Essential Question How can you use division by a mixed number as part of a story?

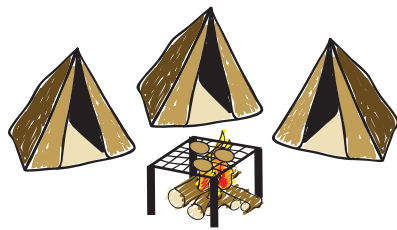Share Your Work at...
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1 EXAMPLE: Writing a Story

**Write a story that uses the division problem $6 \div 1\frac{1}{2}$. Draw pictures for your story.**

There are many possible stories. Here is one about a camping trip.

Joe goes on a camping trip with his aunt, his uncle, and three cousins. They leave at 5:00 P.M. and drive 2 hours to the campground.

Joe helps his uncle put up three tents. His aunt cooks hamburgers on a grill that is over a fire.

In the morning, Joe tells his aunt that he is making pancakes for everyone. He decides to triple the recipe so there will be plenty of pancakes for everyone. A single recipe uses 2 cups of water, so he needs a total of 6 cups.

Pancake Mix
Recipe:
2 cups water
2 cups pancake mix
1/4 cup oil
1 egg
1/4 teaspoon salt
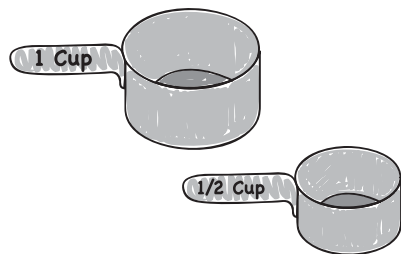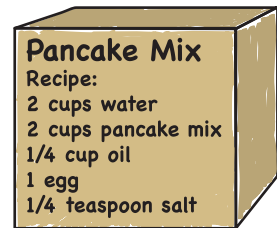
Joe's aunt has a 1-cup measuring cup and a ½-cup measuring cup. The water faucet is about 50 yards from the campsite. Joe tells his cousins that he can get 6 cups of water in only 4 trips.

1 Cup

1/2 Cup

When his cousins ask him how he knows that, he uses a stick to draw a diagram in the dirt. Joe says "This diagram shows that there are four 1½'s in 6." In other words,

$$6 \div 1\frac{1}{2} = 4.$$

Cups — Cups per trip — Trips
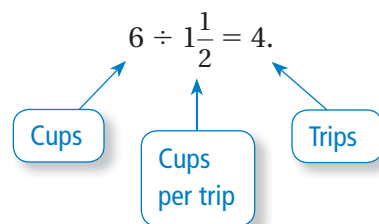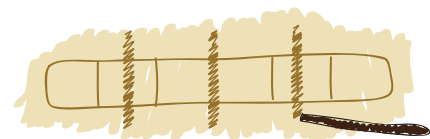
2 EXAMPLE: Dividing by a Mixed Number

Show how Joe solves the division problem in Example 1.

$$6 \div 1\frac{1}{2} = \frac{6}{1} \div \frac{3}{2}$$ Rewrite 6 as $\frac{6}{1}$ and $1\frac{1}{2}$ as $\frac{3}{2}$.

$$= \frac{6}{1} \times \frac{2}{3}$$ Multiply by the reciprocal of $\frac{3}{2}$, which is $\frac{2}{3}$.

$$= \frac{6 \times 2}{1 \times 3}$$ Multiply fractions.

$$= \frac{12}{3}, \text{ or } 4$$ Simplify.

