10.4 Zero and Negative Exponents

Essential Question How can you evaluate a nonzero number with an exponent of zero? How can you evaluate a nonzero number with a negative integer exponent?

1 ACTIVITY: Using the Quotient of Powers Property

Work with a partner.

a. Copy and complete the table.

<table>
<thead>
<tr>
<th>Quotient</th>
<th>Quotient of Powers Property</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{5^1}{5^1})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\frac{6^2}{6^2})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\frac{(-3)^4}{(-3)^4})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\frac{(-4)^5}{(-4)^5})</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. REPEATED REASONING Evaluate each expression in the first column of the table. What do you notice?

c. How can you use these results to define \(a^0\) where \(a \neq 0\)?

2 ACTIVITY: Using the Product of Powers Property

Work with a partner.

a. Copy and complete the table.

<table>
<thead>
<tr>
<th>Product</th>
<th>Product of Powers Property</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3^0 \cdot 3^4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(8^2 \cdot 8^0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>((-2)^3 \cdot (-2)^0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\left(\frac{-1}{3}\right)^0 \cdot \left(\frac{-1}{3}\right)^3)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Do these results support your definition in Activity 1(c)?
Work with a partner.

a. Copy and complete the table.

<table>
<thead>
<tr>
<th>Product</th>
<th>Product of Powers Property</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5^{-3} \cdot 5^3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$6^2 \cdot 6^{-2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(−3)^4 \cdot (−3)^{-4}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(−4)^{-5} \cdot (−4)^5$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. According to your results from Activities 1 and 2, the products in the first column are equal to what value?

c. **REASONING** How does the Multiplicative Inverse Property help you rewrite the numbers with negative exponents?

d. **STRUCTURE** Use these results to define $a^{-n}$ where $a ≠ 0$ and $n$ is an integer.

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**Math Practice**

Use Operations

What operations are used when writing the expanded form?

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Work with a partner. Use the place value chart that shows the number 3452.867.

<table>
<thead>
<tr>
<th>Place Value Chart</th>
</tr>
</thead>
<tbody>
<tr>
<td>thousands</td>
</tr>
<tr>
<td>hundreds</td>
</tr>
<tr>
<td>tens</td>
</tr>
<tr>
<td>ones</td>
</tr>
<tr>
<td>and</td>
</tr>
<tr>
<td>tenths</td>
</tr>
<tr>
<td>hundredths</td>
</tr>
<tr>
<td>thousandths</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$10^3$</th>
<th>$10^2$</th>
<th>$10^1$</th>
<th>10</th>
<th>$10^{-1}$</th>
<th>$10^{-2}$</th>
<th>$10^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4</td>
<td>5</td>
<td>2</td>
<td>.</td>
<td>8</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>

a. **REPEATED REASONING** What pattern do you see in the exponents? Continue the pattern to find the other exponents.

b. **STRUCTURE** Show how to write the expanded form of 3452.867.

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**What Is Your Answer?**

5. **IN YOUR OWN WORDS** How can you evaluate a nonzero number with an exponent of zero? How can you evaluate a nonzero number with a negative integer exponent?

Use what you learned about zero and negative exponents to complete Exercises 5–8 on page 432.
**10.4 Lesson**

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**Key Ideas**

### Zero Exponents

**Words** For any nonzero number \( a \), \( a^0 = 1 \). The power \( 0^0 \) is undefined.

**Numbers** \( 4^0 = 1 \)

**Algebra** \( a^0 = 1 \), where \( a \neq 0 \)

### Negative Exponents

**Words** For any integer \( n \) and any nonzero number \( a \), \( a^{-n} \) is the reciprocal of \( a^n \).

**Numbers** \( 4^{-2} = \frac{1}{4^2} \)

**Algebra** \( a^{-n} = \frac{1}{a^n} \), where \( a \neq 0 \)

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**EXAMPLE 1** Evaluating Expressions

**a.** \( 3^{-4} = \frac{1}{3^4} \) Definition of negative exponent

\[ = \frac{1}{81} \] Evaluate power.

**b.** \( (-8.5)^{-4} \cdot (-8.5)^4 = (-8.5)^{-4+4} \) Product of Powers Property

\[ = (-8.5)^0 \] Simplify.

\[ = 1 \] Definition of zero exponent

**c.** \( \frac{2^6}{2^8} = 2^6 - 8 \) Quotient of Powers Property

\[ = 2^{-2} \] Simplify.

\[ = \frac{1}{2^2} \] Definition of negative exponent

\[ = \frac{1}{4} \] Evaluate power.

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**On Your Own**

Evaluate the expression.

1. \( 4^{-2} \)

2. \( (-2)^{-5} \)

3. \( 6^{-8} \cdot 6^8 \)

4. \( \frac{(-3)^5}{(-3)^6} \)

5. \( \frac{1}{5^7} \cdot \frac{1}{5^{-4}} \)

6. \( \frac{4^5 \cdot 4^{-3}}{4^2} \)
EXAMPLE 2  
Simplifying Expressions

a. $-5x^0 = -5(1)$  
Definition of zero exponent

$= -5$  
Multiply.

b. $\frac{9y^{-3}}{y^5} = 9y^{-3 - 5}$  
Quotient of Powers Property

$= 9y^{-8}$  
Simplify.

$= \frac{9}{y^8}$  
Definition of negative exponent

On Your Own

Simplify. Write the expression using only positive exponents.

7. $8x^{-2}$  
8. $b^0 \cdot b^{-10}$  
9. $\frac{z^6}{15z^9}$

EXAMPLE 3  
Real-Life Application

A drop of water leaks from a faucet every second. How many liters of water leak from the faucet in 1 hour?

