

8.4 Surface Areas and Volumes of Similar Solids

Essential Question: When the dimensions of a solid increase by a factor of $k$, how does the surface area change? How does the volume change?

ACTIVITY: Comparing Surface Areas and Volumes

Work with a partner. Copy and complete the table. Describe the pattern. Are the dimensions proportional? Explain your reasoning.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Radius</th>
<th>Height</th>
<th>Surface Area</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Geometry
In this lesson, you will
- identify similar solids.
- use properties of similar solids to find missing measures.
- understand the relationship between surface areas of similar solids.
- understand the relationship between volumes of similar solids.
- solve real-life problems.

354 Chapter 8 Volume and Similar Solids
2 ACTIVITY: Comparing Surface Areas and Volumes

Work with a partner. Copy and complete the table. Describe the pattern. Are the dimensions proportional? Explain.

<table>
<thead>
<tr>
<th>Base Side</th>
<th>6</th>
<th>12</th>
<th>18</th>
<th>24</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>20</td>
</tr>
<tr>
<td>Slant Height</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>Surface Area</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volume</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What Is Your Answer?

3. **IN YOUR OWN WORDS** When the dimensions of a solid increase by a factor of \( k \), how does the surface area change?

4. **IN YOUR OWN WORDS** When the dimensions of a solid increase by a factor of \( k \), how does the volume change?

5. **REPEATED REASONING** All the dimensions of a prism increase by a factor of 5.
   a. How many times greater is the surface area? Explain.

   - 5
   - 10
   - 25
   - 125

   b. How many times greater is the volume? Explain.

   - 5
   - 10
   - 25
   - 125

Practice

Use what you learned about surface areas and volumes of similar solids to complete Exercise 3 on page 359.

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**Key Vocabulary**

**similar solids**, p. 356

**Similar solids** are solids that have the same shape and proportional corresponding dimensions.

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**EXAMPLE 1**

**Identifying Similar Solids**

Which cylinder is similar to Cylinder A?

Check to see if corresponding dimensions are proportional.

**Cylinder A and Cylinder B**

- Height of A = 4
- Radius of A = 6
- Height of B = 3
- Radius of B = 5

Not proportional

**Cylinder A and Cylinder C**

- Height of A = 4
- Radius of A = 6
- Height of C = 5
- Radius of C = 7.5

Proportional

So, Cylinder C is similar to Cylinder A.

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**EXAMPLE 2**

**Finding Missing Measures in Similar Solids**

The cones are similar. Find the missing slant height $\ell$.

Radius of X = Slant height of X

Radius of Y = Slant height of Y

\[
\frac{5}{7} = \frac{13}{\ell} \quad \text{Substitute.}
\]

\[
5\ell = 91 \quad \text{Cross Products Property}
\]

\[
\ell = 18.2 \quad \text{Divide each side by 5.}
\]

The slant height is 18.2 yards.

---

**On Your Own**

1. Cylinder D has a radius of 7.5 meters and a height of 4.5 meters. Which cylinder in Example 1 is similar to Cylinder D?

2. The prisms at the right are similar. Find the missing width and length.
**Key Ideas**

### Linear Measures

![Images of various 3D shapes including cubes, pyramids, cylinders, and spheres]

**Surface Areas of Similar Solids**

When two solids are similar, the ratio of their surface areas is equal to the square of the ratio of their corresponding linear measures.

\[
\frac{\text{Surface Area of } A}{\text{Surface Area of } B} = \left(\frac{a}{b}\right)^2
\]

### EXAMPLE 3 Finding Surface Area

The pyramids are similar. What is the surface area of Pyramid A?

![Pyramid A and Pyramid B with dimensions]

\[
\frac{S}{600} = \left(\frac{6}{10}\right)^2
\]

Substitute.

\[
\frac{S}{600} = \frac{36}{100}
\]

Evaluate.

\[
\frac{S}{600} \cdot 600 = \frac{36}{100} \cdot 600
\]

Multiplication Property of Equality

\[
S = 216
\]

Simplify.

The surface area of Pyramid A is 216 square feet.

### On Your Own

The solids are similar. Find the surface area of the red solid. Round your answer to the nearest tenth.

3. ![Solid with dimensions]

   Surface Area = 608 m²

4. ![Solid with dimensions]

   Surface Area = 110 cm²

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**Section 8.4 Surface Areas and Volumes of Similar Solids** 357
**Key Idea**

**Volumes of Similar Solids**

When two solids are similar, the ratio of their volumes is equal to the cube of the ratio of their corresponding linear measures.

\[
\frac{\text{Volume of } A}{\text{Volume of } B} = \left(\frac{a}{b}\right)^3
\]

---

**EXAMPLE 4 Finding Volume**

The dimensions of the touch tank at an aquarium are doubled. What is the volume of the new touch tank?

