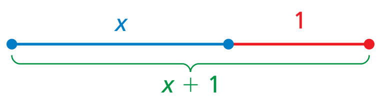
6.4 Simplifying Square Roots

Essential Question How can you use a square root to describe the golden ratio?

Two quantities are in the *golden ratio* if the ratio between the sum of the quantities and the greater quantity is the same as the ratio between the greater quantity and the lesser quantity.



$$\frac{x+1}{x} = \frac{x}{1}$$

In a future algebra course, you will be able to prove that the golden ratio is

$$\frac{1+\sqrt{5}}{2}$$ Golden ratio.

1 ACTIVITY: Constructing a Golden Ratio

Work with a partner.

a. Use grid paper and the given scale to draw a square that is 1 unit by 1 unit (blue).

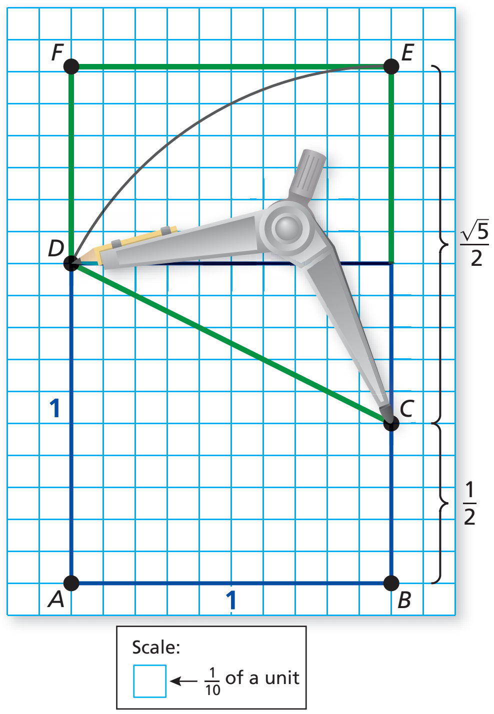
b. Draw a line from midpoint C of one side of the square to the opposite corner D, as shown.

c. Use the Pythagorean Theorem to find the length of segment CD.

d. Set the point of a compass on C. Set the compass radius to the length of segment CD. Swing the compass to intersect line BC at point E.

e. The rectangle $ABEF$ is called a *golden rectangle* because the ratio of its side lengths is the golden ratio.

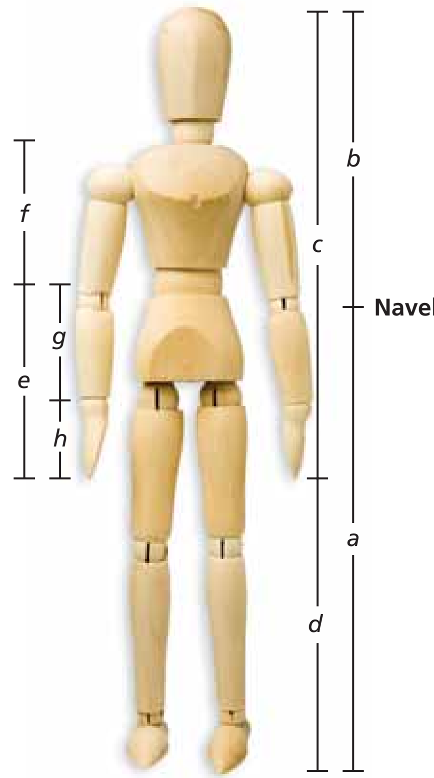
f. Use a calculator to find a decimal approximation of the golden ratio. Round your answer to two decimal places.



2 ACTIVITY: The Golden Ratio and the Human Body

Work with a partner.

Leonardo da Vinci was one of the first to notice that there are several ratios in the human body that approximate the golden ratio.



a. Use a tape measure or two yardsticks to measure the lengths shown in the diagram for both you and your partner. (Take your shoes off before measuring.)

b. Copy the tables below. Record your results in the first two columns.

c. Calculate the ratios shown in the tables.

d. Leonardo da Vinci stated that for many people, the ratios are close to the golden ratio. How close are your ratios?

You		
$a =$	$b =$	$\frac{a}{b} =$
$c =$	$d =$	$\frac{c}{d} =$
$e =$	$f =$	$\frac{e}{f} =$
$g =$	$h =$	$\frac{g}{h} =$

Partner		
$a =$	$b =$	$\frac{a}{b} =$
$c =$	$d =$	$\frac{c}{d} =$
$e =$	$f =$	$\frac{e}{f} =$
$g =$	$h =$	$\frac{g}{h} =$

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you use a square root to describe the golden ratio? Use the Internet or some other reference to find examples of the golden ratio in art and architecture.

Practice Use what you learned about square roots to complete Exercises 3–5 on page 256.

English Spanish

6.4 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

You can add or subtract radical expressions the same way you combine like terms, such as $5x + 4x = 9x$.

