

## 9.2 Product of Powers Property

**Essential Question** How can you multiply two powers that have the same base?

### 1 ACTIVITY: Finding Products of Powers

Work with a partner.

- a. Copy and complete the table.

Product	Repeated Multiplication Form	Power
$2^2 \cdot 2^4$	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	$2^6$
$(-3)^2 \cdot (-3)^4$	$(-3) \cdot (-3) \cdot (-3) \cdot (-3) \cdot (-3) \cdot (-3)$	$(-3)^6$
$7^3 \cdot 7^2$		
$5.1^1 \cdot 5.1^6$		
$(-4)^2 \cdot (-4)^2$		
$10^3 \cdot 10^5$		
$\left(\frac{1}{2}\right)^5 \cdot \left(\frac{1}{2}\right)^5$		

- b. **INDUCTIVE REASONING** Describe the pattern in the table. Then write a rule for multiplying two powers that have the same base.

$$a^m \cdot a^n = a^{\quad}$$

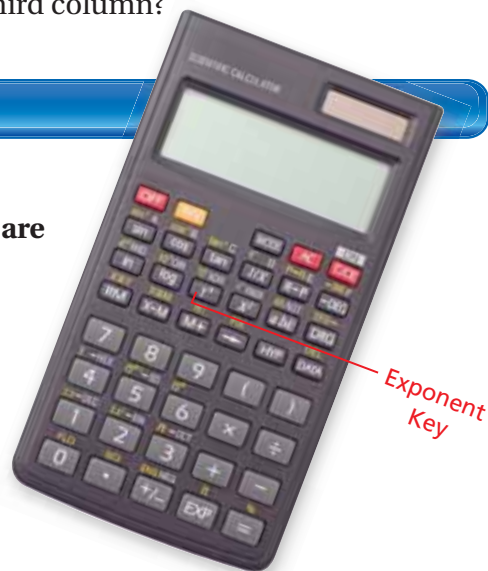
- c. Use your rule to simplify the products in the first column of the table above. Does your rule give the results in the third column?

### 2 ACTIVITY: Using a Calculator

Work with a partner.

Some calculators have *exponent keys* that are used to evaluate powers.

Use a calculator with an exponent key to evaluate the products in Activity 1.

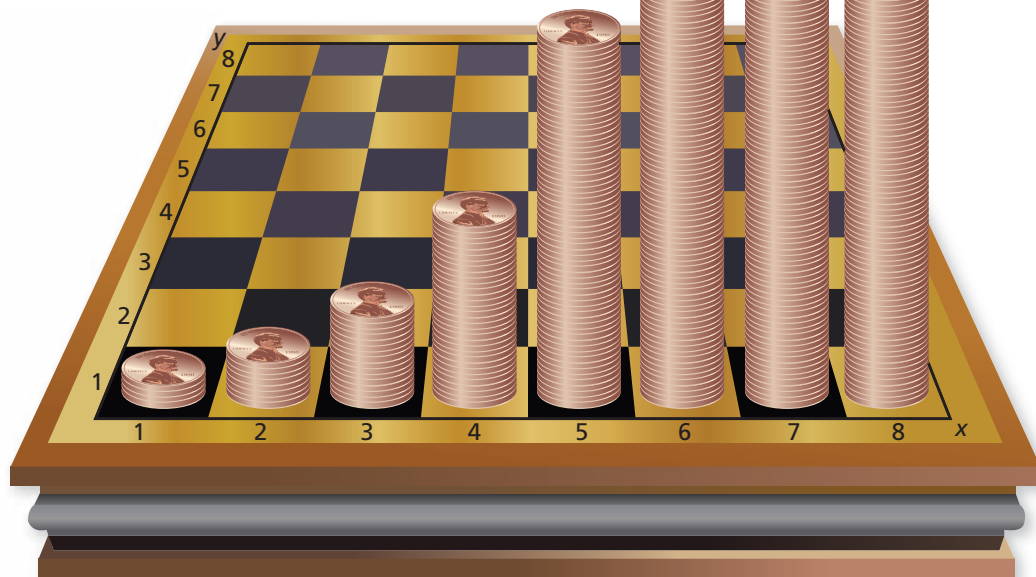


### 3 ACTIVITY: The Penny Puzzle

Work with a partner.

- The rows  $y$  and columns  $x$  of a chess board are numbered as shown.
- Each position on the chess board has a stack of pennies. (Only the first row is shown.)
- The number of pennies in each stack is  $2^x \cdot 2^y$ .

- a. How many pennies are in the stack in location (3, 5)?
- b. Which locations have 32 pennies in their stacks?
- c. How much money (in dollars) is in the location with the tallest stack?
- d. A penny is about 0.06 inch thick. About how tall (in inches) is the tallest stack?



## What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you multiply two powers that have the same base? Give two examples of your rule.