- A) 150 ft³
- B) 4000 ft³
- C) 8000 ft³
- D) 16,000 ft³

The dimensions are doubled, so the ratio of the dimensions of the original tank to the dimensions of the new tank is 1:2.

\[
\frac{\text{Original volume}}{\text{New volume}} = \left(\frac{\text{Original dimension}}{\text{New dimension}}\right)^3
\]

\[
\frac{2000}{V} = \left(\frac{1}{2}\right)^3
\]

Substitute.

\[
\frac{2000}{V} = \frac{1}{8}
\]

Evaluate.

\[
16,000 = V
\]

Cross Products Property

The volume of the new tank is 16,000 cubic feet. So, the correct answer is D.

---

**On Your Own**

The solids are similar. Find the volume of the red solid. Round your answer to the nearest tenth.

5.  

6.
1. **VOCABULARY** What are similar solids?

2. **OPEN-ENDED** Draw two similar solids and label their corresponding linear measures.

3. **NUMBER SENSE** All the dimensions of a cube increase by a factor of $\frac{3}{2}$.
   a. How many times greater is the surface area? Explain.
   b. How many times greater is the volume? Explain.

Determine whether the solids are similar.

4.

5.

6.

7.

The solids are similar. Find the missing dimension(s).

8.

9.
The solids are similar. Find the surface area \( S \) or volume \( V \) of the red solid. Round your answer to the nearest tenth.

10. 
\[
\begin{array}{c}
\text{Surface Area} = 336 \, \text{m}^2 \\
4 \, \text{m} \\
6 \, \text{m}
\end{array}
\]

11. 
\[
\begin{array}{c}
\text{Surface Area} = 1800 \, \text{in}^2 \\
20 \, \text{in.} \\
15 \, \text{in.}
\end{array}
\]

12. 
\[
\begin{array}{c}
\text{Volume} = 5292 \, \text{mm}^3 \\
21 \, \text{mm} \\
21 \, \text{mm} \\
7 \, \text{mm} \\
7 \, \text{mm}
\end{array}
\]

13. 
\[
\begin{array}{c}
\text{Volume} = 7850 \, \text{ft}^3 \\
10 \, \text{ft} \\
12 \, \text{ft}
\end{array}
\]

14. **ERROR ANALYSIS** The ratio of the corresponding linear measures of two similar solids is 3 : 5. The volume of the smaller solid is 108 cubic inches. Describe and correct the error in finding the volume of the larger solid.

\[
\frac{108}{V} = \left( \frac{3}{5} \right)^2
\]

\[
\frac{108}{V} = \frac{9}{25}
\]

\[
300 = V
\]

The volume of the larger solid is 300 cubic inches.

15. **MIXED FRUIT** The ratio of the corresponding linear measures of two similar cans of fruit is 4 to 7. The smaller can has a surface area of 220 square centimeters. Find the surface area of the larger can.

16. **CLASSIC MUSTANG** The volume of a 1968 Ford Mustang GT engine is 390 cubic inches. Which scale model of the Mustang has the greater engine volume, a 1 : 18 scale model or a 1 : 24 scale model? How much greater is it?
17. **MARBLE STATUE** You have a small marble statue of Wolfgang Mozart. It is 10 inches tall and weighs 16 pounds. The original statue is 7 feet tall.
   
   a. Estimate the weight of the original statue. Explain your reasoning.
   
   b. If the original statue were 20 feet tall, how much would it weigh?

18. **REPEATED REASONING** The largest doll is 7 inches tall. Each of the other dolls is 1 inch shorter than the next larger doll. Make a table that compares the surface areas and the volumes of the seven dolls.

19. **Precision** You and a friend make paper cones to collect beach glass. You cut out the largest possible three-fourths circle from each piece of paper.
   
   a. Are the cones similar? Explain your reasoning.
   
   b. Your friend says that because your sheet of paper is twice as large, your cone will hold exactly twice the volume of beach glass. Is this true? Explain your reasoning.

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**Fair Game Review**

What you learned in previous grades & lessons

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**Draw the figure and its reflection in the x-axis. Identify the coordinates of the image.** (Section 2.3)

20. A(1, 1), B(3, 4), C(4, 2)  

21. J(−3, 0), K(−4, 3), L(−1, 4)

22. **MULTIPLE CHOICE** Which system of linear equations has no solution? (Section 5.4)

   A. \[ y = 4x + 1 \]  \[ y = -4x + 1 \]
   
   B. \[ y = 2x - 7 \]  \[ y = 2x + 7 \]
   
   C. \[ 3x + y = 1 \]  \[ 6x + 2y = 2 \]
   
   D. \[ 5x + y = 3 \]  \[ x + 5y = 15 \]