Convert 1 hour to seconds.

$1 \text{ h} \times \frac{60 \text{ min}}{1 \text{ h}} \times \frac{60 \text{ sec}}{1 \text{ min}} = 3600 \text{ sec}$

Water leaks from the faucet at a rate of $50^{-2}$ liter per second. Multiply the time by the rate.

$3600 \text{ sec} \cdot 50^{-2} \frac{1}{\text{sec}} = 3600 \cdot \frac{1}{50^2}$  
Definition of negative exponent

$= 3600 \cdot \frac{1}{2500}$  
Evaluate power.

$= \frac{3600}{2500}$  
Multiply.

$= \frac{1.11}{25} = 1.44 \text{ L}$  
Simplify.

So, 1.44 liters of water leak from the faucet in 1 hour.

On Your Own

10. WHAT IF? The faucet leaks water at a rate of $5^{-5}$ liter per second. How many liters of water leak from the faucet in 1 hour?
10.4 Exercises

Vocabulary and Concept Check:
1. VOCABULARY If \( a \) is a nonzero number, does the value of \( a^0 \) depend on the value of \( a \)? Explain.

2. WRITING Explain how to evaluate \( 10^{-3} \).

3. NUMBER SENSE Without evaluating, order \( 5^0, 5^4, \) and \( 5^{-5} \) from least to greatest.

4. DIFFERENT WORDS, SAME QUESTION Which is different? Find “both” answers.

- Rewrite \( \frac{1}{3 \cdot 3 \cdot 3} \) using a negative exponent.
- Write 3 to the negative third.
- Write \( \frac{1}{3} \) cubed as a power.
- Write \( (-3) \cdot (-3) \cdot (-3) \) as a power.

Practice and Problem Solving
Evaluate the expression.

5. \( \frac{8^7}{8^7} \)  
6. \( 5^0 \cdot 5^3 \)  
7. \( (-2)^{-8} \cdot (-2)^8 \)  
8. \( 9^4 \cdot 9^{-4} \)  
9. \( 6^{-2} \)  
10. \( 158^0 \)  
11. \( \frac{4^3}{4^5} \)  
12. \( \frac{-3}{(-3)^2} \)  
13. \( 4 \cdot 2^{-4} + 5 \)  
14. \( 3^{-3} \cdot 3^{-2} \)  
15. \( \frac{1}{5^{-3}} \cdot \frac{1}{5^6} \)  
16. \( \frac{(1.5)^2}{(1.5)^{-2} \cdot (1.5)^4} \)

17. ERROR ANALYSIS Describe and correct the error in evaluating the expression.

18. SAND The mass of a grain of sand is about \( 10^{-3} \) gram. About how many grains of sand are in the bag of sand?

19. CRITICAL THINKING How can you write the number 1 as 2 to a power? 10 to a power?

Simplify. Write the expression using only positive exponents.

20. \( 6y^{-4} \)  
21. \( 8^{-2} \cdot a^7 \)  
22. \( \frac{9c^3}{c^{-2}} \)  
23. \( \frac{5b^{-2}}{b^{-3}} \)  
24. \( \frac{8x^3}{2x^9} \)  
25. \( 3d^{-4} \cdot 4d^4 \)  
26. \( m^{-2} \cdot n^3 \)  
27. \( \frac{3^{-2} \cdot k^0 \cdot w^0}{w^{-6}} \)
28. **OPEN-ENDED** Write two different powers with negative exponents that have the same value.

**METRIC UNITS** In Exercises 29–32, use the table.

<table>
<thead>
<tr>
<th>Unit of Length</th>
<th>Length (meter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decimeter</td>
<td>$10^{-1}$</td>
</tr>
<tr>
<td>Centimeter</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>Millimeter</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>Micrometer</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>Nanometer</td>
<td>$10^{-9}$</td>
</tr>
</tbody>
</table>

29. How many millimeters are in a decimeter?
30. How many micrometers are in a centimeter?
31. How many nanometers are in a millimeter?
32. How many micrometers are in a meter?

33. **BACTERIA** A species of bacteria is 10 micrometers long. A virus is 10,000 times smaller than the bacteria.
   a. Using the table above, find the length of the virus in meters.
   b. Is the answer to part (a) less than, greater than, or equal to one nanometer?

34. **BLOOD DONATION** Every 2 seconds, someone in the United States needs blood. A sample blood donation is shown. (1 mm$^3 = 10^{-3}$ mL)
   a. One cubic millimeter of blood contains about $10^4$ white blood cells. How many white blood cells are in the donation? Write your answer in words.
   b. One cubic millimeter of blood contains about $5 \times 10^6$ red blood cells. How many red blood cells are in the donation? Write your answer in words.
   c. Compare your answers for parts (a) and (b).

35. **PRECISION** Describe how to rewrite a power with a positive exponent so that the exponent is in the denominator. Use the definition of negative exponents to justify your reasoning.

36. **Reasoning** The rule for negative exponents states that $a^{-n} = \frac{1}{a^n}$. Explain why this rule does not apply when $a = 0$.

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**Fair Game Review** What you learned in previous grades & lessons

Simplify the expression. Write your answer as a power.  **(Section 10.2 and Section 10.3)**

37. $10^3 \cdot 10^6$
38. $10^2 \cdot 10$
39. $\frac{10^8}{10^4}$

40. **MULTIPLE CHOICE** Which data display best orders numerical data and shows how they are distributed?  **(Section 9.4)**
   - [A] bar graph
   - [B] line graph
   - [C] scatter plot
   - [D] stem-and-leaf plot