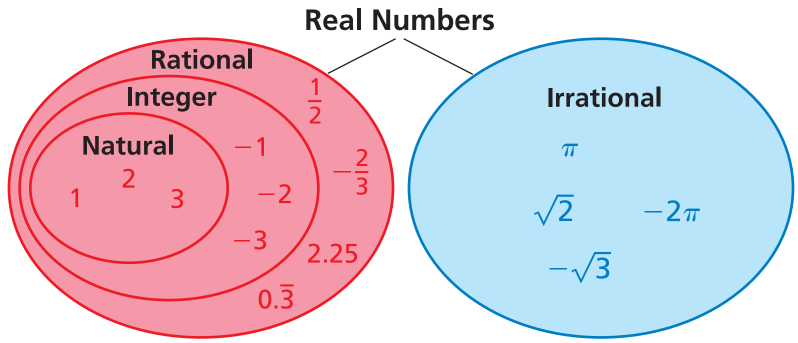
# 6.3 Approximating Square Roots

**Essential Question** How can you find decimal approximations of square roots that are irrational?

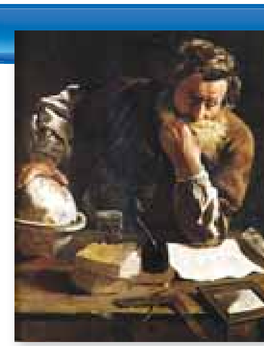
You already know that a rational number is a number that can be written as the ratio of two integers. Numbers that cannot be written as the ratio of two integers are called **irrational**.




## 1 ACTIVITY: Approximating Square Roots

**Work with a partner.**

**Archimedes was a Greek mathematician, physicist, engineer, inventor, and astronomer.**



Archimedes
(c. 287 B.C.–c. 212 B.C.)

**a.** Archimedes tried to find a rational number whose square is 3. Here are two that he tried.

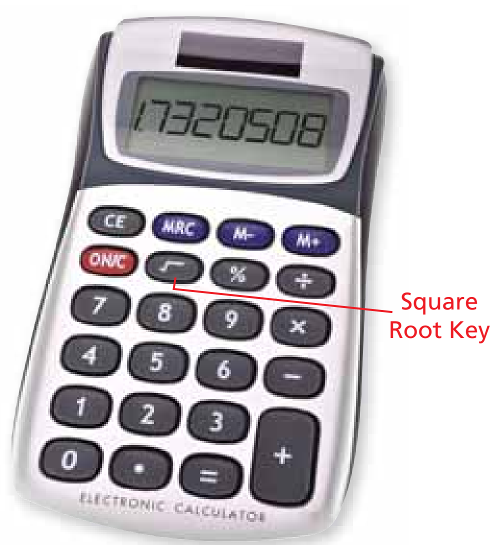
$$\frac{265}{153} \text{ and } \frac{1351}{780}$$

Are either of these numbers equal to $\sqrt{3}$? How can you tell?

**b.** Use a calculator with a square root key to approximate $\sqrt{3}$.

Write the number on a piece of paper. Then enter it into the calculator and square it. Then subtract 3. Do you get 0? Explain.

**c.** Calculators did not exist in the time of Archimedes. How do you think he might have approximated $\sqrt{3}$?



## 2 ACTIVITY: Approximating Square Roots Geometrically

**Work with a partner.**

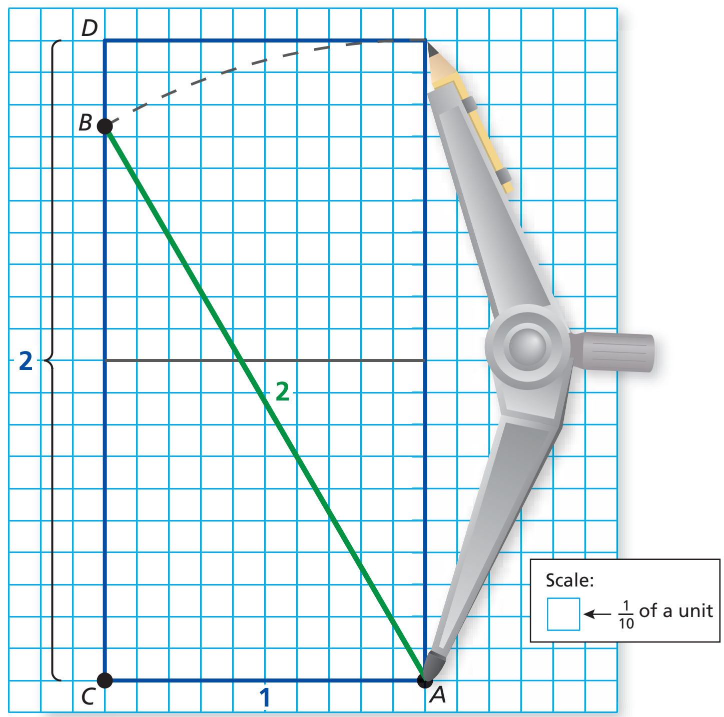
**a.** Use grid paper and the given scale to draw a horizontal line segment 1 unit in length. Label this segment *AC*.