3 ACTIVITY: Writing a Story

Work with a partner. Think of a story that uses division by a mixed number.

$$\boxed{} \div \boxed{\dfrac{}{}}$$

Whole number, fraction, or mixed number
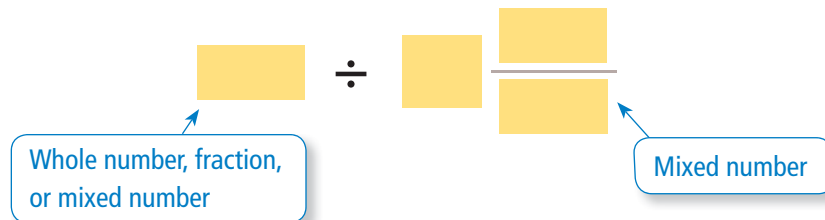
Mixed number

a. Write your story. Then draw pictures for your story.

b. Solve the division problem and use the answer in your story. Include a diagram of the division problem.

What Is Your Answer?

4. IN YOUR OWN WORDS How can you use division by a mixed number as part of a story?

In Example 1, the units of the answer are *trips*.

$$\text{Cups} \div \frac{\text{Cups}}{\text{Trips}} = \text{Cups} \times \frac{\text{Trips}}{\text{Cups}}$$

$$= \text{Cups} \times \frac{\text{Trips}}{\text{Cups}} = \text{Trips}$$

Find the units for the following division problems.

5. $\text{Miles} \div \dfrac{\text{Miles}}{\text{Hour}}$

6. $\text{Dollars} \div \dfrac{\text{Dollars}}{\text{Hour}}$

7. Miles ÷ Hour

8. Dollars ÷ Hour

Practice

Use what you learned about dividing mixed numbers to complete Exercises 5–12 on page 82.

🔑 Key Idea

Dividing Mixed Numbers

Write each mixed number as an improper fraction. Then divide as you would with proper fractions.

EXAMPLE 1 **Dividing a Mixed Number by a Fraction**

Find $4\frac{1}{2} \div \frac{3}{8}$.   **Estimate** $5 \div \frac{1}{2} = 10$

$4\frac{1}{2} \div \frac{3}{8} = \frac{9}{2} \div \frac{3}{8}$ Write $4\frac{1}{2}$ as the improper fraction $\frac{9}{2}$.

$= \frac{9}{2} \times \frac{8}{3}$ Multiply by the reciprocal of $\frac{3}{8}$, which is $\frac{8}{3}$.

$= \frac{\overset{3}{\cancel{9}} \times \overset{4}{\cancel{8}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{3}}}$ Multiply fractions. Divide out common factors.

$= 12$ Simplify.

⋰ So, the quotient is 12.   **Reasonable?** $12 \approx 10$ ✓

EXAMPLE 2 **Dividing Mixed Numbers**

Find $3\frac{5}{6} \div 1\frac{2}{3}$.   **Estimate** $4 \div 2 = 2$

$3\frac{5}{6} \div 1\frac{2}{3} = \frac{23}{6} \div \frac{5}{3}$ Write each mixed number as an improper fraction.

$= \frac{23}{6} \times \frac{3}{5}$ Multiply by the reciprocal of $\frac{5}{3}$, which is $\frac{3}{5}$.

$= \frac{23 \times \overset{1}{\cancel{3}}}{\underset{2}{\cancel{6}} \times 5}$ Multiply fractions. Divide out common factors.

$= \frac{23}{10}$, or $2\frac{3}{10}$ Simplify.

⋰ So, the quotient is $2\frac{3}{10}$.   **Reasonable?** $2\frac{3}{10} \approx 2$ ✓

🔴 On Your Own

Divide. Write the answer in simplest form.

1. $1\frac{3}{7} \div \frac{2}{3}$ **2.** $2\frac{1}{6} \div \frac{3}{4}$ **3.** $8\frac{1}{4} \div 1\frac{1}{2}$ **4.** $6\frac{4}{5} \div 2\frac{1}{8}$

EXAMPLE 3   **Using Order of Operations**

Evaluate $5\frac{1}{4} \div 1\frac{1}{8} - \frac{2}{3}$.

Remember

Be sure to check your answers whenever possible. In Example 3, you can use estimation to check that your answer is reasonable.