EXAMPLE 1 Adding and Subtracting Square Roots

Reading

Do not assume that radicals that have different radicands cannot be simplified.

An expression such as $2\sqrt{4} + \sqrt{1}$ can easily be simplified.

a. Simplify $5\sqrt{2} + 4\sqrt{2}$.

$$5\sqrt{2} + 4\sqrt{2} = (5 + 4)\sqrt{2}$$ Use the Distributive Property.

$$= 9\sqrt{2}$$ Simplify.

b. Simplify $2\sqrt{3} - 7\sqrt{3}$.

$$2\sqrt{3} - 7\sqrt{3} = (2 - 7)\sqrt{3}$$ Use the Distributive Property.

$$= -5\sqrt{3}$$ Simplify.

On Your Own

Now You're Ready
Exercises 6–14

Simplify the expression.

1. $\sqrt{5} + \sqrt{5}$
2. $6\sqrt{10} + 4\sqrt{10}$
3. $2\sqrt{7} - \sqrt{7}$

To simplify square roots that are not perfect squares, use the following property.

Key Idea

Product Property of Square Roots

Algebra $\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$, where $x, y \geq 0$

Numbers $\sqrt{4 \cdot 3} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$

EXAMPLE 2 Simplifying Square Roots

Study Tip

A square root is simplified when the radicand has no perfect square factors other than 1.

Simplify $\sqrt{50}$.

$$\sqrt{50} = \sqrt{25 \cdot 2}$$ Factor using the greatest perfect square factor.

$$= \sqrt{25} \cdot \sqrt{2}$$ Use the Product Property of Square Roots.

$$= 5\sqrt{2}$$ Simplify.

On Your Own

Now You're Ready
Exercises 16–20

Simplify the expression.

4. $\sqrt{24}$
5. $\sqrt{45}$
6. $\sqrt{98}$

Key Idea

Quotient Property of Square Roots

Algebra $\sqrt{\dfrac{x}{y}} = \dfrac{\sqrt{x}}{\sqrt{y}}$, where $x \geq 0$ and $y > 0$

Numbers $\sqrt{\dfrac{7}{9}} = \dfrac{\sqrt{7}}{\sqrt{9}} = \dfrac{\sqrt{7}}{3}$

EXAMPLE 3 Simplifying Square Roots

Simplify $\sqrt{\dfrac{11}{16}}$.

$\sqrt{\dfrac{11}{16}} = \dfrac{\sqrt{11}}{\sqrt{16}}$ — Use the Quotient Property of Square Roots.

$= \dfrac{\sqrt{11}}{4}$ — Simplify.

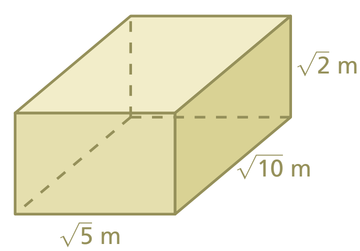
EXAMPLE 4 Finding a Volume

Remember

The volume V of a rectangular prism is the product of the area of its base B and its height h.

$V = Bh$

Find the volume of the rectangular prism.



$V = Bh$ — Write formula for volume.

$= (\sqrt{5})(\sqrt{10})(\sqrt{2})$ — Substitute.

$= \sqrt{5 \bullet 10 \bullet 2}$ — Use the Product Property of Square Roots.

$= \sqrt{100}$ — Multiply.

$= 10$ — Simplify.

The volume is 10 cubic meters.

On Your Own

Now You're Ready Exercises 21–24

Simplify the expression.

7. $\sqrt{\dfrac{35}{36}}$ **8.** $\sqrt{\dfrac{13}{4}}$ **9.** $\sqrt{\dfrac{5}{b^2}}$

10. **WHAT IF?** In Example 4, the height of the rectangular prism is $\sqrt{8}$ meters. Find the volume of the prism.

6.4 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

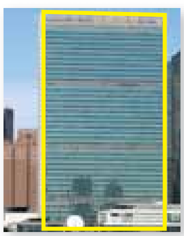
Vocabulary and Concept Check

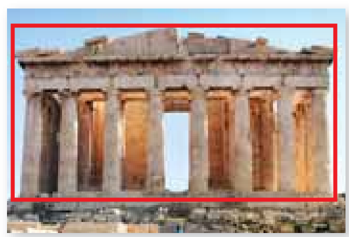
1. **WRITING** Describe how combining like terms is similar to adding and subtracting square roots.

2. **WRITING** How are the Product Property of Square Roots and the Quotient Property of Square Roots similar?

Practice and Problem Solving

Find the ratio of the side lengths. Is the ratio close to the golden ratio?

3.

544 ft
336 ft

4.

21 yd
34 yd

5.

50 m
45 m

Simplify the expression.