**b.** Draw a vertical line segment 2 units in length. Label this segment *DC*.

**c.** Set the point of a compass on *A*. Set the compass to 2 units. Swing the compass to intersect segment *DC*. Label this intersection as *B*.

**d.** Use the Pythagorean Theorem to show that the length of segment *BC* is $\sqrt{3}$ units.

**e.** Use the grid paper to approximate $\sqrt{3}$.




## What Is Your Answer?

**3.** Repeat Activity 2 for a triangle in which segment *CA* is 2 units and segment *BA* is 3 units. Use the Pythagorean Theorem to show that segment *BC* is $\sqrt{5}$ units. Use the grid paper to approximate $\sqrt{5}$.

**4.** **IN YOUR OWN WORDS** How can you find decimal approximations of square roots that are irrational?

**Practice** Use what you learned about approximating square roots to complete Exercises 5–8 on page 249.

English Spanish

# 6.3 Lesson

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**Key Vocabulary**
irrational number, *p. 246*
real numbers, *p. 246*

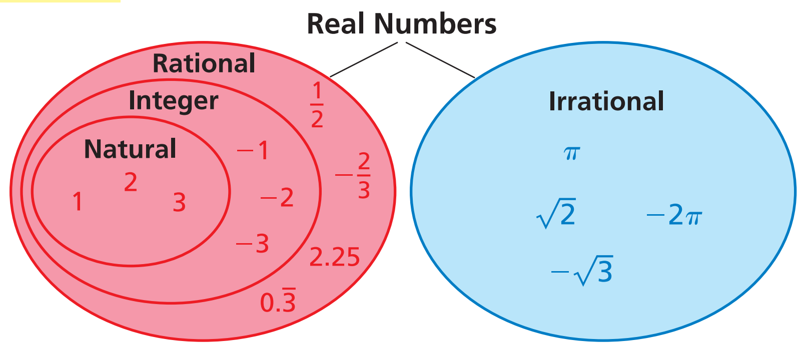
A rational number is a number that can be written as the ratio of two integers. An **irrational number** cannot be written as the ratio of two integers.

- The square root of any whole number that is not a perfect square is irrational.
- The decimal form of an irrational number neither terminates nor repeats.

## Key Idea

**Real Numbers**

Rational numbers and irrational numbers together form the set of **real numbers**.




**Remember**
Decimals that *terminate* or *repeat* are rational.

## EXAMPLE 1 Classifying Real Numbers

**Tell whether the number is *rational* or *irrational*. Explain.**

| | Number | Rational or Irrational | Reasoning |
|---|---|---|---|
| **a.** | $\sqrt{12}$ | Irrational | 12 is not a perfect square. |
| **b.** | $-0.36\overline{4}$ | Rational | $-0.36\overline{4}$ is a repeating decimal. |
| **c.** | $-1\frac{3}{7}$ | Rational | $-1\frac{3}{7}$ can be written as $\frac{-10}{7}$. |
| **d.** | 0.85 | Rational | 0.85 can be written as $\frac{17}{20}$. |

## On Your Own

Now You're Ready
Exercises 9–14

**Tell whether the number is *rational* or *irrational*. Explain.**

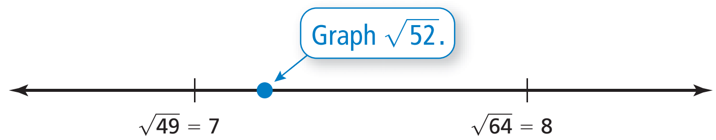
**1.** 0.121221222. . . **2.** $-\sqrt{196}$ **3.** $\sqrt{2}$


Multi-Language Glossary at BigIdeasMath.com.