$5\frac{1}{4} \div 1\frac{1}{8} - \frac{2}{3}$

$\approx 5 \div 1 - 1$

$= 5 - 1$

$= 4$ ✓

$5\frac{1}{4} \div 1\frac{1}{8} - \frac{2}{3} = \frac{21}{4} \div \frac{9}{8} - \frac{2}{3}$ Write each mixed number as an improper fraction.

$= \frac{21}{4} \times \frac{8}{9} - \frac{2}{3}$ Multiply by the reciprocal of $\frac{9}{8}$, which is $\frac{8}{9}$.

$= \frac{\overset{7}{\cancel{21}} \times \overset{2}{\cancel{8}}}{\underset{1}{\cancel{4}} \times \underset{3}{\cancel{9}}} - \frac{2}{3}$ Multiply $\frac{21}{4}$ and $\frac{8}{9}$. Divide out common factors.

$= \frac{14}{3} - \frac{2}{3}$ Simplify.

$= \frac{12}{3}$, or 4   Subtract.

EXAMPLE 4   **Real-Life Application**

One serving of tortilla soup is $1\frac{2}{3}$ cups. A restaurant cook makes 50 cups of soup. Is there enough to serve 35 people? Explain.

Divide 50 by $1\frac{2}{3}$ to find the number of available servings.

$50 \div 1\frac{2}{3} = \frac{50}{1} \div \frac{5}{3}$ Rewrite each number as an improper fraction.

$= \frac{50}{1} \cdot \frac{3}{5}$ Multiply by the reciprocal of $\frac{5}{3}$, which is $\frac{3}{5}$.

$= \frac{\overset{10}{\cancel{50}} \cdot 3}{1 \cdot \underset{1}{\cancel{5}}}$ Multiply fractions. Divide out common factors.

$= 30$ Simplify.

∴ No. Because 30 is less than 35, there is not enough soup to serve 35 people.

On Your Own

Now You're Ready
Exercises 29–37

Evaluate the expression.

5. $1\frac{1}{2} \div \frac{1}{6} - \frac{7}{8}$

6. $3\frac{1}{3} \div \frac{5}{6} + \frac{8}{9}$

7. $\frac{2}{5} + 2\frac{4}{5} \div 1\frac{3}{4}$

8. $\frac{2}{3} - 1\frac{4}{7} \div 4\frac{5}{7}$

9. In Example 4, can 30 cups of tortilla soup serve 15 people? Explain.

Vocabulary and Concept Check

1. **VOCABULARY** What is the reciprocal of $7\frac{1}{3}$?

2. **NUMBER SENSE** Is $5\frac{1}{4} \div 3\frac{1}{2}$ the same as $3\frac{1}{2} \div 5\frac{1}{4}$? Explain.

3. **NUMBER SENSE** Is the reciprocal of an improper fraction *sometimes*, *always*, or *never* a proper fraction? Explain.

4. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

> What is $5\frac{1}{2}$ divided by $\frac{1}{8}$?
>
> Find the quotient of $5\frac{1}{2}$ and $\frac{1}{8}$.
>
> What is $5\frac{1}{2}$ times 8?
>
> Find the product of $5\frac{1}{2}$ and $\frac{1}{8}$.
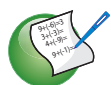
Practice and Problem Solving

Divide. Write the answer in simplest form.

5. $2\frac{1}{4} \div \frac{3}{4}$

6. $3\frac{4}{5} \div \frac{2}{5}$

7. $8\frac{1}{8} \div \frac{5}{6}$

8. $7\frac{5}{9} \div \frac{4}{7}$

9. $7\frac{1}{2} \div 1\frac{9}{10}$

10. $3\frac{3}{4} \div 2\frac{1}{12}$

11. $7\frac{1}{5} \div 8$

12. $8\frac{4}{7} \div 15$

13. $8\frac{1}{3} \div \frac{2}{3}$

14. $9\frac{1}{6} \div \frac{5}{6}$

15. $13 \div 10\frac{5}{6}$

16. $12 \div 5\frac{9}{11}$

17. $\frac{7}{8} \div 3\frac{1}{16}$

18. $\frac{4}{9} \div 1\frac{7}{15}$

19. $4\frac{5}{16} \div 3\frac{3}{8}$

20. $6\frac{2}{9} \div 5\frac{5}{6}$

21. **ERROR ANALYSIS** Describe and correct the error in finding the quotient.