1 6. $\frac{\sqrt{2}}{9} + \frac{1}{9}$

7. $\frac{\sqrt{7}}{3} + \frac{1}{3}$

8. $\frac{1}{4} + \frac{\sqrt{13}}{4}$

9. $2\sqrt{3} + 4\sqrt{3}$

10. $6\sqrt{7} - 2\sqrt{7}$

11. $\frac{3}{4}\sqrt{5} + \frac{5}{4}\sqrt{5}$

12. $\sqrt{6} - 4\sqrt{6}$

13. $1.5\sqrt{15} - 9.2\sqrt{15}$

14. $\frac{7}{8}\sqrt{11} + \frac{3}{8}\sqrt{11}$

15. **ERROR ANALYSIS** Describe and correct the error in simplifying the expression.

✗ $4\sqrt{5} + 3\sqrt{5} = 7\sqrt{10}$

Simplify the expression.

2 3 16. $\sqrt{18}$

17. $\sqrt{200}$

18. $\sqrt{12}$

19. $\sqrt{48}$

20. $\sqrt{125}$

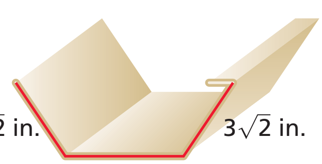
21. $\sqrt{\frac{23}{64}}$

22. $\sqrt{\frac{65}{121}}$

23. $\sqrt{\frac{17}{49}}$

24. $\sqrt{\frac{22}{c^2}}$

25. **RAIN GUTTER** A rain gutter is made from a single sheet of metal. What is the length of the red cross-section?


$3\sqrt{2}$ in.
$3\sqrt{2}$ in.
$4\sqrt{2}$ in.

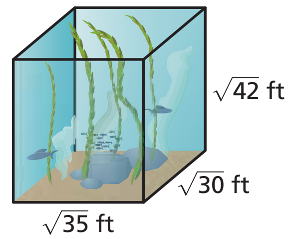
English Spanish

Simplify the expression.

26. $3\sqrt{5} - \sqrt{45}$

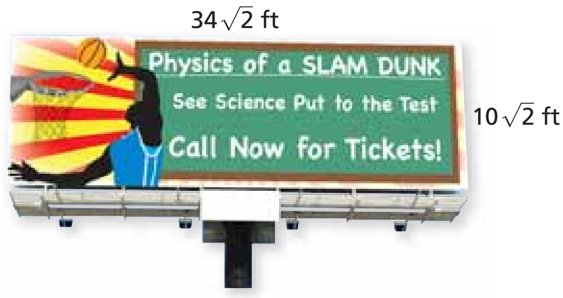
27. $\sqrt{24} + 4\sqrt{6}$

28. $\frac{4}{3}\sqrt{7} + \sqrt{28}$



29. VOLUME What is the volume of the aquarium (in cubic feet)?

30. RATIO The ratio $3 : x$ is equivalent to the ratio $x : 5$. What are the possible values of x?



31. BILLBOARD The billboard has the shape of a rectangle.

a. What is the perimeter of the billboard?

b. What is the area of the billboard?

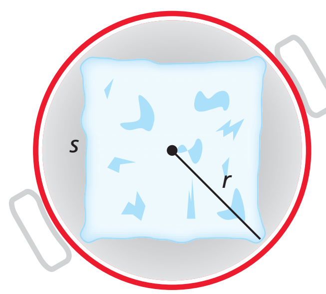
The height of Mt. Fuji is 3.8 kilometers.

32. MT. FUJI Mt. Fuji is in the shape of a cone with a volume of about 475π cubic kilometers. What is the radius of the base of Mt. Fuji?

33. Geometry A block of ice is in the shape of a square prism. You want to put the block of ice in a cylindrical cooler. The equation $s^2 = 2r^2$ represents the minimum radius r needed for the block of ice with side length s to fit in the cooler.

a. Solve the equation for r.

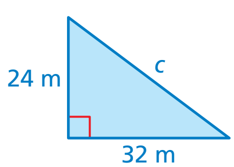
b. Use the equation in part (a) to find the minimum radius needed when the side length of the block of ice is $\sqrt{98}$ inches.



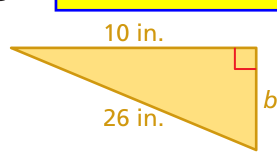
Fair Game Review What you learned in previous grades & lessons

Find the missing length of the triangle. SECTION 6.2

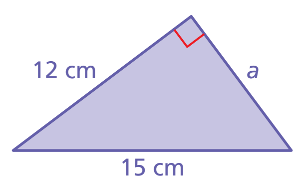
34.



35.



36.



37. MULTIPLE CHOICE Where is $-\sqrt{110}$ on a number line? SECTION 6.3

Ⓐ Between -9 and -10 Ⓑ Between 9 and 10

Ⓒ Between -10 and -11 Ⓓ Between 10 and 11