## EXAMPLE 2 Approximating Square Roots

**Estimate $\sqrt{52}$ to the nearest integer.**

Use a number line and the square roots of the perfect squares nearest to the radicand. The nearest perfect square less than 52 is 49. The nearest perfect square greater than 52 is 64.




Because 52 is closer to 49 than to 64, $\sqrt{52}$ is closer to 7 than to 8.

So, $\sqrt{52} \approx 7$.

### On Your Own

Now You're Ready
Exercises 18–23

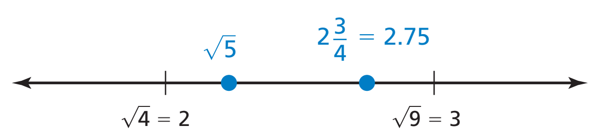
**Estimate to the nearest integer.**

**4.** $\sqrt{33}$ **5.** $\sqrt{85}$ **6.** $\sqrt{190}$ **7.** $-\sqrt{7}$

## EXAMPLE 3 Comparing Real Numbers

**a. Which is greater, $\sqrt{5}$ or $2\frac{3}{4}$?**

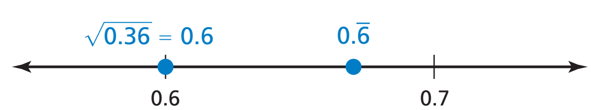
Graph the numbers on a number line.




$2\frac{3}{4}$ is to the right of $\sqrt{5}$. So, $2\frac{3}{4}$ is greater.

**b. Which is greater, $0.\overline{6}$ or $\sqrt{0.36}$?**

Graph the numbers on a number line.




$0.\overline{6}$ is to the right of $\sqrt{0.36}$. So, $0.\overline{6}$ is greater.

### On Your Own

Now You're Ready
Exercises 25–30

**Which number is greater? Explain.**

**8.** $4\frac{1}{5}, \sqrt{23}$ **9.** $\sqrt{10}, -\sqrt{5}$ **10.** $-\sqrt{2}, -2$

## EXAMPLE 4 Approximating an Expression

**The radius of a circle with area $A$ is approximately $\sqrt{\frac{A}{3}}$. The area of a circular mouse pad is 51 square inches. Estimate its radius.**

$$\sqrt{\frac{A}{3}} = \sqrt{\frac{51}{3}} \qquad \text{Substitute 51 for } A.$$

$$= \sqrt{17} \qquad \text{Divide.}$$

The nearest perfect square less than 17 is 16. The nearest perfect square greater than 17 is 25.


Because 17 is closer to 16 than to 25, $\sqrt{17}$ is closer to 4 than to 5.

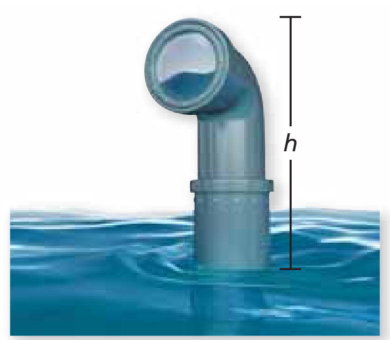
∴ The radius is about 4 inches.

### On Your Own

11. **WHAT IF?** The area of a circular mouse pad is 64 square inches. Estimate its radius.

## EXAMPLE 5 Real-Life Application

**The distance (in nautical miles) you can see with a periscope is $1.17\sqrt{h}$, where $h$ is the height of the periscope above the water. Can a periscope that is 6 feet above the water see twice as far as a periscope that is 3 feet above the water? Explain.**




Use a calculator to find the distances.

***3 feet above water***

$$1.17\sqrt{h} = 1.17\sqrt{3} \qquad \text{Substitute for } h.$$

$$\approx 2.03 \qquad \text{Use a calculator.}$$

***6 feet above water***

$$1.17\sqrt{h} = 1.17\sqrt{6}$$

$$\approx 2.87$$

You can see $\frac{2.87}{2.03} \approx 1.41$ times farther with the periscope that is 6 feet above the water than with the periscope that is 3 feet above the water.

∴ No, the periscope that is 6 feet above the water cannot see twice as far.

### On Your Own

12. You use a periscope that is 10 feet above the water. Can you see farther than 4 nautical miles? Explain.

# 6.3 Exercises

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## Vocabulary and Concept Check

1. **VOCABULARY** What is the difference between a rational number and an irrational number?

2. **WRITING** Describe a method of approximating $\sqrt{32}$.

