Essential Question How are the lengths of the sides of a right triangle related?

Pythagoras was a Greek mathematician and philosopher who discovered one of the most famous rules in mathematics. In mathematics, a rule is called a **theorem**. So, the rule that Pythagoras discovered is called the Pythagorean Theorem.

Pythagoras
(c. 570 B.C.–c. 490 B.C.)

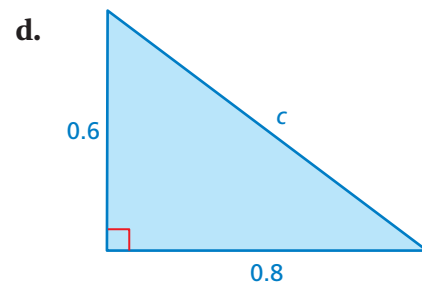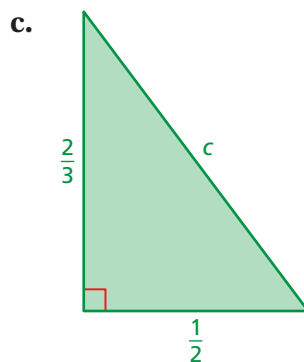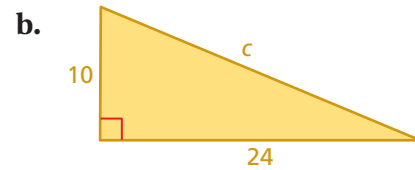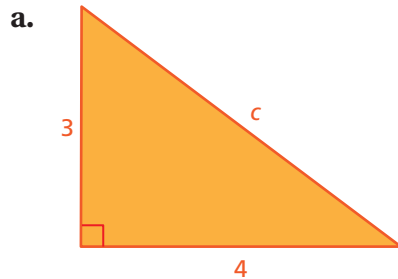### 1 ACTIVITY: Discovering the Pythagorean Theorem

Work with a partner.

a. On grid paper, draw any right triangle. Label the lengths of the two shorter sides (the **legs**) a and b.

b. Label the length of the longest side (the **hypotenuse**) c.

c. Draw squares along each of the three sides. Label the areas of the three squares a^2, b^2, and c^2.

d. Cut out the three squares. Make eight copies of the right triangle and cut them out. Arrange the figures to form two identical larger squares.

e. What does this tell you about the relationship among a^2, b^2, and c^2?
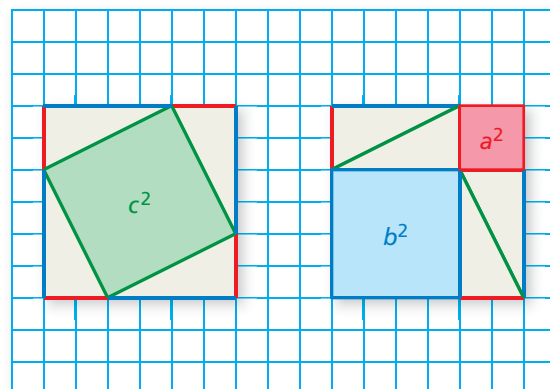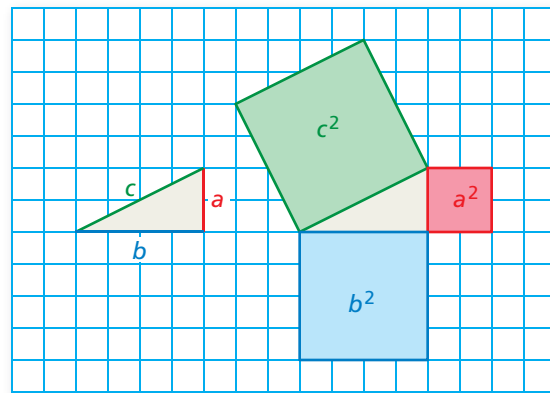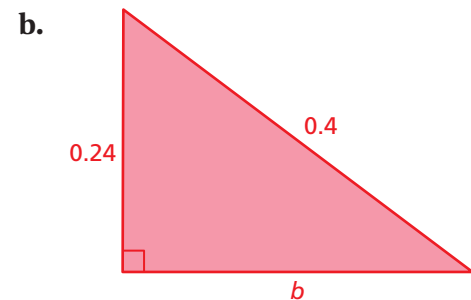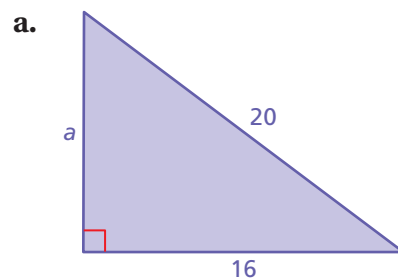
2 ACTIVITY: Finding the Length of the Hypotenuse

Work with a partner. Use the result of Activity 1 to find the length of the hypotenuse of each right triangle.

a.