$$✗ \quad 3\frac{1}{2} \div 1\frac{2}{3} = 3\frac{1}{2} \times 1\frac{3}{2} = \frac{7}{2} \times \frac{5}{2} = \frac{35}{4} = 8\frac{3}{4}$$

22. **DOG FOOD** A bag contains 42 cups of dog food. Your dog eats $2\frac{1}{3}$ cups of dog food each day. How many days does the bag of dog food last?

23. **HAMBURGERS** How many $\frac{1}{4}$-pound hamburgers can be made from $3\frac{1}{2}$ pounds of ground beef?

24. **BOOKS** How many $1\frac{3}{5}$-inch thick books can fit on a $14\frac{1}{2}$-inch long bookshelf?

ALGEBRA Evaluate the expression when $x = 5\frac{5}{8}$ and $y = 2\frac{2}{9}$.

25. $y \div 4\frac{1}{6}$
26. $x \div y$
27. $4\frac{7}{12} \div x$
28. $y \div x$

Evaluate the expression.

❸ 29. $5\frac{5}{6} \div 3\frac{3}{4} - \frac{2}{9}$

30. $6\frac{1}{2} - \frac{7}{8} \div 5\frac{11}{16}$

31. $9\frac{1}{6} \div 5 + 3\frac{1}{3}$

32. $3\frac{3}{5} + 4\frac{4}{15} \div \frac{4}{9}$

33. $\frac{3}{5} \times \frac{7}{12} \div 2\frac{7}{10}$

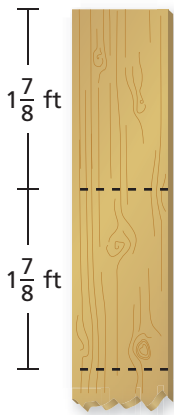**34.** $4\frac{3}{8} \div \frac{3}{4} \times \frac{4}{7}$

35. $1\frac{9}{11} \times 4\frac{7}{12} \div \frac{2}{3}$

36. $3\frac{4}{15} \div \left(8 \times 6\frac{3}{10}\right)$

37. $2\frac{5}{14} \div \left(2\frac{5}{8} \times 1\frac{3}{7}\right)$

38. TRAIL MIX You have 12 cups of granola and $8\frac{1}{2}$ cups of peanuts to make trail mix. What is the greatest number of full batches of trail mix you can make? Explain how you found your answer.

> Trail Mix
> $2\frac{3}{4}$ cups granola
> $1\frac{1}{3}$ cups peanuts

$1\frac{7}{8}$ ft

$1\frac{7}{8}$ ft

39. RAMPS You make skateboard ramps by cutting pieces from a board that is $12\frac{1}{2}$ feet long.

  a. Estimate how many ramps you can cut from the board. Is your estimate reasonable? Explain.

  b. How many ramps can you cut from the board? How much wood is left over?

40. **Reasoning** At a track meet, the longest shot put throw by a boy is 25 feet and 8 inches. The longest shot put throw by a girl is 19 feet and 3 inches. How many times greater is the longest shot put throw by a boy than by a girl?

Fair Game Review What you learned in previous grades & lessons

Write the number as a decimal.

41. forty-three hundredths

42. thirteen thousandths

43. three and eight tenths

44. seven and nine thousandths

45. MULTIPLE CHOICE The winner in a vote for class president received $\frac{3}{4}$ of the 240 votes. How many votes did the winner receive?

Ⓐ 60
Ⓑ 150
Ⓒ 180
Ⓓ 320