3. **VOCABULARY** What are real numbers? Give three examples.

4. **WHICH ONE DOESN'T BELONG?** Which number does *not* belong with the other three? Explain your reasoning.

$-\frac{11}{12}$     $25.075$     $\sqrt{8}$     $-3.\overline{3}$

## Practice and Problem Solving

**Tell whether the rational number is a reasonable approximation of the square root.**

5. $\frac{559}{250}, \sqrt{5}$

6. $\frac{3021}{250}, \sqrt{11}$

7. $\frac{678}{250}, \sqrt{28}$

8. $\frac{1677}{250}, \sqrt{45}$

**Tell whether the number is *rational* or *irrational*. Explain.**

1 9. $3.66666\overline{6}$

10. $\frac{\pi}{6}$

11. $-\sqrt{7}$

12. $-1.125$

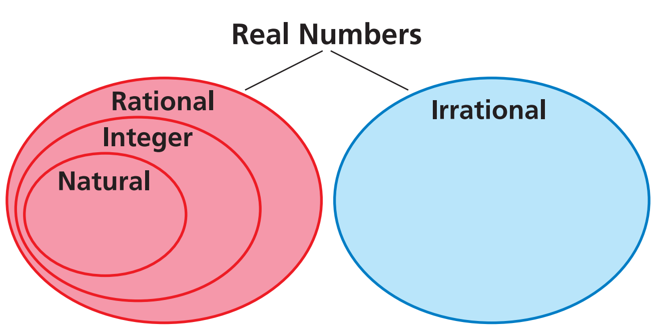
13. $-3\frac{8}{9}$

14. $\sqrt{15}$

15. **ERROR ANALYSIS** Describe and correct the error in classifying the number.

✗ $\sqrt{144}$ is irrational.

16. **SCRAPBOOKING** You cut a picture into a right triangle for your scrapbook. The lengths of the legs of the triangle are 4 inches and 6 inches. Is the length of the hypotenuse a rational number? Explain.




17. **VENN DIAGRAM** Place each number in the correct area of the Venn Diagram.

   a. Your age

   b. The square root of any prime number

   c. The ratio of the circumference of a circle to its diameter

**Estimate to the nearest integer.**

2 **18.** $\sqrt{24}$ **19.** $\sqrt{685}$ **20.** $-\sqrt{61}$

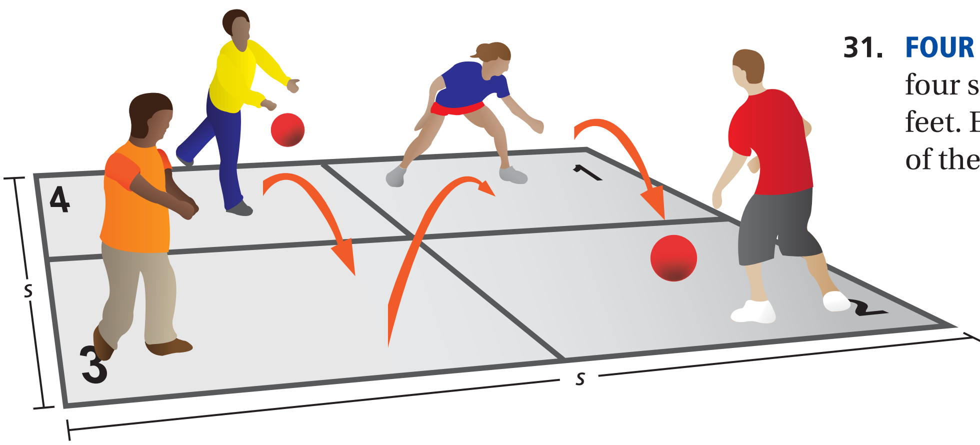
**21.** $-\sqrt{105}$ **22.** $\sqrt{\frac{27}{4}}$ **23.** $-\sqrt{\frac{335}{2}}$

**24. CHECKERS** A checkerboard is 8 squares long and 8 squares wide. The area of each square is 14 square centimeters. Estimate the perimeter of the checkerboard.