A right triangle with legs labeled 3 and 4, and hypotenuse labeled c.

b.

A right triangle with legs labeled 10 and 24, and hypotenuse labeled c.

c.

A right triangle with legs labeled $\frac{2}{3}$ and $\frac{1}{2}$, and hypotenuse labeled c.

d.

A right triangle with legs labeled 0.6 and 0.8, and hypotenuse labeled c.

3 ACTIVITY: Finding the Length of a Leg

Work with a partner. Use the result of Activity 1 to find the length of the leg of each right triangle.

a.

A right triangle with leg labeled a, base labeled 16, and hypotenuse labeled 20.

b.

A right triangle with leg labeled 0.24, hypotenuse labeled 0.4, and base labeled b.

What Is Your Answer?

4. **IN YOUR OWN WORDS** How are the lengths of the sides of a right triangle related? Give an example using whole numbers.

Practice

Use what you learned about the Pythagorean Theorem to complete Exercises 3–5 on page 240.

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Key Vocabulary 🔊
theorem, *p. 236*
legs, *p. 238*
hypotenuse, *p. 238*
Pythagorean
    Theorem, *p. 238*

🔑 Key Ideas

Sides of a Right Triangle

The sides of a right triangle have special names.

leg, *a*

hypotenuse, *c*

The **hypotenuse** is the side opposite the right angle.

The **legs** are the two sides that form the right angle.

leg, *b*

Study Tip

In a right triangle, the legs are the shorter sides and the hypotenuse is always the longest side.
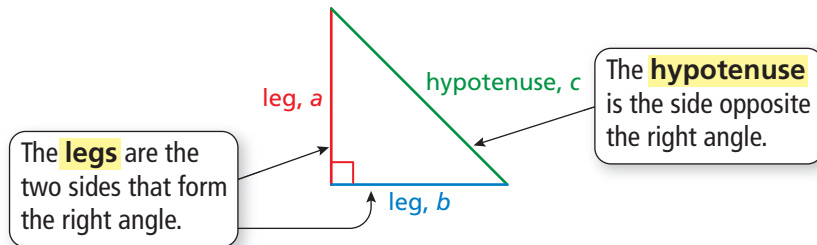
The Pythagorean Theorem

Words   In any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

Algebra $a^2 + b^2 = c^2$

EXAMPLE ① **Finding the Length of a Hypotenuse**

Find the length of the hypotenuse of the triangle.

5 m

c   12 m
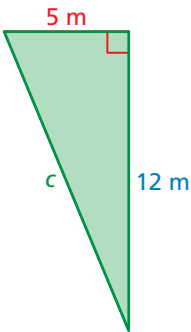
$$a^2 + b^2 = c^2 \qquad \text{Write the Pythagorean Theorem.}$$
$$5^2 + 12^2 = c^2 \qquad \text{Substitute 5 for } a \text{ and 12 for } b.$$
$$25 + 144 = c^2 \qquad \text{Evaluate powers.}$$
$$169 = c^2 \qquad \text{Add.}$$
$$\sqrt{169} = \sqrt{c^2} \qquad \text{Take positive square root of each side.}$$
$$13 = c \qquad \text{Simplify.}$$
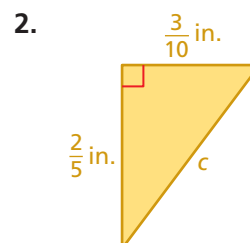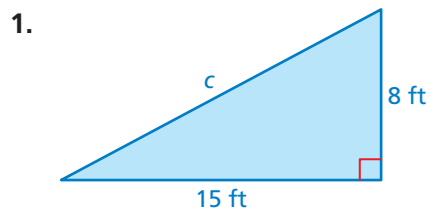
⫶ The length of the hypotenuse is 13 meters.

🔴 On Your Own

Find the length of the hypotenuse of the triangle.

1.

c

8 ft

15 ft

2.

$\frac{3}{10}$ in.

$\frac{2}{5}$ in.

c

🔊 Multi-Language Glossary at BigIdeasMath✓com.

EXAMPLE 2   **Finding the Length of a Leg**

Find the missing length of the triangle.

a | 2.1 cm

2.9 cm
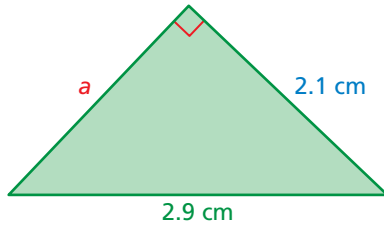
$a^2 + b^2 = c^2$ Write the Pythagorean Theorem.

$a^2 + 2.1^2 = 2.9^2$ Substitute 2.1 for b and 2.9 for c.