### Practice

Use what you learned about the Product of Powers Property to complete Exercises 3–5 on page 360.



## Key Idea

### Product of Powers Property

**Words** To multiply powers with the same base, add their exponents.

**Numbers**  $4^2 \cdot 4^3 = 4^{2+3} = 4^5$       **Algebra**  $a^m \cdot a^n = a^{m+n}$

## EXAMPLE 1 Multiplying Powers with the Same Base

a.  $2^4 \cdot 2^5 = 2^{4+5}$   
 $= 2^9$

The base is 2. Add the exponents.

Simplify.

b.  $-5 \cdot (-5)^6 = (-5)^1 \cdot (-5)^6$   
 $= (-5)^{1+6}$   
 $= (-5)^7$

Rewrite  $-5$  as  $(-5)^1$ .

The base is  $-5$ . Add the exponents.

Simplify.

c.  $x^3 \cdot x^7 = x^{3+7}$   
 $= x^{10}$

The base is  $x$ . Add the exponents.

Simplify.



### Study Tip

When a number is written without an exponent, its exponent is 1.

### On Your Own

Simplify the expression. Write your answer as a power.

1.  $6^2 \cdot 6^4$

2.  $\left(-\frac{1}{2}\right)^3 \cdot \left(-\frac{1}{2}\right)^6$

3.  $z \cdot z^{12}$

## EXAMPLE 2 Raising a Power to a Power

a.  $(3^4)^3 = 3^4 \cdot 3^4 \cdot 3^4$   
 $= 3^{4+4+4}$   
 $= 3^{12}$

Write as repeated multiplication.

The base is 3. Add the exponents.

Simplify.

b.  $(w^5)^4 = w^5 \cdot w^5 \cdot w^5 \cdot w^5$   
 $= w^{5+5+5+5}$   
 $= w^{20}$

Write as repeated multiplication.

The base is  $w$ . Add the exponents.

Simplify.

### On Your Own

Simplify the expression. Write your answer as a power.

4.  $(4^4)^3$

5.  $(y^2)^4$

6.  $(\pi^3)^3$

7.  $((-4)^3)^2$

### EXAMPLE 3 Raising a Product to a Power

- a.  $(2x)^3 = 2x \cdot 2x \cdot 2x$  Write as repeated multiplication.  
 $= (2 \cdot 2 \cdot 2) \cdot (x \cdot x \cdot x)$  Group like bases using properties of multiplication.  
 $= 2^{1+1+1} \cdot x^{1+1+1}$  The bases are 2 and  $x$ . Add the exponents.  
 $= 2^3 \cdot x^3 = 8x^3$  Simplify.
- b.  $(xy)^2 = xy \cdot xy$  Write as repeated multiplication.  
 $= (x \cdot x) \cdot (y \cdot y)$  Group like bases using properties of multiplication.  
 $= x^{1+1} \cdot y^{1+1}$  The bases are  $x$  and  $y$ . Add the exponents.  
 $= x^2y^2$  Simplify.

### On Your Own

Now You're Ready  
Exercises 17–22

Simplify the expression.

8.  $(5y)^4$       9.  $(0.5n)^2$       10.  $(ab)^5$

### EXAMPLE 4 Standardized Test Practice

Details
Local Disk (C:) Local Disk
Free Space: 16GB
Total Space: 64GB

A gigabyte (GB) of computer storage space is  $2^{30}$  bytes. The details of a computer are shown. How many bytes of total storage space does the computer have?

- (A)  $2^{34}$       (B)  $2^{36}$       (C)  $2^{180}$       (D)  $128^{30}$

The computer has 64 gigabytes of total storage space. Notice that 64 can be written as a power,  $2^6$ . Use a model to solve the problem.

$$\begin{aligned} \text{Total number of bytes} &= \text{Number of bytes in a gigabyte} \cdot \text{Number of gigabytes} \\ &= 2^{30} \cdot 2^6 && \text{Substitute.} \\ &= 2^{30+6} && \text{Add exponents.} \\ &= 2^{36} && \text{Simplify.} \end{aligned}$$

∴ The computer has  $2^{36}$  bytes of total storage space. The correct answer is (B).

### On Your Own

11. How many bytes of free storage space does the computer have?

## Vocabulary and Concept Check

- REASONING** When should you use the Product of Powers Property?
- CRITICAL THINKING** Can you use the Product of Powers Property to multiply powers with different bases? Explain.

## Practice and Problem Solving

Simplify the expression. Write your answer as a power.

