

# 1.3b Solutions of Linear Equations

Linear equations do not always have one solution. Linear equations can also have no solution or infinitely many solutions.

When solving a linear equation that has no solution, you will obtain an equivalent equation that is not true for any value of  $x$ , such as  $0 = 2$ .

## EXAMPLE 1 Solving Equations with No Solution

a. Solve  $3 - 4x = -7 - 4x$ .

$$3 - 4x = -7 - 4x \quad \text{Write the equation.}$$

Undo the subtraction.

$$\rightarrow +4x \quad +4x \quad \text{Add } 4x \text{ to each side.}$$

$$3 = -7 \quad \text{Simplify.}$$

∴ The equation  $3 = -7$  is never true. So, the equation has no solution.

b. Solve  $\frac{1}{2}(10x + 7) = 5x$ .

$$\frac{1}{2}(10x + 7) = 5x \quad \text{Write the equation.}$$

$$5x + \frac{7}{2} = 5x \quad \text{Distributive Property}$$

Undo the addition.

$$\rightarrow -5x \quad -5x \quad \text{Subtract } 5x \text{ from each side.}$$

$$\frac{7}{2} = 0 \quad \text{Simplify.}$$

∴ The equation  $\frac{7}{2} = 0$  is never true. So, the equation has no solution.

## Practice

Solve the equation.

1.  $x + 6 = x$

2.  $2x + 1 = 2x - 1$

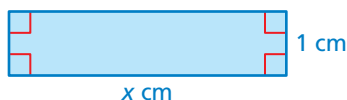
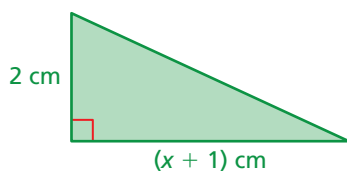
3.  $3x - 1 = 1 - 3x$

4.  $4x - 9 = 3.5x - 9$

5.  $\frac{1}{3}(2x + 9) = \frac{2}{3}x$

6.  $6(5 - 2x) = -4(3x + 1)$

7. **GEOMETRY** Are there any values of  $x$  for which the areas of the figures are the same? Explain.



When solving a linear equation that has infinitely many solutions, you will obtain an equivalent equation that is true for all values of  $x$ , such as  $-5 = -5$ .

## EXAMPLE 2 Solving Equations with Infinitely Many Solutions

a. Solve  $3(4x - 1) = 12x - 3$ .

$$3(4x - 1) = 12x - 3 \quad \text{Write the equation.}$$

$$12x - 3 = 12x - 3 \quad \text{Distributive Property}$$

Undo the addition.  $\rightarrow$   $\underline{-12x} \quad \underline{-12x}$   $\quad$  Subtract  $12x$  from each side.

$$-3 = -3 \quad \text{Simplify.}$$

∴ The equation  $-3 = -3$  is always true. So, the equation has infinitely many solutions.

b. Solve  $2(2 - 3x) = 4\left(1 - \frac{3}{2}x\right)$ .

$$2(2 - 3x) = 4\left(1 - \frac{3}{2}x\right) \quad \text{Write the equation.}$$

$$4 - 6x = 4 - 6x \quad \text{Distributive Property}$$

Undo the subtraction.  $\rightarrow$   $\underline{+6x} \quad \underline{+6x}$   $\quad$  Add  $6x$  to each side.

$$4 = 4 \quad \text{Simplify.}$$

∴ The equation  $4 = 4$  is always true. So, the equation has infinitely many solutions.

## Practice

Solve the equation.

8.  $x + 8 - x = 9$

9.  $\frac{1}{2}x + \frac{1}{2}x = x + 1$

10.  $3x + 15 = 3(x + 5)$

11.  $\frac{1}{2}(6x - 4) = 3x - 2$

12.  $5x - 7 = 4x - 1$

13.  $2x + 4 = -(-7x + 6)$

14.  $5.5 - x = -4.5 - x$

15.  $10x - \frac{8}{3} - 4x = 6x$

16.  $-3(2x - 3) = -6x + 9$

17.  $6(7x + 7) = 7(6x + 6)$

18.  $\frac{3}{4}(4x - 8) = -10$

19.  $-\frac{1}{8} = 2(x - 1)$