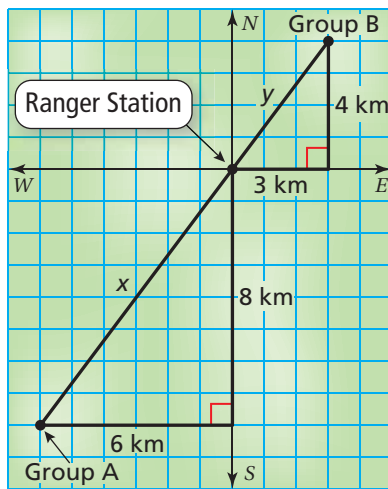$a^2 + 4.41 = 8.41$ Evaluate powers.

$a^2 = 4$ Subtract 4.41 from each side.

$a = 2$ Take positive square root of each side.

∴   The length of the leg is 2 centimeters.

EXAMPLE 3   **Standardized Test Practice**

Hiking Group A leaves a ranger station and hikes 8 kilometers south then 6 kilometers west. Group B leaves the station and hikes 3 kilometers east then 4 kilometers north. Using the figure, how far apart are the two groups of hikers?

Ⓐ   5 km   Ⓑ   10 km   Ⓒ   15 km   Ⓓ   21 km

The distance between the groups is the sum of the hypotenuses, x and y. Use the Pythagorean Theorem to find x and y.

$a^2 + b^2 = c^2$ Write the Pythagorean Theorem. $a^2 + b^2 = c^2$

$6^2 + 8^2 = x^2$ Substitute. $3^2 + 4^2 = y^2$

$36 + 64 = x^2$ Evaluate powers. $9 + 16 = y^2$

$100 = x^2$ Add. $25 = y^2$

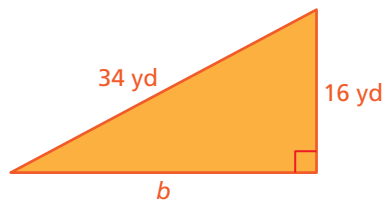$10 = x$ Take positive square root of each side. $5 = y$

∴   The distance between the groups of hikers is $10 + 5 = 15$ kilometers. So, the correct answer is Ⓒ.
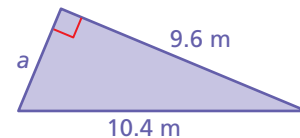
🔴 **On Your Own**

Now You're Ready
Exercises 3–8

Find the missing length of the triangle.

3.

34 yd

16 yd

b

4.

a   9.6 m

10.4 m

5. **WHAT IF?** In Example 3, Group A hikes 12 kilometers south and 9 kilometers west. How far apart are the hikers?
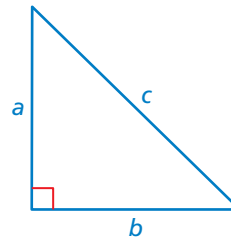
✓ Vocabulary and Concept Check

1. **VOCABULARY** In a right triangle, how can you tell which sides are the legs and which side is the hypotenuse?

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

   Which side is the hypotenuse?

   Which side is the longest?
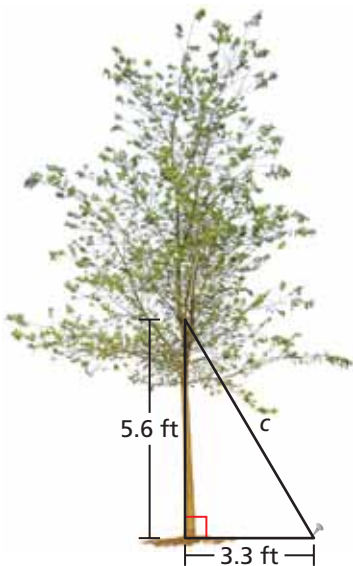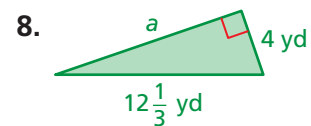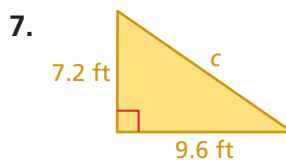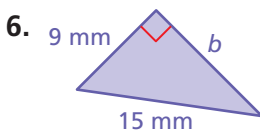
   Which side is a leg?

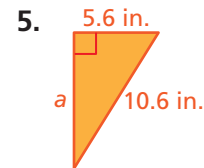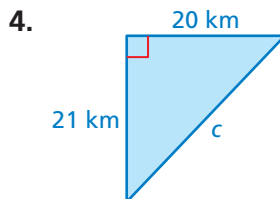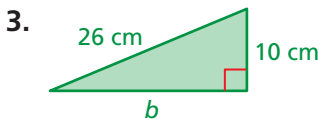   Which side is opposite the right angle?