- |   |   |  |                             |  |
|---|---|--|-----------------------------|--|
| 1 | 2 | 3. $3^2 \cdot 3^2$   | 4. $8^{10} \cdot 8^4$       | 5. $(-4)^5 \cdot (-4)^7$   |
|   |   | 6. $a^3 \cdot a^3$   | 7. $h^6 \cdot h$            | 8. $\left(\frac{2}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^6$ |
|   |   | 9. $\left(-\frac{5}{7}\right)^8 \cdot \left(-\frac{5}{7}\right)^9$ | 10. $(-2.9) \cdot (-2.9)^7$ | 11. $(5^4)^3$  |
|   |   | 12. $(b^{12})^3$   | 13. $(3.8^3)^4$             | 14. $\left(\left(-\frac{3}{4}\right)^5\right)^2$                 |

**ERROR ANALYSIS** Describe and correct the error in simplifying the expression.

15.  $5^2 \cdot 5^9 = (5 \cdot 5)^{2+9} = 25^{11}$

16.  $(r^6)^4 = r^{6+4} = r^{10}$

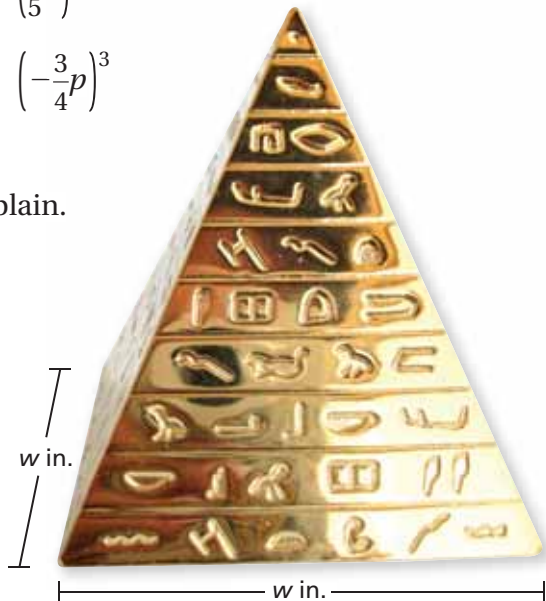
Simplify the expression.

- |   |                |                 |                                    |
|---|----------------|-----------------|------------------------------------|
| 3 | 17. $(6g)^3$   | 18. $(-3v)^5$   | 19. $\left(\frac{1}{5}k\right)^2$  |
|   | 20. $(1.2m)^4$ | 21. $(rt)^{12}$ | 22. $\left(-\frac{3}{4}p\right)^3$ |

23. **CRITICAL THINKING** Is  $3^2 + 3^3$  equal to  $3^5$ ? Explain.

24. **ARTIFACT** A display case for the artifact is in the shape of a cube. Each side of the display case is three times longer than the width of the artifact.

- Write an expression for the volume of the case. Write your answer as a power.
- Simplify the expression.



**Simplify the expression.**

25.  $2^4 \cdot 2^5 - (2^2)^2$

26.  $16\left(\frac{1}{2}x\right)^4$

27.  $5^2(5^3 \cdot 5^2)$

28. **CLOUDS** The lowest altitude of an altocumulus cloud is about  $3^8$  feet. The highest altitude of an altocumulus cloud is about 3 times the lowest altitude. What is the highest altitude of an altocumulus cloud? Write your answer as a power.

29. **PYTHON EGG** The volume  $V$  of a python egg is given by the formula  $V = \frac{4}{3}\pi abc$ . For the python egg shown,  $a = 2$  inches,  $b = 2$  inches, and  $c = 3$  inches.



- a. Find the volume of the python egg.
- b. Square the dimensions of the python egg. Then evaluate the formula. How does this volume compare to your answer in part (a)?



30. **PYRAMID** The volume of a square pyramid is  $V = \frac{1}{3}b^2h$ , where  $b$  is the length of one side of the base and  $h$  is the height of the pyramid. The length of each side of the base increases by 50%. Write a formula for the volume of the new pyramid.

31. **MAIL** The United States Postal Service delivers about  $2^6 \cdot 5^3$  pieces of mail each second. There are  $2^8 \cdot 3^4 \cdot 5^2$  seconds in 6 days. How many pieces of mail does the United States Postal Service deliver in 6 days? Write your answer as a power.

32. **Critical Thinking** Find the value of  $x$  in the equation without evaluating the power.

a.  $2^5 \cdot 2^x = 256$

b.  $\left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^x = \frac{1}{729}$



## Fair Game Review what you learned in previous grades & lessons

**Simplify.** (*Skills Review Handbook*)

33.  $\frac{4 \cdot 4}{4}$

34.  $\frac{5 \cdot 5 \cdot 5}{5}$

35.  $\frac{2 \cdot 3}{2}$

36.  $\frac{8 \cdot 6 \cdot 6}{6 \cdot 8}$

37. **MULTIPLE CHOICE** What is the measure of each angle of the regular polygon? (*Section 5.3*)

(A)  $45^\circ$

(B)  $135^\circ$

(C)  $1080^\circ$

(D)  $1440^\circ$

