.

$8000 = P$ Solve for P.

∴ The principal is $8000.

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On Your Own

Now You're Ready
Exercises 5 and 6

1. You deposit $2500 in a savings account. The account earns 2% interest compounded annually. (a) What is the balance after 3 years? (b) What is the interest earned?
2. An account earns 1% interest compounded annually. The balance after 2 years is $10,201. What is the principal?

EXAMPLE 3 Choosing an Account

A business owner wants to invest $10,000 for 10 years. Which account should the business owner choose? Explain.

Account A

Account B

Account A

$B = P(1 + rt)$ Write simple interest balance formula.

$= 10{,}000(1 + 0.05(10))$ Substitute.

$= 15{,}000$ Simplify.

The balance after 10 years is $15,000.

Account B

$B = P(1 + r)^t$ Write compound interest formula.

$= 10{,}000(1 + 0.05)^{10}$ Substitute.

$= 10{,}000(1.05)^{10}$ Add.

$\approx 16{,}288.95$ Use a calculator.

The balance after 10 years is $16,288.95.

∴ The business owner should choose Account B because its balance is greater than the balance of Account A after 10 years.

On Your Own

Now You're Ready
Exercises 8 and 9

3. **WHAT IF?** In Example 3, which account should the business owner choose if the money is being invested for 5 years? Explain.

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B.2 Exercises

Check It Out
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Vocabulary and Concept Check

1. **WRITING** For a given interest rate, how does compound interest differ from simple interest?

2. **DIFFERENT WORDS, SAME QUESTION** You deposit $1000 in an account that earns 4% interest compounded annually. Which question is different? Find "both" answers.

What is the balance after 1 year?

What is the interest earned after 1 year?

By how much money did your account increase after 1 year?

What is the difference of the balance and the principal after 1 year?

Practice and Problem Solving

Copy and complete the table for an account that earns interest compounded annually.

3. Principal: $500

Annual Interest Rate	Balance After 5 Years
3.0%	
3.5%	
4.0%	
4.5%	

4. Principal: $1000

Annual Interest Rate	Balance After 10 Years
3.5%	
4.0%	
4.5%	
5.0%	

1 5. You deposit $600 in a savings account. The account earns 3.5% interest compounded annually.

   a. What is the balance after 4 years?

   b. What is the interest earned?

2 6. An account earns 1.5% interest compounded annually. The balance after 2 years is $8241.80. What is the principal?

7. **ERROR ANALYSIS** Describe and correct the error in finding the balance of a savings account with interest compounded annually.

Principal: $2000
Annual Interest Rate: 4%
Time: 2 years

$B = P(1 + r)^t$
$= 2000(1 + 4)^2$
$= 2000(5)^2$
$= 50{,}000$

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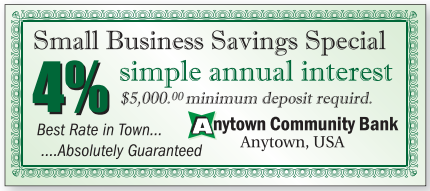
A business owner wants to invest $7000 for 8 years. Which account should the business owner choose? Explain.

8. Account A

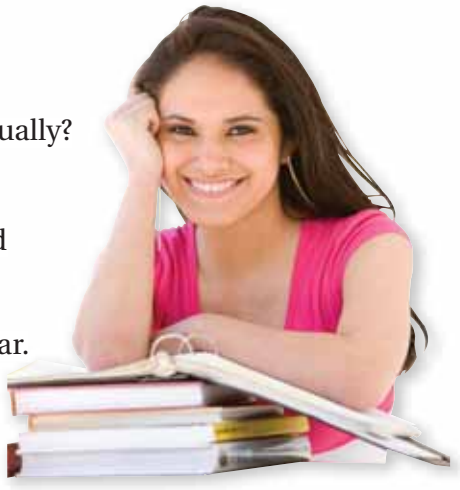
Account B

9. Account A



Account B

10. COLLEGE You want to save $13,000 in 4 years for college tuition. About how much money should you deposit in a savings account that earns 5% interest compounded annually?



11. CRITICAL THINKING Interest can be compounded more than once per year. The formula for interest compounded more than once per year is $B = P\left(1 + \frac{r}{n}\right)^{nt}$, where n is the number of times the balance is compounded each year.

a. You deposit $1000 in a savings account. The account earns 3% interest compounded 4 times per year. What is the balance after 2 years?

b. What happens to the balance after 2 years as n increases?

12. Critical Thinking Two accounts have the same interest rate, but one account earns interest compounded annually and the other earns interest compounded monthly. Which account earns more interest? Explain.

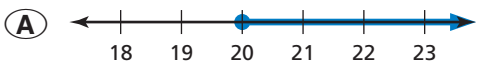
Fair Game Review What you learned in previous grades & lessons

Simplify the expression. SKILLS REVIEW HANDBOOK

13. $465.25 \cdot 60$ **14.** $\frac{1}{12} \cdot 0.036$ **15.** $\frac{4.8}{12}$ **16.** $268.68 \cdot 72$

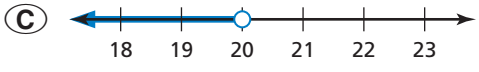
17. MULTIPLE CHOICE Which graph represents the solution of the inequality $-\frac{c}{4} \geq -5$? SECTION 8.3

A



B

C



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