Practice and Problem Solving

Find the missing length of the triangle.

① ② 3. 26 cm, 10 cm, b

4. 20 km, 21 km, c

5. 5.6 in., a, 10.6 in.

6. 9 mm, b, 15 mm

7. 7.2 ft, c, 9.6 ft

8. a, 4 yd, $12\frac{1}{3}$ yd

9. **ERROR ANALYSIS** Describe and correct the error in finding the missing length of the triangle.
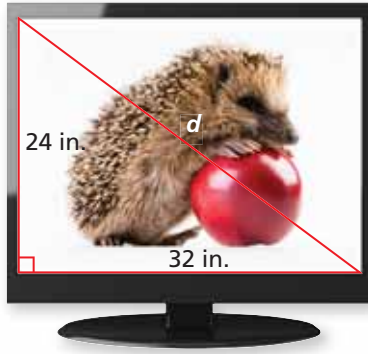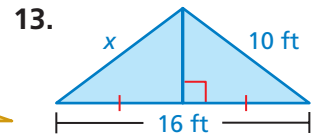
   ✗ 7 ft, 25 ft
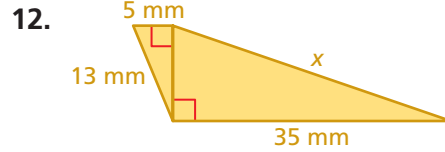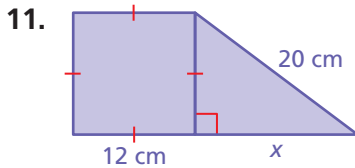
   $$a^2 + b^2 = c^2$$
   $$7^2 + 25^2 = c^2$$
   $$674 = c^2$$
   $$\sqrt{674} = c$$

10. **TREE SUPPORT** How long is the wire that supports the tree?

    5.6 ft, c, 3.3 ft

Find the value of x.

11.

20 cm

12 cm

x

12.

5 mm

13 mm

x

35 mm

13.

x

10 ft

16 ft

14. FLAT SCREEN Televisions are advertised by the lengths of their diagonals. A store has a sale on televisions 40 inches and larger. Is the television on sale? Explain.

24 in.

d

32 in.

15. BUTTERFLY Approximate the wingspan of the butterfly.

Wingspan

4 cm

5.8 cm

Hole 13
Par 3
181 Yards

Hole

180 yd

x

Tee
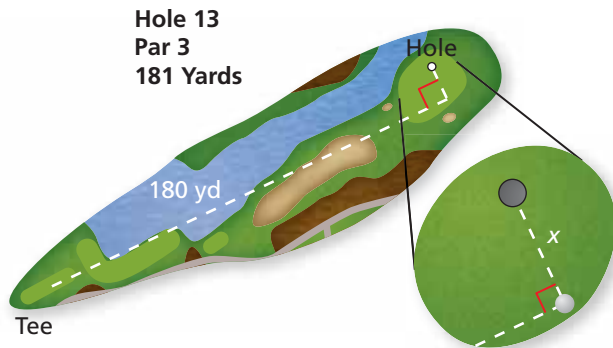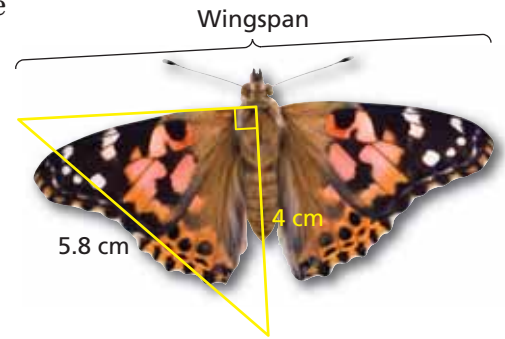
16. GOLF The figure shows the location of a golf ball after a tee shot. How many feet from the hole is the ball?

17. SNOWBALLS You and a friend stand back-to-back. You run 20 feet forward then 15 feet to your right. At the same time, your friend runs 16 feet forward then 12 feet to her right. She stops and hits you with a snowball.

    a. Draw the situation in a coordinate plane.

    b. How far does your friend throw the snowball?

18. **Algebra** The legs of a right triangle have lengths of 28 meters and 21 meters. The hypotenuse has a length of $5x$ meters. What is the value of x?

Fair Game Review What you learned in previous grades & lessons

Find the square root(s). *(Section 6.1)*

19. $\pm\sqrt{36}$
    20. $-\sqrt{121}$
    21. $\sqrt{169}$
    22. $-\sqrt{225}$

23. MULTIPLE CHOICE Which type of triangle can have an obtuse angle? *(Section 5.2)*

    Ⓐ equiangular
    Ⓑ right
    Ⓒ isosceles
    Ⓓ equilateral