**Which number is greater? Explain.**

3 **25.** $\sqrt{20}$, 10 **26.** $\sqrt{15}$, $-3.5$ **27.** $\sqrt{133}$, $10\frac{3}{4}$

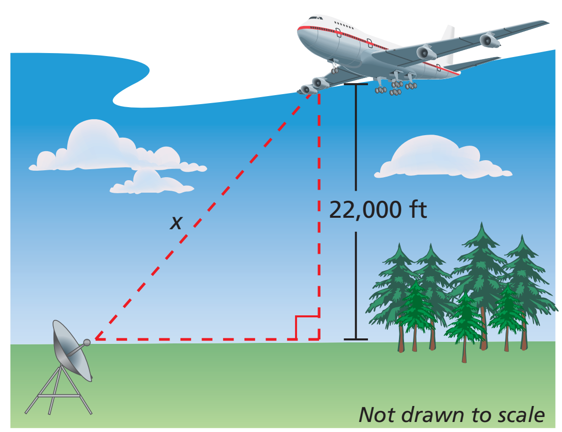
**28.** $\frac{2}{3}$, $\sqrt{\frac{16}{81}}$ **29.** $-\sqrt{0.25}$, $-0.25$ **30.** $-\sqrt{182}$, $-\sqrt{192}$




**31. FOUR SQUARE** The area of a four square court is 66 square feet. Estimate the length $s$ of one of the sides of the court.

**32. RADIO SIGNAL** The maximum distance (in nautical miles) that a radio transmitter signal can be sent is represented by the expression $1.23\sqrt{h}$, where $h$ is the height (in feet) above the transmitter.

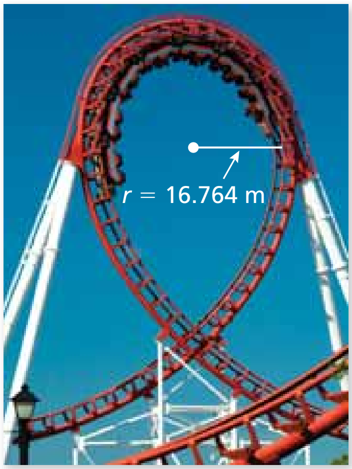
Estimate the maximum distance $x$ (in nautical miles) between the plane that is receiving the signal and the transmitter. Round your answer to the nearest tenth.




**33. OPEN-ENDED** Find two numbers $a$ and $b$ that satisfy the diagram.

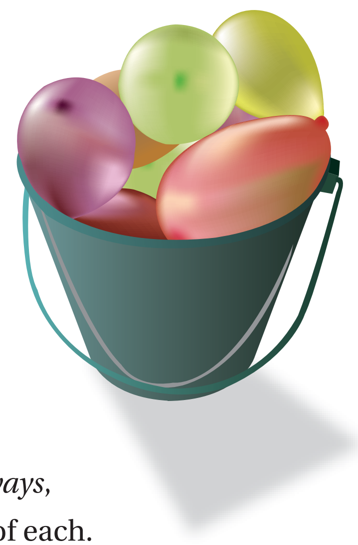
**Estimate to the nearest tenth.**

**34.** $\sqrt{0.39}$ **35.** $\sqrt{1.19}$ **36.** $\sqrt{1.52}$




**37. ROLLER COASTER** The velocity $v$ (in meters per second) of a roller coaster is represented by the equation $v = 3\sqrt{6r}$, where $r$ is the radius of the loop. Estimate the velocity of a car going around the loop. Round your answer to the nearest tenth.

**38.** Is $\sqrt{\frac{1}{4}}$ a rational number? Is $\sqrt{\frac{3}{16}}$ a rational number? Explain.



**39. WATER BALLOON** The time $t$ (in seconds) it takes a water balloon to fall $d$ meters is represented by the equation $t = \sqrt{\frac{d}{4.9}}$. Estimate the time it takes the balloon to fall to the ground from a window that is 14 meters above the ground. Round your answer to the nearest tenth.

**40.** Number Sense Determine if the statement is *sometimes*, *always*, or *never* true. Explain your reasoning and give an example of each.

**a.** A rational number multiplied by a rational number is rational.

**b.** A rational number multiplied by an irrational number is rational.

**c.** An irrational number multiplied by an irrational number is rational.

## Fair Game Review What you learned in previous grades & lessons

**Simplify the expression.** SKILLS REVIEW HANDBOOK

**41.** $2x + 3y - 5x$ **42.** $3\pi + 8(t - \pi) - 4t$ **43.** $17k - 9 + 23k$

**44. MULTIPLE CHOICE** What is the ratio (red to blue) of the corresponding side lengths of the similar triangles? SKILLS REVIEW HANDBOOK


Ⓐ 1 : 3 Ⓑ 5 : 2 Ⓒ 3 : 4 Ⓓ 2 : 5